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Analysis of approximate solutions of initial value problems (IVP) for ordinary differential equations (ODE)

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Abstract

In this paper, We have used Euler method and Runge-kutta method for finding approximate solutions of ordinary differential equations(ODE) in initial value problems(IVP). Numerical examples are considered to illustrate the efficiency and convergence ofthe two methods. Numerical results show that the proposed two method are very effective and efficient.We have investigated and computed the error of the proposed two methods. The approximated solutions with different step-size of the methods and analytical solutions are computed in Mathematica software. Approximation accuracy comparison between Euler method and Runge-Kutta methods for ordinary differential equations are done by finding the absolute error. The absolute errors produced by the proposed methods can then be analyzed to determine which one provides more accurate results.

Keywords: Euler method, Runge-kutta method, Ordinary differential equation (ODE).

1. Introduction

A number of problems in science and engineering can be formulated in terms of differential equations. A differential equation is an equation involving a relation between an unknown function and one or more derivatives. Equations involving derivatives of only one independent variable are called ordinary differential equations and can be classified as either initial value problems (IVP) or boundary value problem (BVP). In most real life situations, the differential equation that models the problem is too complicated to solve exactly. Only a limited number of differential equations can be solved analytically. There are many analytical methods for finding the solution of ordinary differential equations.Even then, there exists a large number of ordinary differential equations whose solutions cannot be obtained in closed form by using well known analytical methods. There, we have to use the numerical methods to get the approximate solution of a differential equation under the prescribed initial condition or conditions. There are many types of practical numerical methods for solving initial value problems for ordinary differential equations. In this paper we present two standard numerical methods, Euler and Runge-Kutta method for solving initial value problems of ordinary differential equations.

We may realize that several works in numerical solutions of initial value problems using Euler method and Runge-Kutta method have been carried out. Many authors have attempted to solve initial value problems (IVP) to obtain high accuracy rapidly by using a numerous methods, including the Euler method and Runge-Kutta method.In [1] the author discussed accuracy analysis of Numerical solutions of initial value problems (IVP) for ordinary differential equations (ODE). In [2] the author discussed accurate solutions of initial value problems for ordinary differential equations with fourth order Runge-kutta method. In [3] studied the Comparative study of the accuracy of an implicit linear multistep method of order six and classical Runge-Kutta method for the solution of initial value problems in ordinary differential equations.In [4,5,6,7,8,9,10,11,12,13,14,15] also numerical solutions of initial value problems for ordinary differential equations are solved using various numerical methods. In this paper Euler method and Runge-kutta method are applied without any discretization, transformation or restrictive assumptions for solving ordinary differential equations in initial value problems. The Euler Method is traditionally the first numerical technique. It is very simple to understand and geometrically easy to articulate but not very practical, the method has limited accuracy for more complicated functions.A more robust and intricate numerical technique is the Runge–Kuttamethod. This method is the most widely used one since it gives reliable

starting values and is particularly suitable when the computation of higher derivatives is complicated. The numerical results are very encouraging. Finally, two examples of different kinds of ordinary differential equations are given to verify the proposed formulae. The results of each numerical example indicateconvergence and error analysis are discussed to illustrate the efficiency of the methods. The use of Euler method to solve the differential equation numerically is less efficient since it requires h to be small for obtaining reasonable accuracy.But in Runge-Kutta method, the derivatives of higher order are not required and they are designed to give greater accuracy with the advantage of requiring only the functional values at some selected points on the sub-interval. Runge-Kutta method is a more general and improvised method as compared to that of the Euler method. Euler method uses excessively small step size to converge to analytical solution. So, large number of computation is needed. Runge-kutta method gives more accurate solutions because it requires four evaluations per step and the approximated solutions converge faster to exact solution and involves less iteration to obtain the accurate solution.

2. Initial Value problem (IVP)

In this paper we consider a simple first order differential equation in an initial value problemthat can be defined as $y' = f(x, y(x))$ with initial condition $y(x_0) = y_0$ where $x_0 \le x \le x_n$ (1)

Analytical solution is defined by $y(x)$ and approximate solution is defined by y_n. For solving (1) we divide the interval $[x_0, x_n]$ into n equally spaced subintervals such that $x_n = x_0 + n\alpha$ for each $n = 0,1,2,...,n$ .The parameter α is called the step size. Numerical solutions of the initial value problem are defined to be a set of points $\{(x_n, y_n) : n = 0,1,2,...,n\}$ and each point (x_n, y_n) is an approximation to the corresponding point $(x_n, y(x_n))$ on the solution curve.

