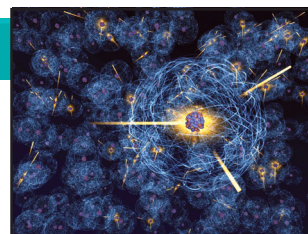


4

Nuclear Chemistry

CHAPTER

KEY CONCEPTS AND EQUATIONS



RADIOACTIVITY AND TYPES OF RADIATIONS

The disintegration or decay of unstable atoms accompanied by emission of radiation is called **Radioactivity**. The substances having this property are called **Radioactive substances**. The radioactive emissions are of three types - α , β , or γ radiations as shown in Fig. 4.1.

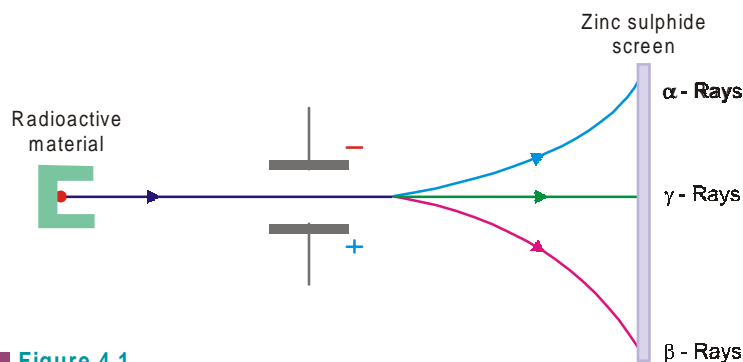


Figure 4.1
Detection of α , β and γ -Rays.

The charge and mass of α , β and γ - particles are given in Table 4.1

| TABLE 4.1 CHARGE AND MASS OF α , β AND γ -EMISSIONS | | | | |
|---|--------|------|---------------------------------------|--|
| Particle | Charge | Mass | Representation | |
| α | 2 | 4 | ${}^2_2\text{He}^4$ or ${}_2\alpha^4$ | |
| β | -1 | 0 | ${}_{-1}e^0$ or ${}_{-1}\beta^0$ | |
| γ | 0 | 0 | ${}_0\gamma^0$ | |

RATE OF RADIOACTIVE DECAY (No. of disintegrations per unit time)

The rate of decay is characteristic of an isotope and depends only on the number of atoms present. It is given by

$$-\frac{dN}{dt} = \lambda N$$

where λ is disintegration constant and N is number of atoms present.
On integration we have

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N} \quad \dots(i)$$

where N^0 is the number of atoms at $t = 0$

Radioactive disintegration follows the first order kinetics and the equation

$$\lambda = \frac{2.303}{t} \log \frac{a}{a-x} \quad \dots(ii)$$

is also applicable.

HALF LIFE PERIOD

The half life period of a radioactive isotope is the time required for one half of the isotope to decay. It is represented by $t_{1/2}$ or $t_{0.5}$

$$\text{When } t = t_{1/2} \quad N = \frac{N^0}{2}$$

Putting in equation (i) we have

$$t_{1/2} = \frac{2.303}{\lambda} \log 2 = \frac{0.693}{\lambda}$$

It is independent of initial concentration of radioactive substance.

AVERAGE LIFE

The average life of a radioactive substance is the reciprocal of disintegration constant *i.e.*,

$$\lambda' = \frac{1}{\lambda} = \frac{t_{1/2}}{0.693} = 1.44 \times t_{1/2}$$

UNITS OF RADIOACTIVITY

The unit of radioactivity is disintegration per second (dps). It is also expressed in curie or rutherford or becquerel. These are related to dps as follows :

$$\begin{aligned} 1 \text{ curie} &= 3.7 \times 10^{10} \text{ dps} \\ 1 \text{ rutherford} &= 1 \times 10^6 \text{ dps} \\ 1 \text{ becquerel} &= 1 \text{ dps} \end{aligned}$$

RADIOACTIVE EQUILIBRIUM

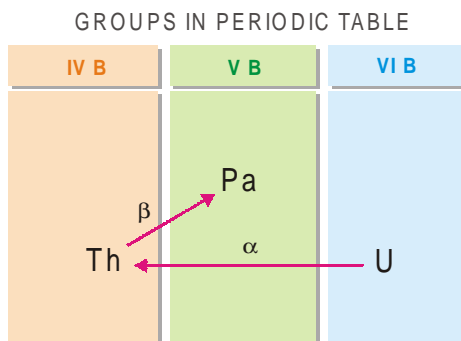
Let a radioactive substance A decay to give another radioactive substance B which decays to form the substance C. If λ_A and λ_B are their decay constants then we have



$$\frac{N_A}{N_B} = \frac{\lambda_B}{\lambda_A} = \frac{(t_{1/2})_B}{(t_{1/2})_A}$$

GROUP DISPLACEMENT LAW

In an α emission the parent element is displaced to a group **two** places to left and in β emission it will be displaced to a group **one** place to the right in the periodic table as illustrated in Fig.4.2. This is called Group Displacement Law. It was first stated by Fajans and Soddy (1913) and is often named as Fajans-Soddy Group Displacement law.



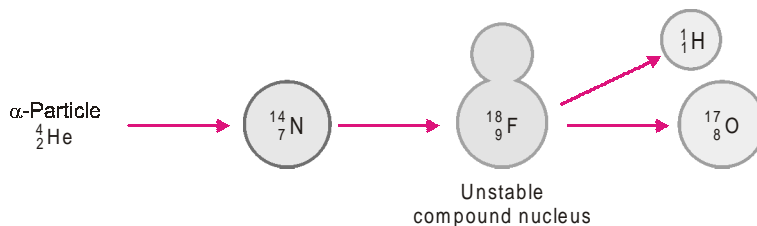
■ **Figure 4.2**
Illustration of Group Displacement Law.

NUCLEAR REACTIONS

A nuclear reaction is one which proceeds with a change in the composition of the nucleus so as to produce an atom of a new element. The conversion of one element to another by a nuclear change is called transmutation.

NUCLEAR FISSION REACTIONS

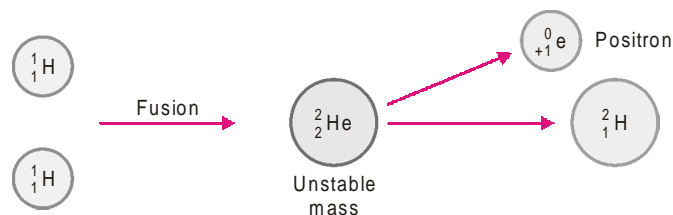
In these reactions an atomic nucleus is broken or fissioned into two or more fragments. For example, ${}^7_7\text{N}^{14}$ when struck by an α - particle first forms an intermediate unstable compound nucleus, ${}^9_9\text{F}^{18}$, which at once cleaves to form stable ${}^8_8\text{O}^{17}$ and ${}^1_1\text{H}^1$.



■ **Figure 4.3**
Mechanism of a Nuclear fission reaction.

NUCLEAR FUSION REACTIONS

These reactions take place by combination or fusion of two small nuclei into a larger nucleus. For example two hydrogen nuclei, ${}^1_1\text{H}^1$, fuse to produce a deuterium nucleus, H^2 as shown in Fig. 4.4.