3. Numerical Evaluation Procedures

In this section we use two different procedures to solve initial value problems (IVP) for ordinary differential equations (ODE).

3.1. Procedure-1

Euler method is the simplest one-step method. It is a basic explicit method for numerical integration of ordinary differential equations. Euler proposed his method for initial value problems (IVP) in 1768. It is the first numerical method for solving IVP and serves to illustrate the concepts involved in the advanced methods. It is important to study it because it makes the error analysis much easier to understand. The general formula for Euler approximation is $y_{n+1}(x) = y_n(x) + \alpha f(x_n, y_n),\ n = 0,1,2,3...$

3.2. Procedure-2

In Numerical Analysis, the Runge-Kutta method is an important family of implicit and explicit iterative methods, which are used in temporal discretization for the approximation of solutions of ordinary differential equations. These techniques were developed around 1900 by the German mathematicians C. Runge and M. W. Kutta.The fourth order RungeKuttamethod (RK4) is most popular for solving initial value problems (IVP) for ordinary differential equation (ODE).The general formula for Runge-Kutta approximation is

$$y_{n+1}(x) = y_n(x) + \frac{1}{6}(t_1 + 2t_2 + 2t_3 + t_4),\ n = 0,1,2,3...$$

Where $t_1 = \alpha f(x, y),\ k_2 = \alpha f(x + \frac{\alpha}{2}, y + \frac{t_1}{2}),\ t_3 = \alpha f(x + \frac{\alpha}{2}, y + \frac{t_2}{2}),\ t_4 = \alpha f(x + \alpha, y + t_3)$

4. Error Analysis

There are two types of errors in numerical solutions. Round-off errors andTruncation errors occur when ordinary differential equations are solved numerically.Rounding errors originate from the fact that computers can only represent numbers using a fixed and limited number of significant figures. Thus, such numbers or cannot be represented exactly in computer memory. The discrepancy introduced by this limitation is call Round-off error.Truncation errors in numerical analysisarise when approximations are used to estimate some quantity. The accuracy of the solution will depend on how small we make the step size, α . A

numerical method is said to be convergent if the numerical solution approaches the exact solution as the step size α goes to 0. In this paper we consider two IVP problems to verify accuracy of the proposed methods. Then using these methods we find numerical approximations for desired IVP. All the computations are performed by Mathematicasoftware. The convergence of initial value problem (IVP) is calculated by $|y(x_n) - y_n| < \delta$ where $y(x_n)$ denotes the approximate solution and y_n denotes the exact solution and δ depends on the problem which varies from 10^{-7} .The absolute error for this formula is defined by $|y(x_n) - y_n|$.

5. Numerical Examples

In this section,we perform numerical experiments using two proposed methods on illustrative examples of the initial value problems for ordinary differential equations to verify accuracy.

Example 1: we consider the initial value problem $y'(x) = xy - e^{-x}$, $y(0) = 1$ on the interval $0 \le x \le 1$.

The exact solution of the given problem is given $y(x) = \frac{1}{2} e^{\frac{x^2}{2}} (2 + \sqrt{2e\pi}\ erf(\frac{1}{\sqrt{2}}) - \sqrt{2e\pi}\ erf(\frac{x+1}{\sqrt{2}}))$ .The numerical approximations and Absolute errors using two proposed method are shown in Tables5.1-5.4