■ **Figure 4.4**
Mechanism of a Nuclear fission reaction.

MASS-ENERGY EQUIVALENCE

Mass and energy are equivalent and are related to each other by the equation.

$$E = mc^2$$

where c is the speed of light, m sec^{-1} .

This equation is known as Einstein equation.

MASS DEFECT AND BINDING ENERGY

The difference between the experimental and calculated mass of the nucleus is called the mass defect.

Mass defect = Experimental mass of the nucleus – (mass of protons + electrons + neutrons).

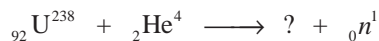
It is equal to the total nucleon mass minus the nuclear mass. This loss of mass is converted into energy which stabilizes the nucleus. This energy is known as binding energy. It is also equal to the energy needed to break a nucleus into its individual protons and neutrons. Einstein equation is used to calculate the binding energy. With mass defect of 1 amu 931.5 MeV of energy is produced *i.e.*

$$1 \text{ amu} = 931.5 \text{ MeV}$$

To compare the stabilities of various nuclei, the binding energy per nucleon is compared.

BALANCING OF NUCLEAR EQUATIONS

The equations involving the nuclei of the reactants and products are called nuclear equations. The nuclear reactions occur by redistribution of protons and neutrons present in the reactants so as to form the products. Thus the total number of protons and neutrons in the reactants and products is the same. For example in the nuclear equation



the atomic number of missing atom is 94 and its mass no. is $238 + 4 - 1 = 241$. Hence the atom is ${}_{94}\text{Pu}^{241}$. The balanced equation becomes.



ADDITIONAL SOLVED PROBLEMS

SOLVED PROBLEM 1. It is found that a sample of radioactive matter is half integrated in 18 hours. How much of it will remain after 42.5 hours ?

SOLUTION :

(i) To calculate disintegration constant

Formula used $\lambda = \frac{0.693}{t_{1/2}}$

Quantities given $t_{1/2} = 18 \text{ hr}$

Substitution of values $\lambda = \frac{0.693}{18 \text{ hr}} = 0.0385 \text{ hr}^{-1}$

(ii) To calculate the amount of sample left after 42.5 hours

Formula used

$$\lambda = \frac{1}{t} \log \frac{a}{a-x}$$

Quantities given

$$\lambda = 0.0385 \text{ hr}^{-1} \quad t = 42.5 \text{ hours} \quad a = 100 \quad a-x = 100-x$$

Substitution of values

$$0.0385 \text{ hr}^{-1} = \frac{1}{42.5 \text{ hr}} \log \frac{100}{100-x}$$

$$\text{or} \quad 1.63625 = \log \frac{100}{100-x}$$

$$\text{or} \quad \frac{100}{100-x} = \text{Antilog } 1.63625$$

$$= 5.13$$

$$\text{or} \quad 100 = 5.13 \times (100-x)$$

$$100 = 513 - 5.13x$$

$$\text{or} \quad x = \frac{413}{5.13}$$

$$= 80.51$$

$$\therefore \text{Amount left} = 100 - 80.51$$

$$= \mathbf{19.49\%}$$

SOLVED PROBLEM 2. Radium has atomic mass 226 and a half life of 1600 years. Calculate the number of disintegration produced per second from one gram of radium.

SOLUTION :

(i) To calculate the disintegration constant

Formula used $\lambda = \frac{0.693}{t_{1/2}}$

Quantities given

$$t_{1/2} = 1600 \text{ years} = 1600 \times 365 \times 24 \times 60 \times 60 \text{ sec}$$

$$= 5.04576 \times 10^{10} \text{ sec}$$

Substitution of values

$$\therefore \lambda = \frac{0.693}{5.04576 \times 10^{10} \text{ sec}}$$

$$= 0.1373 \times 10^{-10} \text{ sec}^{-1}$$

(ii) To calculate the rate of disintegration

Formula used $-\frac{dN}{dt} = \lambda \times N$

Quantities given $\lambda = 0.1373 \times 10^{-10} \text{ sec}^{-1}$

No. of nuclei of Ra^{226} in 1g, $N = \frac{\text{Avogadro's No.}}{226}$

$$= \frac{6.023 \times 10^{23}}{226}$$

$$= 0.0266 \times 10^{-23} \text{ atoms}$$

Substitution of values

Rate of disintegration, $-\frac{dN}{dt} = 0.1373 \times 10^{-10} \text{ sec}^{-1} \times 0.0266 \times 10^{-23} \text{ atoms}$

$$= 0.003652 \times 10^{-13} \text{ atom sec}^{-1}$$

$$= \mathbf{3.652 \times 10^{16} \text{ atom sec}^{-1}}$$

SOLVED PROBLEM 3. The activity of a radioactive isotope reduces by 25% after 100 minutes. Calculate the decay constant and half life period.

SOLUTION :

(i) To calculate decay constant

Formula used $\lambda = \frac{2.303}{t} \log \frac{a}{a-x}$

Quantities given

$$t = 100 \text{ min} \qquad a = 100 \qquad a - x = 25$$

Substitution of values

$$\lambda = \frac{2.303}{100 \text{ min}} \log \frac{100}{25}$$

$$= 0.02303 \text{ min}^{-1} \times \log 4$$

$$= 0.02303 \text{ min}^{-1} \times 0.6021$$

$$= \mathbf{0.01387 \text{ min}^{-1}}$$

(ii) To calculate Half Life period

$$\begin{aligned} \text{Formula used} \quad t_{1/2} &= \frac{0.693}{\lambda} \\ \therefore t_{1/2} &= \frac{0.693}{0.01386 \text{ min}^{-1}} \\ &= \mathbf{49.96 \text{ min}} \end{aligned}$$

SOLVED PROBLEM 4. Calculate the number of α and β particles emitted in the conversion of thorium ${}_{90}\text{Th}^{232}$ to ${}_{82}\text{Pb}^{208}$.

SOLUTION :

Let x and y be the number of α and β particles emitted during the change



Comparing the mass numbers, we have

$$232 = 208 + 4x + y \times 0$$

or $4x = 24$

or $x = \mathbf{6}$

Now comparing the atomic numbers, we have

$$90 = 82 + 2x - 1y$$

or $90 = 82 + 2 \times 6 - y$ ($\because x=6$)

or $y = 94 - 90$

$= \mathbf{4}$

SOLVED PROBLEM 5. Half life period of thorium is 24.5 minutes. How much thorium would be left after 30 minutes if the initial amount of thorium is one gram ?

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantities given

$$t_{1/2} = 24.5 \text{ min}$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{0.693}{24.5 \text{ min}} \\ &= \mathbf{0.02828 \text{ min}^{-1}} \end{aligned}$$

(ii) To calculate the amount of Thorium left

Formula used

$$\lambda = \frac{1}{t} \log \frac{a}{a-x}$$

Quantities given

$$\lambda = 0.02828 \text{ min}^{-1}$$

$$t = 30 \text{ minutes}$$

$$a = 1 \text{ g}$$

$$a-x = 1-x \text{ g}$$

Substitution of values

$$0.02828 \text{ min}^{-1} = \frac{1}{30 \text{ min}} \log \frac{1}{1-x}$$

$$\begin{aligned} \text{or} \quad \log \frac{1}{1-x} &= 30 \times 0.02828 \\ &= 0.8484 \end{aligned}$$

$$\begin{aligned} \text{or} \quad \frac{1}{1-x} &= \text{Antilog } 0.8484 \\ &= 2.3359 \end{aligned}$$

$$\begin{aligned} \text{or} \quad 1 &= (1-x) 2.3359 \\ 1 &= 2.3359 - 2.3359x \end{aligned}$$

$$\begin{aligned} \text{or} \quad x &= \frac{2.3359 - 1}{2.3359} \\ &= 0.572 \text{ g} \end{aligned}$$

$$\begin{aligned} \therefore \text{Amount left} = 1 - x &= 1 - 0.572 \text{ g} \\ &= \mathbf{0.428 \text{ g}} \end{aligned}$$

SOLVED PROBLEM 6. How many α and β particles will be emitted by an element ${}_{84}\text{A}^{218}$ is changing to a stable isotope of ${}_{82}\text{B}^{206}$?

SOLUTION :

Let x and y be the number of α and β particles respectively emitted during the change



Comparing the mass number

$$218 = 206 + 4x + 0y$$

$$\text{or} \quad 4x = 12$$

$$\text{or} \quad x = 3$$

Now comparing the atomic numbers

$$84 = 82 + 2x - y$$

$$84 = 82 + 2 \times 3 - y$$

$$[\because x = 3]$$

$$\text{or} \quad y = 4$$

SOLVED PROBLEM 7. Calculate the decay constant for Ag^{108} , if its half life is 2.31 minutes.

SOLUTION :**Formula used**

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantities given

$$t_{1/2} = 2.31 \text{ minutes}$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{0.693}{2.31 \text{ min}} \\ &= \mathbf{0.3 \text{ min}^{-1}} \end{aligned}$$

SOLVED PROBLEM 8. A radioactive isotope has half life period of 20 days. What is the amount of the isotope left over after 40 days if the initial concentration is 5 g ?

SOLUTION :

(i) To calculate the decay constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantities given

$$t_{1/2} = 20 \text{ days}$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{0.693}{20 \text{ days}} \\ &= \mathbf{0.03465 \text{ day}^{-1}} \end{aligned}$$

(ii) To calculate amount of radioactive isotope

Formula used

$$\lambda = \frac{1}{t} \log \frac{a}{a-x}$$

Quantities given

$$t = 40 \text{ days}$$

$$a = 5 \text{ g}$$

$$a-x = 5-x \text{ g}$$

Substitution of values

$$0.03465 \text{ day}^{-1} = \frac{1}{40 \text{ days}} \log \frac{5}{5-x}$$

$$\text{or} \quad \log \frac{5}{5-x} = 1.386$$

$$\text{or} \quad \frac{5}{5-x} = \text{Antilog } 1.386$$

$$= 3.9988$$

$$5 = (5-x) \times 3.9988$$

$$\text{or} \quad x = 3.75 \text{ g}$$

$$\therefore \text{Amount left} = \text{Initial concentration} - \text{Amount used}$$

$$= 5 - 3.75 \text{ g}$$

$$= \mathbf{1.25 \text{ g}}$$

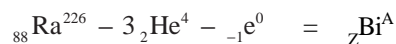
SOLVED PROBLEM 9. Calculate the mass number, atomic number and group in the periodic table for Bi in the following disintegration series.



(Ra : Mass No. 226, at. no. 88, group no. II A)

SOLUTION :

We can write



Comparing the mass numbers

$$226 - 3 \times 4 - 0 = A$$

$$\text{or} \quad A = \mathbf{214}$$

Comparing the atomic numbers

$$88 - 3 \times 2 - 1 \times (-1) = Z$$

$$\text{or} \quad Z = \mathbf{83}$$

$$\text{Group} = \mathbf{VA}$$

SOLVED PROBLEM 10. The half life of Th^{233} is 1.4×10^{10} years. Calculate the disintegration constant.

SOLUTION :

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantities given

$$t_{1/2} = 1.4 \times 10^{10} \text{ years}$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{0.693}{1.4 \times 10^{10} \text{ years}} \\ &= 0.495 \times 10^{10} \text{ year}^{-1} \\ &= \mathbf{4.95 \times 10^{-11} \text{ year}^{-1}} \end{aligned}$$

SOLVED PROBLEM 11. A radioactive substance having half life of 3.8 days, emitted initially alpha particles per second. In what time will its rate of emission reduces to alpha particles per second ?

SOLUTION :

(i) To calculate disintegration constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantities given

$$t_{1/2} = 3.8 \text{ days}$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{0.693}{3.8 \text{ days}} \\ &= \mathbf{0.1824 \text{ day}^{-1}} \end{aligned}$$

(ii) To calculate the time t

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N}$$

or

$$t = \frac{2.303}{\lambda} \log \frac{N^0}{N}$$

Quantities given

$$\lambda = 0.1824 \text{ days}^{-1}$$

$$N^0 = 7 \times 10^4 \text{ dps}$$

$$N = 2 \times 10^4 \text{ dps}$$

Substitution of values

$$\begin{aligned} t &= \frac{2.303}{0.1824 \text{ day}^{-1}} \log \frac{7 \times 10^4 \text{ dps}}{2 \times 10^4 \text{ dps}} \\ &= \frac{2.303}{0.1824 \text{ day}^{-1}} \log 3.5 \\ &= \frac{2.303}{0.1824 \text{ day}^{-1}} \times 0.5440 \\ &= \mathbf{6.872 \text{ days}} \end{aligned}$$

SOLVED PROBLEM 12. A freshly cut piece of plant gives 20.4 counts per minute per gram. A piece of wood antique gives 12.18 counts per minute per gram. What is the age in years of antique? It is assumed that the radioactivity is entirely due to C^{14} . The half life period of C^{14} is 5760 years.

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantities given

$$t_{1/2} = 5760 \text{ years}$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{0.693}{5790 \text{ years}} \\ &= 1.203 \times 10^{-4} \text{ year}^{-1} \end{aligned}$$

(ii) To calculate the age of antique

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N}$$

or

$$t = \frac{2.303}{\lambda} \log \frac{N^0}{N}$$

Quantities given

$$\lambda = 2.303 \times 10^{-4} \text{ year}^{-1} N^0 = 20.4 \text{ counts per minutes per g}$$

$$N = 12.18 \text{ counts per minutes per g}$$

Substitution of values

$$\begin{aligned} t &= \frac{2.303}{2.303 \times 10^{-4} \text{ year}^{-1}} \log \frac{20.4}{12.18} \\ &= \frac{2.303}{2.303 \times 10^{-4} \text{ year}^{-1}} \log 1.6749 \\ &= \frac{2.303}{2.303 \times 10^{-4} \text{ year}^{-1}} \times 0.2339 \\ &= 4287.43 \text{ years} \end{aligned}$$

SOLVED PROBLEM 13. 2 g of a radioactive element degraded to 0.5 g in 60 hours. In what time will it be reduced 10% of its original amount?