Table 5.1: Numerical Approximations for different step size using Euler method

x_n	Approximate Solutions				Exact Solutions
	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.0125$	
0.1	0.9000000000000000	0.9048135287749642	0.9071825281966918	0.9083581709844192	0.9095281318269378
0.2	0.8185162581964041	0.8275410446371659	0.8320031870900734	0.8342227388971348	0.8364350145966888
0.3	0.753013508052534	0.7658758772439387	0.7722657265605345	0.7754518320655703	0.7786326527713157
0.4	0.7015220912259382	0.7180443542090251	0.7262931266506466	0.730416546408438	0.7345401670168167
0.5	0.6625509702714119	0.682732839287642	0.692861373058415	0.6979380435871402	0.7030241932058348
0.6	0.6350254528137191	0.6590376890154925	0.6711555310054594	0.6772467069626616	0.6833611456192851
0.7	0.6182458163731397	0.646435416774346	0.6607459484358202	0.6679614594504808	0.6752197247016554
0.8	0.6118644931401185	0.6447717407037723	0.6615838022541796	0.6700887376162563	0.6786634544456545
0.9	0.6158807561796058	0.6542695024189834	0.6740165662901743	0.6840420202068181	0.6941744857956834
1.0	0.6306530582617104	0.675556795352521	0.6988254527170644	0.7106843582710795	0.7227015389486589

Table 5.2: Numerical Approximations for different step size using Runge-Kutta method

x_n	Approximate Solutions				Exact Solutions
	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.0125$	
0.1	0.9095281891559677	0.9095281354086854	0.9095281320508142	0.9095281318409314	0.9095281318269378
0.2	0.8364351249694311	0.8364350215043004	0.8364350150287971	0.8364350146237085	0.8364350145966888
0.3	0.7786328110327301	0.7786326627020703	0.7786326533933213	0.7786326528102341	0.7786326527713157
0.4	0.7345403673890025	0.7345401796338673	0.73454016780841	0.7345401670663868	0.7345401670168167
0.5	0.7030244293805993	0.7030242081423681	0.7030241941449501	0.7030241932647049	0.7030241932058348
0.6	0.6833614105929027	0.6833611624671793	0.6833611466813637	0.6833611456859505	0.6833611456192851
0.7	0.675220010215858	0.6752197429755409	0.6752197258573754	0.675219724774316	0.6752197247016554
0.8	0.6786637498272825	0.678663473508537	0.6786634556562307	0.6786634545219193	0.6786634544456545
0.9	0.694174775862907	0.6941745047240107	0.6941744870043147	0.6941744858720337	0.6941744857956834
1.0	0.7227018004483059	0.7227015563011094	0.722701540065817	0.72270153901952	0.7227015389486589

Table 5.3: Observed Absolute Error for different step size using Euler method

x_n	Absolute errors			
	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.0125$
0.1	9.52813E-03	4.71460E-03	2.34560E-03	1.16996E-03
0.2	1.79188E-02	8.89397E-03	4.43183E-03	2.21228E-03
0.3	2.56191E-02	1.27568E-02	6.36693E-03	3.18082E-03
0.4	3.30181E-02	1.64958E-02	8.24704E-03	4.12362E-03
0.5	4.04732E-02	2.02914E-02	1.01628E-02	5.08615E-03
0.6	4.83357E-02	2.43235E-02	1.22056E-02	6.11444E-03
0.7	5.69739E-02	2.87843E-02	1.44738E-02	7.25827E-03
0.8	6.67990E-02	3.38917E-02	1.70797E-02	8.57472E-03
0.9	7.82937E-02	3.99050E-02	2.01579E-02	1.01325E-02
1.0	9.20485E-02	4.71447E-02	2.38761E-02	1.20172E-02

Table 5.4: Observed Absolute Error for different step size using Runge-kuttamethod

x_n	Absolute errors			
	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.0125$
0.1	5.73290E-08	3.58175E-09	2.23877E-10	1.39939E-11
0.2	1.10373E-07	6.90761E-09	4.32109E-10	2.70199E-11
0.3	1.58261E-07	9.93075E-09	6.22006E-10	3.89190E-11
0.4	2.00372E-07	1.26171E-08	7.91594E-10	4.95700E-11
0.5	2.36175E-07	1.49365E-08	9.39116E-10	5.88700E-11
0.6	2.64974E-07	1.68479E-08	1.06208E-09	6.66650E-11
0.7	2.85514E-07	1.82739E-08	1.15572E-09	7.26610E-11
0.8	2.95382E-07	1.90629E-08	1.21058E-09	7.62650E-11
0.9	2.90067E-07	1.89283E-08	1.20863E-09	7.63500E-11
1.0	2.61500E-07	1.73525E-08	1.11716E-09	7.08620E-11