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{a}{a-x}$$

Quantities given

$$t = 60 \text{ hours}$$

$$a = 2 \text{ g}$$

$$a-x = 0.5 \text{ g}$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{2.303}{60 \text{ hours}} \times \log \frac{2}{0.5} \\ &= \frac{2.303}{60 \text{ hours}} \times \log 4 \end{aligned}$$

$$= \frac{2.303}{60 \text{ hours}} \times 0.6021$$

$$= 2.311 \times 10^{-2} \text{ hour}^{-1}$$

(ii) To calculate the time t

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{a}{a-x}$$

or

$$t = \frac{2.303}{\lambda} \log \frac{a}{a-x}$$

Quantities given

$$\lambda = 2.311 \times 10^{-2} \text{ hour}^{-1} \quad a = 2 \text{ g} \quad a-x = 90\% \text{ of } a = 1.8 \text{ g}$$

Substitution of values

$$t = \frac{2.303}{2.311 \times 10^{-2} \text{ hour}^{-1}} \times \log \frac{2}{1.8}$$

$$= \frac{2.303}{2.311 \times 10^{-2} \text{ hour}^{-1}} \times \log 1.11$$

$$= \frac{2.303}{2.311 \times 10^{-2} \text{ hour}^{-1}} \times 0.0453$$

$$= 4.51 \text{ hours}$$

SOLVED PROBLEM 14. A radioactive isotope has half life of 20 days. What is the amount of isotope left over after 40 days if the initial amount is 5 g ?

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantity given

$$t_{1/2} = 20 \text{ days}$$

Substitution of values

$$\lambda = \frac{0.693}{20 \text{ days}}$$

$$= 0.03465 \text{ day}^{-1}$$

(ii) To calculate the amount decayed, x

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{a}{a-x}$$

Quantities given

$$t = 40 \text{ days}$$

$$\lambda = 0.03465 \text{ day}^{-1}$$

$$a = 5 \text{ g}$$

Substitution of values

$$0.03465 \text{ day}^{-1} = \frac{2.303}{40 \text{ days}} \log \frac{5}{5-x}$$

or

$$\log \frac{5}{5-x} = \frac{0.03465 \text{ day}^{-1} \times 40 \text{ days}}{2.303}$$

$$= 0.6018$$

or

$$\frac{5}{5-x} = \text{Antilog } 0.6018$$

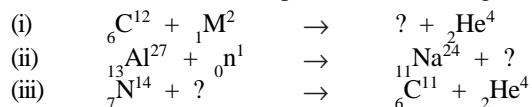
$$= 3.9978$$

or

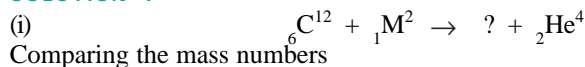
$$5 = (5-x) \times 3.9978$$

$$\begin{aligned}
 & 5 = 5 \times 3.9978 - 3.9978x \\
 \text{or} & \quad x = \frac{19.989 - 5}{3.9978} \\
 & = 3.75 \text{ g} \\
 \therefore & \quad \text{Amount left} = 5 - 3.75 \text{ g} \\
 & = \mathbf{1.25 \text{ g}}
 \end{aligned}$$

SOLVED PROBLEM 15. Complete the following nuclear reactions :



SOLUTION :



$$12 + 2 = x + 4$$

$$\text{or} \quad x = 10$$

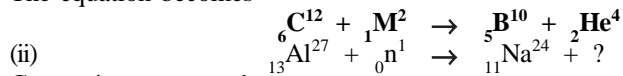
Comparing the atomic numbers

$$6 + 1 = y + 2$$

$$\text{or} \quad y = 5$$

Therefore the missing particle is ${}_5\text{B}^{10}$

The equation becomes



Comparing mass numbers

$$27 + 1 = 24 + x$$

$$\text{or} \quad x = 4$$

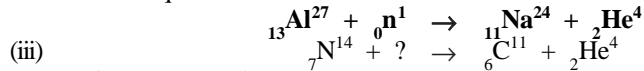
Comparing atomic numbers

$$13 + 0 = 11 + y$$

$$\text{or} \quad y = 2$$

\therefore the missing particle is ${}_2\text{He}^4$

The nuclear equation becomes



Comparing mass numbers

$$14 + x = 11 + 4$$

$$\text{or} \quad x = 1$$

Comparing atomic numbers

$$7 + y = 6 + 2$$

$$\text{or} \quad y = 1$$

\therefore the missing particle is ${}_1\text{H}^1$

The nuclear equation becomes



SOLVED PROBLEM 16. Calculate the rate of disintegration of one gram of Th^{232} if its decay constant is .

SOLUTION :

(i) To calculate the number of nuclei in 1 g of Th

$$\text{No. of nuclei in 1 g of Th} = \frac{\text{Avogadro's No.}}{\text{at. mass}}$$

$$= \frac{6.02 \times 10^{23}}{232}$$

$$= 2.5948 \times 10^{21} \text{ atoms}$$

(ii) To calculate the rate of disintegration

Formula used

$$\frac{dN}{dt} = \lambda \times N$$

Quantities given

$$\lambda = 1.58 \times 10^{-18} \text{ sec}^{-1}$$

$$N = 2.5948 \times 10^{21} \text{ atoms}$$

Substitution of values

$$\frac{dN}{dt} = 1.58 \times 10^{-18} \text{ sec}^{-1} \times 2.5948 \times 10^{21} \text{ atoms}$$

$$= \mathbf{4.099 \times 10^3 \text{ dps}}$$

SOLVED PROBLEM 17. Calculate the mass defect, binding energy and the binding energy per nucleon of ${}^4_2\text{He}$ which has an isotopic mass of 4.0026 amu (${}_1\text{H}^1 = 1.0081 \text{ amu}$; ${}_0\text{H}^1 = 1.0089 \text{ amu}$).

SOLUTION :

$$\begin{aligned} \text{Mass of 2 protons + mass of 2 electron} &= 2 \times \text{mass of } {}_1\text{H}^1 \\ &= 2 \times 1.0081 \text{ amu} \\ &= 2.0162 \text{ amu} \end{aligned}$$

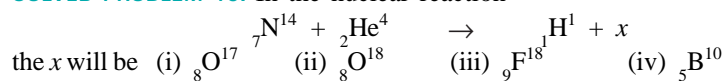
$$\begin{aligned} \text{and mass of 2 neutrons} &= 2 \times 1.0089 \text{ amu} \\ &= 2.0178 \text{ amu} \\ \text{mass of He atom} &= 2.0162 + 2.0178 \text{ amu} \\ &= 4.034 \text{ amu} \end{aligned}$$

$$\begin{aligned} \text{But Actual mass of He atom} &= 4.0026 \text{ amu} \\ \text{Mass defect (i.e. loss of mass)} &= 4.034 \text{ amu} - 4.0026 \text{ amu} \\ &= \mathbf{0.0314 \text{ amu}} \end{aligned}$$

$$\begin{aligned} \text{Binding Energy} &= \text{Mass defect} \times 931.5 \text{ MeV} \\ &= 0.0314 \times 931.5 \text{ MeV} \\ &= \mathbf{29.249 \text{ MeV}} \end{aligned}$$

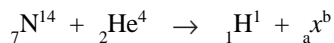
$$\begin{aligned} \text{and Binding energy per nucleon} &= \frac{29.249 \text{ MeV}}{4} \\ &= \mathbf{7.3122 \text{ MeV}} \end{aligned}$$

SOLVED PROBLEM 18. In the nuclear reaction



SOLUTION :

The nuclear reaction is



Comparing the mass numbers

$$14 + 4 = 1 + b$$

or

$$b = 17$$

Comparing the atomic numbers

$$7 + 2 = 1 + a$$

or

$$a = 8$$

\therefore

$$x \text{ will be } {}_8\text{O}^{17}$$

SOLVED PROBLEM 19. The activity of a radioactive sample falls to 85% of the initial value in four years. What is the half life of the sample ? Calculate the time by which activity will fall by 85%.

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{a}{a-x}$$

Quantities given

$$t = 4 \text{ years}$$

$$a = 100 \quad a-x = 85$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{2.303}{4 \text{ years}} \log \frac{100}{85} \\ &= \frac{2.303}{4 \text{ years}} \log 1.1765 \\ &= \frac{2.303}{4 \text{ years}} \times 0.0705 \\ &= \mathbf{4.059 \times 10^{-2} \text{ year}^{-1}} \end{aligned}$$

(ii) To calculate half life period

Formula used

$$\begin{aligned} t_{1/2} &= \frac{0.693}{\lambda} \\ \therefore \lambda &= \frac{0.693}{4.059 \times 10^{-2} \text{ year}^{-1}} \\ &= \mathbf{17.07 \text{ years}} \end{aligned}$$

(iii) To calculate the time

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{a}{a-x}$$

or

$$t = \frac{2.303}{\lambda} \log \frac{a}{a-x}$$

Quantities given

$$\lambda = 4.059 \times 10^{-2} \text{ year}^{-1}$$

$$a = 100 \quad a-x = 15$$

Substitution of values

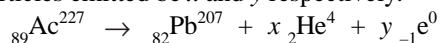
$$\begin{aligned} t &= \frac{2.303}{4.059 \times 10^{-2} \text{ year}^{-1}} \log \frac{100}{15} \\ &= \frac{2.303}{4.059 \times 10^{-2} \text{ year}^{-1}} \times \log 6.66 \\ &= \mathbf{46.69 \text{ years}} \end{aligned}$$

SOLVED PROBLEM 20. The mass number and atomic number of a radioactive element Actinium are 227 and 89 respectively. Calculate the number of α and β particles emitted, if the mass number and atomic number of the new element lead are 207 and 82 respectively.

SOLUTION :

Let the number of α and β particles emitted be x and y respectively.

Let the number of α and β particles emitted be x and y respectively.



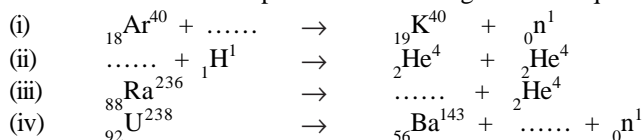
Comparing the mass numbers, we have

$$\begin{aligned}
 227 &= 207 + x \times 4 + y \times 0 \\
 \text{or } 4x &= 20 \\
 x &= 5
 \end{aligned}$$

Now comparing the atomic numbers, we have

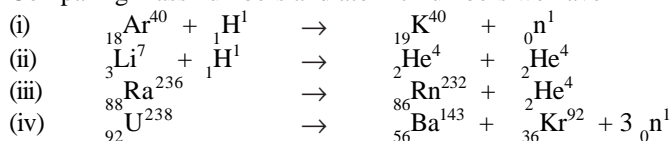
$$\begin{aligned}
 89 &= 82 + 2x + y(-1) \\
 89 - 82 &= 2x - y \\
 \text{or } 7 &= 2 \times 5 - y \\
 \text{or } y &= 3 \qquad (\because x = 5)
 \end{aligned}$$

SOLVED PROBLEM 21. Complete the following nuclear equations :



SOLUTION :

Comparing mass numbers and atomic numbers we have



SOLVED PROBLEM 22. Calculate the age of the tooth in which C^{14} activity is 20% of the activity found at the present time ($t_{1/2}$ for $\text{C}^{14} = 5580$ years).

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantity given

$$t_{1/2} = 5580 \text{ years}$$

Substitution of values

$$\begin{aligned}
 \lambda &= \frac{0.693}{5580 \text{ years}} \\
 &= 1.2419 \times 10^{-4} \text{ year}^{-1}
 \end{aligned}$$

(ii) To calculate the time

Formula used

$$\begin{aligned}
 \lambda &= \frac{2.303}{t} \log \frac{N^0}{N} \\
 \text{or } t &= \frac{2.303}{\lambda} \log \frac{N^0}{N}
 \end{aligned}$$

Quantities given

$$\lambda = 1.1785 \times 10^{-4} \text{ year}^{-1}$$

$$N^0 = 100$$

$$N = 20$$

Substitution of values

$$\begin{aligned}
 t &= \frac{2.303}{1.2419 \times 10^{-4} \text{ year}^{-1}} \log \frac{100}{20} \\
 &= \frac{2.303}{1.2419 \times 10^{-4} \text{ year}^{-1}} \log 5 \\
 &= \frac{2.303 \times 0.6990}{1.2419 \times 10^{-4} \text{ year}^{-1}} \\
 &= \mathbf{12962 \text{ years}}
 \end{aligned}$$

SOLVED PROBLEM 23. An old wooden article shows 2.0 counts per minute per gram. A fresh sample of wood shows 15.2 counts per minute per gram. Calculate the age of the wooden article ($t_{1/2}$ for $C^{14} = 5760$ years)..

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantity given

$$t_{1/2} = 5760 \text{ years}$$

Substitution of values

$$\lambda = \frac{0.693}{5760 \text{ years}} = 1.203 \times 10^{-4} \text{ year}^{-1}$$

(ii) To calculate the age of the wooden article

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N}$$

or

$$t = \frac{2.303}{\lambda} \log \frac{N^0}{N}$$

Quantities given

$$\lambda = 1.203 \times 10^{-4} \text{ year}^{-1}$$

$$N^0 = 15.2 \text{ Counts per min per g}$$

$$N = 2.0 \text{ Counts per min per g}$$

Substitution of values

$$\begin{aligned} t &= \frac{2.303}{1.203 \times 10^{-4} \text{ year}^{-1}} \log \frac{15.2}{2} \\ &= \frac{2.303}{1.203 \times 10^{-4} \text{ year}^{-1}} \log 7.6 \\ &= \frac{2.303}{1.203 \times 10^{-4} \text{ year}^{-1}} \times 0.8808 \\ &= 1.943 \times 10^4 \text{ year} \times 0.8808 \\ &= \mathbf{16862 \text{ years}} \end{aligned}$$

SOLVED PROBLEM 24. ${}_{92}\text{U}^{238}$ by successive radioactive decay changes to ${}_{82}\text{Pb}^{206}$. A sample of uranium ore was analyzed and found to contain 1 g of U^{238} and 0.1 g of Pb^{206} . Assuming that all Pb^{206} had accumulated due to decay of U^{238} find out age of the ore. (Half life of $\text{U}^{238} = 4.5 \times 10^9$ years).