Example 2: we consider the initial value problem $y'(x) = xy + e^x$, $y(0) = 1$ on the interval $0 \le x \le 1$. The exact solution of the given problem is given by $y(x) = \frac{1}{2} e^{\frac{x^2}{2}} (\sqrt{2e\pi}\ erf(\frac{x-1}{\sqrt{2}}) + 2 + \sqrt{2e\pi}\ erf(\frac{1}{\sqrt{2}}))$. The numerical approximations and Absolute errors using two proposed method are shown in Tables5.5-5.8

Table 5.5: Numerical Approximations for different step size using Euler method

x_n	Approximate Solutions				Exact Solutions
	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.0125$	
0.1	1.100000000000000	1.1051885548188012	1.1078393561464583	1.1091796928070503	1.110530301632017
0.2	1.2215170918075648	1.2328095534593166	1.2386044746837532	1.2415411421210325	1.2445047509926233
0.3	1.3680877094597332	1.3867366550725795	1.3963492404510334	1.4012315340835104	1.4061660780777825
0.4	1.5441162215011255	1.571797058999487	1.5861293379826054	1.5934254824102	1.600811124250039
0.5	1.7550633401252975	1.7939908896249332	1.8142391785103749	1.8245713153222383	1.8350469141081591
0.6	2.007688634201575	2.060774486242448	2.08851857525606	2.102710305819003	2.117123064869392
0.7	2.3103618322927204	2.3814260712775424	2.418749937102355	2.437890885365414	2.457363933871341
0.8	2.6734624313002584	2.767518505694381	2.8171738817509677	2.842707519203811	2.8687320948873327
0.9	3.109893518653526	3.2335322273678173	3.2991608375712325	3.333004318701614	3.3675659276217007
1.0	3.6357442464480383	3.797652829447975	3.884089401273545	3.928797715378466	3.974549507166877

Table 5.6: Numerical Approximations for different step size using Runge-Kutta method

x_n	Approximate Solutions				Exact Solution
	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.0125$	
0.1	1.1105302410717133	1.110530297851038	1.1105303013958143	1.1105303016172572	1.110530301632017
0.2	1.2445046270887068	1.2445047432709144	1.2445047505107112	1.2445047509625244	1.2445047509926233
0.3	1.4061658878477006	1.4061660662618751	1.4061660773416873	1.4061660780318517	1.4061660780777825
0.4	1.6008108631429778	1.6008111081125787	1.6008111232475102	1.6008111241875747	1.600811124250039
0.5	1.8350465731825811	1.8350468931762551	1.8350469128126372	1.8350469140276	1.8350469141081591
0.6	2.117122625482692	2.1171230380943378	2.117123063219478	2.117123064767038	2.117123064869392
0.7	2.4573633582017687	2.4573638990299127	2.457363931733263	2.457363933739007	2.457363933871341
0.8	2.8687313095751765	2.8687320475362137	2.8687320919889703	2.8687320947082036	2.8687320948873327
0.9	3.3675647961671404	3.367565859316478	3.3675659234396402	3.367565927363227	3.3675659276217007
1.0	3.974547784090327	3.974549402480526	3.9745495007365212	3.974549506768798	3.974549507166877

Table 5.7: Observed Absolute error for different step size using Euler method

x_n	Absolute errors			
	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.0125$
0.1	1.05303E-02	5.34175E-03	2.69095E-03	1.35061E-03
0.2	2.29877E-02	1.16952E-02	5.90028E-03	2.96361E-03
0.3	3.80784E-02	1.94294E-02	9.81684E-03	4.93454E-03
0.4	5.66949E-02	2.90141E-02	1.46818E-02	7.38564E-03
0.5	7.99836E-02	4.10560E-02	2.08077E-02	1.04756E-02
0.6	1.09434E-01	5.63486E-02	2.86045E-02	1.44128E-02
0.7	1.47002E-01	7.59379E-02	3.86140E-02	1.94730E-02
0.8	1.95270E-01	1.01214E-01	5.15582E-02	2.60246E-02
0.9	2.57672E-01	1.34034E-01	6.84051E-02	3.45616E-02
1.0	3.38805E-01	1.76897E-01	9.04601E-02	4.57518E-02