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantity given

$$t_{1/2} = 4.5 \times 10^9 \text{ years}$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{0.693}{4.5 \times 10^9 \text{ years}} \\ &= \mathbf{0.154 \times 10^{-9} \text{ year}^{-1}} \end{aligned}$$

(ii) To calculate the initial amount of U^{238} , N^0

Since the whole of Pb^{206} comes from U^{238}

$$\begin{aligned} \text{Amount of } U^{238} \text{ decayed} &= \text{Amount of } Pb^{206} \text{ formed} \\ \therefore \text{Amount of } U^{238} \text{ decayed} &= \frac{0.1 \times 238}{206} \text{ g} \\ &= 0.1155 \text{ g} \\ \text{and the initial amount of } U^{238} &= 1 \text{ g} + 0.1155 \text{ g} \\ N^0 &= 1.1155 \text{ g} \end{aligned}$$

(iii) To calculate the age of the ore

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N}$$

or

$$t = \frac{2.303}{\lambda} \log \frac{N^0}{N}$$

Quantities given

$$\lambda = 0.154 \times 10^{-9} \text{ year}^{-1} \quad N^0 = 1.1155 \text{ g} \quad N = 1 \text{ g}$$

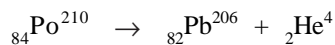
Substitution of values

$$\begin{aligned} t &= \frac{2.303}{0.154 \times 10^{-9} \text{ year}^{-1}} \log \frac{1.1155}{1} \\ &= 14.95 \times 10^9 \text{ year} \times 0.0474 \\ &= \mathbf{7.09 \times 10^8 \text{ years}} \end{aligned}$$

SOLVED PROBLEM 25. Po^{210} decays with alpha to $_{82}Pb^{206}$ with a half life of 138.4 days. If 1.0 g of Po^{210} is placed in a sealed tube, how much helium will accumulate in 69.2 days? Express the answer in cm^3 at STP.

SOLUTION :

The nuclear equation is



(i) To calculate the decay constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantity given

$$t_{1/2} = 138.4 \text{ days}$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{0.693}{138.4 \text{ days}} \\ &= 5.0 \times 10^{-3} \text{ day}^{-1} \end{aligned}$$

(ii) To calculate the amount of $_{84}Po^{210}$ left after 69.2 days

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N}$$

Quantities given

$$\lambda = 5.0 \times 10^{-3} \text{ day}^{-1} \quad t = 69.2 \text{ days}$$

Substitution of values

$$5.0 \times 10^{-3} \text{ day}^{-1} = \frac{2.303}{69.2 \text{ days}} \log \frac{N^0}{N}$$

or

$$\log \frac{N^0}{N} = \frac{5.0 \times 10^{-3} \text{ day}^{-1} \times 69.2 \text{ day}}{2.303}$$

$$\log \frac{N^0}{N} = 1.502 \times 10^{-1} = 0.1502$$

$$\text{or } \frac{N^0}{N} = \text{Antilog } 0.1502$$

$$= 1.413$$

$$\text{or } \frac{N}{N^0} = \frac{1}{1.413} = 0.707$$

$$\text{or } N = N^0 \times 0.707$$

$$= 1 \text{ g} \times 0.707 \quad [\because N^0 = 1 \text{ g given}]$$

$$= 0.707 \text{ g}$$

$$\text{Mass of } {}_{84}\text{Po}^{210} \text{ decomposed to } {}_2\text{He}^4$$

$$= 1 \text{ g} - 0.707 \text{ g}$$

$$= 0.293 \text{ g}$$

(iii) To calculate the volume of ${}_2\text{He}^4$ accumulated at STP

$$\text{i.e. } 210 \text{ g of } {}_{84}\text{Po}^{210} = {}_2\text{He}^4$$

$$= 4 \text{ g of } {}_2\text{He}^4$$

$$\therefore 0.293 \text{ g of } {}_{84}\text{Po}^{210} = \frac{4}{210} \times 0.293 \text{ g of } {}_2\text{He}^4$$

$$= 5.580 \times 10^{-3} \text{ g}$$

$$\text{At STP } 4 \text{ g of } {}_2\text{He}^4 = 22400 \text{ cm}^3$$

$$\therefore 5.580 \times 10^{-3} \text{ g of } {}_2\text{He}^4 = \frac{22400}{4} \times 5.580 \times 10^{-3} \text{ cm}^3$$

$$= \mathbf{31.248 \text{ cm}^3}$$

SOLVED PROBLEM 26. In nature a decay chain series start with ${}_{90}\text{Th}^{232}$ and finally terminates at ${}_{82}\text{Pb}^{208}$. A thorium ore sample was found to contain 8×10^5 ml of helium at STP and of Th^{232} . Find the age of the sample assuming the source of helium to be only due to the decay of Th^{232} . Also assume complete retention of helium within the ore. (Half life of $\text{Th}^{232} = 1.39 \times 10^{10}$ ml).

SOLUTION :

(i) To calculate the decay constant

$$\text{Formula used } \lambda = \frac{0.693}{t_{1/2}}$$

Quantity given

$$t_{1/2} = 1.39 \times 10^{10} \text{ year}$$

Substitution of values

$$\lambda = \frac{0.693}{1.39 \times 10^{10} \text{ year}}$$

$$= \mathbf{4.986 \times 10^{-11} \text{ year}^{-1}}$$

(ii) To calculate the no. of Th^{232} used up and left

$$\text{No. of helium atoms in } 8 \times 10^5 \text{ ml at STP} = \frac{8 \times 10^5 \times 6.02 \times 10^{23}}{22400}$$

$$[\because 22400 \text{ ml} = 6.02 \times 10^{23} \text{ atoms}]$$

$$= 2.15 \times 10^{15} \text{ atoms}$$

$$\begin{aligned} \text{No. of Th}^{232} \text{ atoms used up} &= \frac{2.15 \times 10^{15} \text{ atoms}}{6} \\ &= 3.583 \times 10^{14} \text{ atoms} \\ \text{No. of Th}^{232} \text{ atoms left (N)} &= \frac{5 \times 10^{-7} \times 6.02 \times 10^{23}}{232} \\ &= 1.297 \times 10^{15} \text{ atoms} \end{aligned}$$

[5" 232 g of Th = 6.02×10^{23} atoms]

$$\begin{aligned} \text{No. of Th}^{232} \text{ atoms at the beginning (N}^0\text{)} &= 3.583 \times 10^{14} + 1.297 \times 10^{15} \text{ atoms} \\ &= 16.553 \times 10^{14} \text{ atoms} \end{aligned}$$

(iii) To calculate the age of the sample**Formula used**

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N}$$

or

$$t = \frac{2.303}{\lambda} \log \frac{N^0}{N}$$

Quantities given

$$\lambda = 4.985 \times 10^{-11} \text{ year}^{-1} \quad N^0 = 16.553 \times 10^{14} \text{ atoms} \quad N = 1.297 \times 10^{14} \text{ atoms}$$

Substitution of values

$$\begin{aligned} t &= \frac{2.303}{4.985 \times 10^{-11} \text{ year}^{-1}} \times \log \frac{16.553 \times 10^{14} \text{ atom}}{1.297 \times 10^{14} \text{ atom}} \\ &= 4.6199 \times 10^{10} \text{ year} \times 0.1059 \\ &= \mathbf{4.892 \times 10^9 \text{ years}} \end{aligned}$$

SOLVED PROBLEM 27. A sample of U^{238} (half life = 4.5×10^9 years) ore is found to contain 23.8 g of U^{238} and 20.6 g of Pb^{206} . Calculate the age of the ore.

SOLUTION :**Formula used**

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantity given

$$t_{1/2} = 4.5 \times 10^9 \text{ years}$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{0.693}{4.5 \times 10^9 \text{ years}} \\ &= 0.154 \times 10^{-9} \text{ year}^{-1} \\ &= 1.54 \times 10^{-10} \text{ year}^{-1} \end{aligned}$$

(ii) To calculate the number of moles of U^{238} disintegrated and left

$$\begin{aligned} \text{Since } \text{U}^{238} &= \text{Pb}^{206} \\ \text{and } 20.6 \text{ g of Pb}^{206} &= \frac{20.6 \text{ g}}{206 \text{ g mol}^{-1}} = 0.1 \text{ mole} \\ \therefore \text{ no. of mole of U}^{238} \text{ disintegrated} &= 0.1 \text{ mole} \\ \text{and the initial no. of moles of U}^{238} &= \text{no. of moles of U}^{238} \text{ left} + \\ &= \text{no. of moles of U}^{238} \text{ integrated} \\ &= 0.1 + 0.1 \text{ mol} \\ &= 0.2 \text{ mol} \end{aligned}$$

(iii) To calculate the age of the ore
Formula used

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N}$$

or

$$t = \frac{2.303}{\lambda} \log \frac{N^0}{N}$$

Quantities given

$$\lambda = 1.54 \times 10^{-10} \text{ year}^{-1}$$

$$N^0 = 0.2 \text{ mol}$$

$$N = 0.1 \text{ mol}$$

Substitution of values

$$\begin{aligned} t &= \frac{2.303}{1.54 \times 10^{-10} \text{ year}^{-1}} \log \frac{0.2 \text{ mol}}{0.1 \text{ mol}} \\ &= 1.4955 \times 10^{10} \text{ year} \times \log 2 \\ &= 1.4955 \times 10^{10} \times 0.3010 \text{ years} \\ &= 0.450 \times 10^{10} \text{ years} \\ &= \mathbf{4.50 \times 10^9 \text{ years}} \end{aligned}$$

SOLVED PROBLEM 28. An experiment requires minimum Beta activity produced at the rate of 346 Beta particles per minute. The half life of ${}_{42}\text{Mo}^{99}$, which is a Beta emitter is 66.6 hours. Find the minimum amount of ${}_{42}\text{Mo}^{99}$ required to carry out the experiment 6.909 hours.

SOLUTION :
(i) To calculate the disintegration constant
Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantity given

$$t_{1/2} = 66.6 \text{ hours} = 66.6 \times 60 \text{ min} = 3996 \text{ min}$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{0.693}{3996 \text{ min}} \\ &= 1.734 \times 10^{-4} \text{ min}^{-1} \end{aligned}$$

(ii) To calculate the activity required

Time required for the completion of experiment

$$\begin{aligned} &= 6.909 \text{ hours} \\ &= 6.909 \times 60 \text{ min} \\ &= 414.54 \text{ min} \end{aligned}$$

$$\begin{aligned} \text{Total activity required} &= 346 \times 414.54 \text{ Beta particles} \\ &= 1.4343 \times 10^5 \text{ Beta particles} \end{aligned}$$

(iii) To calculate the amount of ${}_{42}\text{Mo}^{99}$ required
Formula used

$$\begin{aligned} \text{Activity} &= \lambda \times N \\ \text{or} \quad N &= \frac{\text{Activity}}{\lambda} \end{aligned}$$

Quantities given

$$\text{Activity} = 1.4343 \times 10^5 \beta \text{ particles} \quad \lambda = 1.734 \times 10^{-4} \text{ min}^{-1}$$

Substitution of values

$$N = \frac{1.4343 \times 10^5 \beta \text{ particles}}{1.734 \times 10^{-4} \text{ min}^{-1}}$$

$$\begin{aligned} \therefore \text{No. of moles of } {}_{42}\text{Mo}^{99} \text{ required} &= 8.27 \times 10^8 \beta \text{ particles} \\ &= \frac{8.27 \times 10^8}{6.02 \times 10^{23} \text{ mol}^{-1}} \\ &= 1.37 \times 10^{-15} \text{ mol} \end{aligned}$$

$$\begin{aligned} \text{and Amount of } {}_{42}\text{Mo}^{99} \text{ required} &= 1.37 \times 10^{-15} \text{ mol} \times 99 \text{ g mol}^{-1} \\ &= \mathbf{135.63 \times 10^{-15} \text{ g}} \end{aligned}$$

SOLVED PROBLEM 29. The nucleic ratio ${}_1\text{H}^3$ to ${}_1\text{H}^1$ in a sample of water is . Tritium undergoes decay with half life period of 12.3 years. How many tritium atoms would 10 g of such a sample contain 40 years after the original sample is collected.

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantity given

$$t_{1/2} = 12.3 \text{ years}$$

Substitution of values

$$\begin{aligned} \lambda &= \frac{0.693}{12.3 \text{ years}} \\ &= 5.634 \times 10^{-2} \text{ year}^{-1} \end{aligned}$$

(ii) To calculate the number of Tritium atoms

$$\text{No. of moles of water in 10 g sample} = \frac{10}{18} \text{ mole}$$

$$\text{No. of moles of H atoms in 10 g sample} = \frac{2 \times 10}{18} \text{ mole}$$

$$\text{As } {}_1\text{H}^3 : {}_1\text{H}^1 = 8 \times 10^{-18} : 1$$

$$\begin{aligned} \therefore \text{the no. of tritium atoms} &= \frac{8 \times 10^{-18} \times 2 \times 10 \times 6.02 \times 10^{23}}{18} \\ &= 5.35 \times 10^6 \text{ atoms} \end{aligned}$$

(iii) To calculate the number of Tritium atoms left after 40 years

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N}$$

Quantities given

$$\lambda = 5.634 \times 10^{-2} \text{ year}^{-1} \quad t = 40 \text{ years} \quad N^0 = 5.35 \times 10^6 \text{ atoms}$$

Substitution of values

$$5.634 \times 10^{-2} \text{ year}^{-1} = \frac{2.303}{40 \text{ years}} \log \frac{N^0}{N}$$

$$\begin{aligned} \text{or } \log \frac{N^0}{N} &= \frac{5.634 \times 10^{-2} \text{ year}^{-1} \times 40 \text{ years}}{2.303} \\ &= 0.9785 \end{aligned}$$

$$\text{or } \log \frac{N}{N^0} = 0.9785$$

$$\begin{aligned}
 \text{or} \quad \frac{N}{N^0} &= \text{Antilog}(-0.9785) \\
 &= 0.1050 \\
 \text{or} \quad N &= N^0 \times 0.1050 \\
 &= 5.35 \times 10^6 \times 0.1050 \\
 &= \mathbf{5.6175 \times 10^5 \text{ atoms}}
 \end{aligned}$$

SOLVED PROBLEM 30. One of the hazards of nuclear explosion is the generation of Sr^{90} and its subsequent incorporation in bones. This nuclide has half life of 28.1 years. Suppose one microgram was absorbed by a new born child, how much Sr^{90} will remain in his bones after 20 years.