Table 5.8: Observed Absolute errorfor different step size using Runge-Kutta method

x_n	Absolute errors			
	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.0125$
0.1	6.05603E-08	3.78098E-09	2.36200E-10	1.47600E-11
0.2	1.23904E-07	7.72171E-09	4.81910E-10	3.00999E-11
0.3	1.90230E-07	1.18159E-08	7.36100E-10	4.59301E-11
0.4	2.61107E-07	1.61375E-08	1.00252E-09	6.24600E-11
0.5	3.40926E-07	2.09319E-08	1.29552E-09	8.05500E-11
0.6	4.39387E-07	2.67751E-08	1.64992E-09	1.02360E-10
0.7	5.75670E-07	3.48414E-08	2.13808E-09	1.32340E-10
0.8	7.85312E-07	4.73511E-08	2.89836E-09	1.79130E-10
0.9	1.13145E-06	6.83052E-08	4.18206E-09	2.58480E-10
1.0	1.72308E-06	1.04686E-07	6.43035E-09	3.98080E-10

6. Discussion of Results

Tables (5.1) and (5.5) show a comparison between the numerical approximation results and the exact solution using Euler method for the different step size with various selected values of x and Tables (5.3) and (5.7) are the computed absolute errors .we can see that the approximate solution when the step sizes $\alpha = 0.1$ and $\alpha = 0.05$ does not converge to exact solution but the step sizes reduces gradually to $\alpha = 0.025$ and $\alpha = 0.0125$ the approximate solution tends to converge slowly to the exact solution with

the reduction of the step sizes α, Also the accuracy of the method is not impressive. Hence the Euler method is less accurate. Also Tables (5.2) and (5.6) shows that the approximate solution using Runge-Kutta method at $\alpha = 0.1$, $\alpha = 0.05$, $\alpha = 0.025$ and $\alpha = 0.0125$ is more accurate, efficient and sufficient than the Euler method and Tables (5.4) and (5.8) show the computed absolute errors. We comparing this method to the Euler method, this method give more efficient and accurate results as the approximate solution tends to converge faster to the exact solution. .

7. Conclusion

In this paper, we present Euler method and Runge-Kutta method for solving ordinary differential equation in initial value problems (IVP). To achieve the desired accuracy of numerical solution it is necessary to take step size very, very small. The application of two proposed methods were illustrated by solving an initial value problems (IVP) which is a first order differential equation and using uniform step sizes to obtain numerical results of the approximate solution and compare it with the exact solution. The simple Euler method was found to be less accurate due to the inaccurate numerical results that were obtained from the approximate solution in comparison to the exact solution. In terms of convergence, the approximate solution of the Runge-kutta method was found to converge faster to the exact solution compared to the Euler method. Finally we observe that the Runge-kutta method is more powerful and more efficient in finding numerical solutions of initial value problems compared to the Euler method.

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Promote Sustainable and Healthy Environment of Rajshahi City Corporation through Effective Management of Solid Waste

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Abstract

The urban population of Rajshahi City Corporation is increasing very rapidly. The increase in populations is solely responsible for high increasing rate of solid waste thus its proper management is prerequisite. Management of waste is a costly and troublesome problem for most of the local authorities of developing countries. Rajshahi City Corporation also lacks proper management facilities therefore dumping of solid waste in water bodies, drains and vacant lands are very common. The unplanned and poor management of waste pollutes water, air and land that consequently creates a number of health related problems. Therefore, the study was conducted to explore the existing waste management system and the deficiencies in the sustainable waste management within the study area. This paper will examine various techniques and technologies on solid waste management in light of structural, management and behavioral aspect with the aim to ensure healthy environment of Rajshahi City Corporation.

Keywords: Solid waste, Management, Environment, Sustainable development, Rajshahi City

1. Introduction

Waste is an unavoidable by product of human activities, economic development, urbanization and improving living standards in cities, have led to an increase in the quantity and complexity of generated waste [1]. Rajshahi is the 4th largest Metropolitan Cities of Bangladesh and one of the prominent cities in the northern region of Bangladesh. The city was declared as City Corporation in 1991 with 284056 urban populations that have reached to 449757 urban populations by the year 2011 [2 & 3]. The increase in populations is solely responsible for high increasing rate of solid waste. The waste generation rate in the RCC area is highly increased from 100 tons to 350M.tons by the year 2014 compared to 1994 [4] however the waste collection and management system is not improved significantly thus its proper management is prerequisite. In RCC still a large proportion (54%) of urban populations disposes their household wastes alongside the road (Field survey, 2014). Up to 2003 in RCC area the solid waste disposal bins covered only 19.52% of the area [5]. Due to shortage of dustbins and inadequate collection facilities dumping of solid waste in water bodies, drains and vacant lands was very common [4]. The unplanned and poor management of the waste pollutes water, air and land that consequently creates a number of health related problems [9]. Thus its proper management is prerequisite to enhance environmental sustainability. Management of waste is a costly and troublesome problem for most of the local authorities of developing countries [6]. Inefficient management and disposal of solid waste is an obvious cause of degradation of the environment in most cities of the developing countries. In particular, the City Corporations of the developing countries are not able to handle increasing quantities of waste, which results in uncollected waste on roads and in other public places [10]. There is a need to work towards a sustainable waste management system that requires environmental, institutional, financial, economical and social sustainability [11]. This paper looks in brief the current waste- generation, collection and management scenario in RCC area, along with the associated environmental impacts.

Finally a number of strategic options are proposed to improve the overall waste management system that may lead to healthy environment in the Rajshahi City Corporation area.

2. Materials and Method

Waste management is a costly and troublesome problem in most of the cities of Bangladesh and considered as burden to the environments and daily lives [6]. In the developing countries a very little considerations are given to manage the waste properly therefore, the environments are degrading day by day. Proper management of waste not only brings environmental benefits but also economic benefits to the society and ensures long term sustainability of any city. In the Rajshahi City Corporation area the solid wastes are not managed properly therefore the city is facing a lot of environmental problems. Thus the study has been carried out with the aim to achieve the following objectives:

- To investigate the solid waste management system within the study area (RCC).
- To identify problems and prospects in practicing the hygienic method of waste management.
- To formulate guidelines to improve the overall waste management of the City.

To achieve the objectives of the study an extensive literature review was conducted for background study and better understanding of the topic. The study was conducted based on the secondary data collection and official opinion survey of RCC. The secondary data have been collected from the conservancy department of RCC, Bangladesh Bureau of Statistics, journals, books and reports. Official opinion survey was conducted to know the existing waste management system of the city and the problems faced to ensure hygienic waste management system. Simple statistical technique is used to analyze the data and the final outcomes are presented in the form of tables, figures and photographs etc.

3. Results and Discussions

3.1 Existing Waste Generation, Collection and Management System in RCC Area

Rajshahi City Corporation (RCC) is the designated authority to manage solid waste and to keep the city clean. The cleaning systems can be classified as household level, solid waste disposal from house, collection and transportation by CBO, dumping to the nearest bin/point and collection by RCC to main landfill site/final disposal sites (Fig. 1).

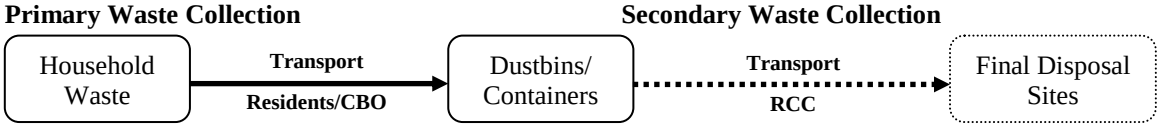

Fig. 1. Waste Collection System in RCC area

With increase in population the waste generation rate has increased in RCC area. The table 1 shows that in 1994 the waste generation rate was only 100 tons that has reached to 350 M. tons by the year 2014. It is also apparent that RCC is unable to collect of all the generated waste and it collects only 210 M. tons (Table 1).

Table 1. Waste Generation and Collection Rate in RCC area

Type \ Year	**1994**	**2014**
Waste Generation/Day	100 tons	350 M. tons
Waste Collection/Day	40-50 tones	210 M. tons

Source: RCC, 1994 & Official Opinion, 2014

A questionnaire survey was conducted in 2014 among all the 300 citizens of RCC and the result depicts that still the waste dumping system is not satisfactory in RCC area. No hygienic method is followed to dump the household waste therefore; the majority of the citizens 54% dump their household waste alongside the road without any segregation (Fig. 2). This open air dumping system creates an unpleasant and unhygienic environment with odor, air and land pollution.