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantity given

$$t_{1/2} = 28.1 \text{ years}$$

Substitution of values

$$\begin{aligned}
 \lambda &= \frac{0.693}{28.1 \text{ years}} \\
 &= 0.0246 \text{ year}^{-1}
 \end{aligned}$$

(ii) To calculate the amount of Sr^{90} left after 20 years

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N}$$

Quantities given

$$\lambda = 0.0246 \text{ year}^{-1}$$

$$N^0 = 1 \text{ g}$$

$$t = 20 \text{ years}$$

Substitution of values

$$0.0246 \text{ year}^{-1} = \frac{2.303}{20 \text{ years}} \times \log \frac{N^0}{N}$$

$$\begin{aligned}
 \text{or} \quad \log \frac{N^0}{N} &= \frac{0.0246 \text{ year}^{-1} \times 20 \text{ years}}{2.303} \\
 &= 0.2136
 \end{aligned}$$

$$\text{or} \quad \frac{N^0}{N} = \text{Antilog } 0.2136$$

$$\text{or} \quad \frac{N^0}{N} = 1.635$$

$$\text{or} \quad \frac{N}{N^0} = \frac{1}{1.635} = 0.61$$

$$\begin{aligned}
 \text{or} \quad N &= N^0 \times 0.61 \\
 &= 1 \mu\text{g} \times 0.61 \\
 &= \mathbf{0.61 \mu\text{g}}
 \end{aligned}$$

SOLVED PROBLEM 31. Ac^{227} has a half life of 21.8 years with respect to radioactive decay. The decay follows two parallel paths, one leading to Th^{227} and the other leading to Fr^{223} . The percentage yield of these two daughter nucleides are 1.2% and 98.8% respectively. What is the rate constant, in year^{-1} , for each of the separate path.

SOLUTION :**(i) To calculate the rate constant for Ac²²⁷ decay****Formula used**

$$k = \lambda = \frac{0.693}{t_{1/2}}$$

Quantity given

$$t_{1/2} = 21.8 \text{ years}$$

Substitution of values

$$\begin{aligned} k &= \frac{0.693}{21.8 \text{ years}} \\ &= 3.178 \times 10^{-2} \text{ year}^{-1} \end{aligned}$$

(ii) To calculate the rate constants for Th²²⁷ and Fr²²³.

For a first order parallel reaction, the overall rate constant is equal to the sum of rate constant for separate paths *i.e.*

$$\begin{aligned} K_{Ac} &= k_{Th} + k_{Fr} \\ k_{Th} &= \% \text{ age yield} \times K_{Ac} \\ &= \frac{1.2}{100} \times 3.178 \times 10^{-2} \text{ year}^{-1} \\ &= \mathbf{3.814 \times 10^{-2} \text{ year}^{-1}} \end{aligned}$$

and

$$\begin{aligned} k_{Fr} &= \% \text{ age yield} \times K_{Ac} \\ &= \frac{98.8}{100} \times 3.178 \times 10^{-2} \text{ year}^{-1} \\ &= \mathbf{3.139 \times 10^{-2} \text{ year}^{-1}} \end{aligned}$$

ADDITIONAL PRACTICE PROBLEMS

- Calculate the number of λ and β particles emitted in the conversion of Thorium, ${}_{90}\text{Th}^{232}$, to lead, ${}_{82}\text{Pb}^{206}$.
Answer: 6 α , 4 β
- The mass number and atomic number of a radioactive element Actinium are 227 and 89 respectively. Calculate the number of α and β particles emitted if the mass number and atomic number of the new element lead are 207 and 82 respectively.
Answer: 5 α , 3 β
- ${}_{92}\text{U}^{238}$ undergoes a series of change emitting α and β particles and finally ${}_{82}\text{Pb}^{206}$ is formed. Calculate the number of α and β particles which must have been ejected during the series.
Answer: 8 α , 6 β
- The half life of cobalt-60 is 5.26 years. Calculate the percentage activity after eight years.
Answer: 34.87%

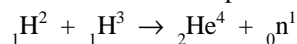
5. An old wooden article shows 2.0 counts per minute per gram. A fresh sample of wood shows 15.2 counts per minute per gram. Calculate the age of the wooden article. (of $C^{14} = 5760$ years)

Answer. 1686.6 years

6. Calculate the age of the tooth in which C^{14} activity is 20% of the activity found at the present time (for $C^{14} = 5580$ years)

Answer. 12961.4 years

7. Calculate the energy released in the fusion reaction per atom of helium produced.



Given the following atomic masses

$${}_1H^2 = 2.014; \quad {}_1H^3 = 3.016; \quad {}_2He^4 = 4.003; \quad {}_0n^1 = 1.009 \text{ amu} \quad \text{and}$$

$$1 \text{ amu} = 1.51 \times 10^{-10} \text{ J}.$$

Answer. $2.72 \times 10^{-12} \text{ J}$

8. Calculate the rate of disintegration of one gram of Th^{232} if its decay constant is $1.58 \times 10^{-18} \text{ sec}^{-1}$.

Answer. $4.0998 \times 10^3 \text{ dps}$

9. The activity of a radioactive sample falls to 85% of the initial value in four years. What is the half life of the sample? Calculate the time by which activity will fall to 85%.

Answer. 17.05 years; 46.735 years

10. 2 g of a radioactive element degraded to 0.5 g in 60 hours. In what time will it be reduced to 10% of its original amount?

Answer. 4.56 hours

11. Calculate the time required for a radioactive sample to lose one-third of the atoms of its parent isotope. The half life is 33 min.

Answer. 19.31 min

12. A piece of wood recovered in excavation has 30% as much ${}_6C^{14}$ as a fresh wood today. Calculate the age of excavated piece assuming half life period of ${}_6C^{14}$ as 5700 years.

Answer. 9908 years

13. Radioisotope ${}_{15}P^{32}$ has a half life of 15 days. Calculate the time in which the radioactivity of its 1 mg quantity will fall to 10% of the initial value.

Answer. 49.84 days

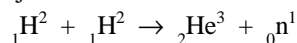
14. A natural isotope of potassium has a half life of years. Out of 1 mole of this isotope, how many atoms will remain after 10^{11} years?

Answer. 1.258×10^{23} atoms

15. In the fission of ${}_{92}U^{235}$ with thermal neutron, the products are Mo^{95} , La^{139} and two neutrons. Calculate the energy released in MeV in the fission of 1 g ${}_{92}U^{235}$. The atomic masses in amu are $U = 235$; $Mo = 94.936$; $La = 138.950$; ${}_0n^1 = 1.009$.

Answer. $5.32 \times 10^{23} \text{ MeV}$

16. Calculate the energy released in joules and MeV in the following nuclear reaction:



Assume that masses of ${}_1H^2$, ${}_2He^3$, and ${}_0n^1$ are 2.0141, 3.0160 and 1.0087 in amu.

Answer. $5.22 \times 10^{-3} \text{ J}$; 3.2585 MeV