In terms of waste collection, RCC is not in a satisfactory condition though a dramatic improvement is occurred in the provision of waste collection accessories. The table 2 represents that the waste collection facilities including rubbish bins and landfill sites are increased over the last 20 years in RCC area. But these are a crude dumping sites and not engineered or sanitary. No soil cover is provided at the landfill

sites hence the environmental pollution is increasing day by day that pose to health related hazards. There is no enforcement rule for the effective management of solid waste in Rajshahi City Corporation that leads to dumping wastes here and there. In addition, RCC does not follow any recycling and waste segregation practice. Therefore, the unhygienic waste collection and dumping facilities is deteriorating the environment significantly.

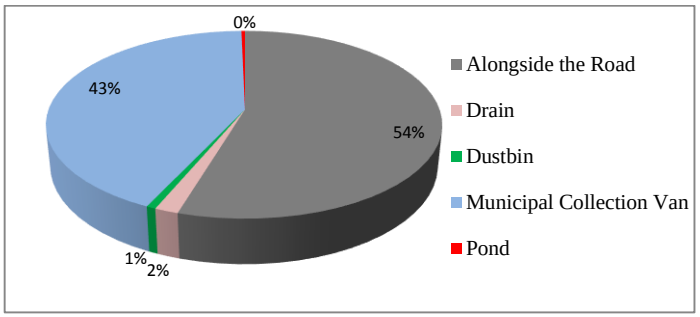

Fig. 2. Waste dumping points used by the citizens of RCC

Table 2. Solid Waste Collection facilities in RCC area

Facilities	**Quantities in Different Years**	
	1994	**2014**
Rubbish bins	325	1050
Push carts	30	150
Rickshaw van	20	228
Trailer	1	3
Garbage Truck	7	12
Sweepers	372	1200
Landfill sites	1acres	12 acres

3.2 Impacts of Open Air Solid Waste Disposal on the Environment

Improper management of solid waste is one of the main causes of environmental pollution and degradation in many cities. Deteriorating soil quality and decrease in vegetation abundance are grave consequences of open waste dumping which have resulted in growing public concern. The menace of environmental pollution has been haunting the human world since early times and is still growing due to excessive growth in developing countries. Solid waste disposal possesses a greater problem as it leads to land pollution because of open dumping, water pollution because of dumping in low lands and air pollution because of burning. The decomposition of waste into constituent chemicals is a common source of local environmental pollution. This problem is especially acute in developing nations. Very few existing landfills in the world's poorest countries meet environmental standards. The problem is again compounded by the issues associated with rapid urbanization. A major environmental concern is gas release by decomposing of garbage. Methane is a by-product of the anaerobic respiration of bacteria, and these bacteria thrive in landfills with high amounts of moisture. Methane concentrations can reach up to 50% of the composition of landfill gas at maximum anaerobic decomposition [8]. A second problem with these gasses is their contribution to the enhanced greenhouse gas effect and climate change. Liquid leachate management varies throughout the landfills of the developing world. Leachate poses a threat to local land surface and ground water systems.

3.3 Problems of Waste Management within the Study Area

Bangladesh is a developing country with a large number of populations. The major cities of Bangladesh are becoming over populated due to rapid urbanization. The city corporations and the city development authorities are unable to provide the basic municipal services and a very little consideration is given to waste management. Wastes are always considered as unwanted and burden to human life and never considered its prospects and economic benefits. Beyond this, the city corporations lack adequate resources and finance to facilitate its functions and activities smoothly. Unlike other City Corporations, the RCC also faces a lot of problems regarding the effective waste management.

3.3.1 Financial

Though Rajshahi is one of the prominent cities in the northern region of Bangladesh still it is an agrarian city and lacks of adequate industrial and commercial activities. Industrialization and commercialization accelerates to generate revenue of the local authorities that in turns help to develop the municipal services. As the rate of industrialization is very sluggish in RCC area therefore it leads to low revenue generation. Lack of finance and dependency on the central government for fund are the major impediments in the proper management of solid wastes.

3.3.2 Institutional

The RCC lacks skilled and adequate manpower to handle the increasing volume of wastes. The authority has no monitoring cell at the field level therefore the workers are indifferent to carry out their responsibilities. In addition, lack of training, modern office and equipments are also the impediments in the effective management of wastes. There is also no installed weighbridge to automatically calculate the amount of daily wastes.

3.3.3 Management

In RCC area it is a common scenario to dump and throw waste here and there openly. In these consequences RCC fails to impose fine as there are no strict rules and regulations. There is no regular record keeping format for generated wastes and no policy to encourage recycling practice. It lacks synchronization and segregation between primary and secondary collection system and lacking of proper handling rules and standard.

3.3.4 Others

RCC is one of the key local government institutions in Rajshahi that works independently without cooperation and coordination with other organizations, NGO's and the local peoples. There is insufficient public education and community participation about waste management. In many cases dustbins are provided but not in appropriate locations and these are not attractive that's why people dump waste outside the dustbins that in turns pollutes the environment and creates a number of health problems.

3.4 Recommendations for Effective Management of Waste

Proper solid waste management have to be undertaken to ensure that it does not affect the environment and does not cause health hazards to the people living there. At the household-level proper segregation of waste has to be done and it should be ensured that all organic matter is kept aside for composting, which is undoubtedly the best method for the correct disposal of this segment of the waste. Including the above all the effective management of waste is only possible if the structural, management and behavioral issues are considered and promoted collectively.

3.4.1 Provision of Structural Facilities

To ensure effective management of solid wastes first of all the structural facilities should be extended in terms of dustbin coverage, collection points and collection vehicles. Attractive and innovative dustbins should be provided to motivate the peoples for using these. Ensure scientific collection practices by providing sufficient equipment, manpower and other resources. To minimize recycling cost separate dustbins should be provided at the dumping sites and transfer stations need to be provided to transfer waste from the smaller collection vehicle to larger transport.

3.4.2 Adopt Appropriate Management Policy and Proposals

RCC is weak in terms of management and imposition of strict rules and regulations. Therefore it should adopt appropriate management policy and proposals to enrich the conservancy department of RCC. Strengthen the managerial efforts by providing modern equipments; forming special monitoring cell; appointing skilled and sufficient man powers; and organizing training programs at regular basis. In addition to these the billboards can be set to encourage people for using dustbins.

The strict rules and regulations should be formed at the national and local level. Peoples violating the rules will be fined and the fine will be multiplied by 2 times if a person is captured for free dumping wastes for 3 times or more. A citizen who is given penalty his name should be announced publicly, so that he may feel shy to violate the waste management rules in the next time.

3.4.3 Change the Behavioral Aspects of the Citizens

To motivate the local peoples for using dustbins and fixed disposal sites the RCC should ensure public participation. The RCC should organize community based meeting to increase public awareness regarding proper management of waste. The community meeting will also help to understand the negative impact of unauthorized dumping. To change the behavioral aspects of the citizens - attractive dustbins should be provided to encourage people using these; a best community can be declared yearly on the basis of proper waste management; in the provision of utility facilities the declared best clean community will be given priority from RCC so that other communities are motivated by it; the civic taxes and other charges might be reduced for the citizens who keep clean the surroundings of their home.

3.4.4 Adopt Appropriate Techniques of Waste Management

In today's world due to rapid industrialization, population is on the rise therefore the creation of large volume of waste material is a common phenomenon. Wastes are not always harmful to the environment. Its harmfulness depends on how these are disposed off. Proper disposal of waste material helps to keep the environment free from disease causing pathogens and keeps it green. The methods described below should be followed for a sustainable and clean environment (Fig. 3).

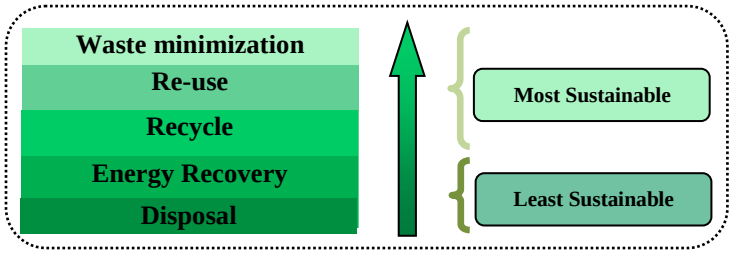

Fig. 3. Sustainable Waste Management Practices

Recycle

Recycling is one of the most well know method of waste management. It is not expensive and can be easily done by anyone. Practice of recycling will save lot of energy and resources that in turn reduces pollution. The recycling techniques should be adopted to minimize the negative environmental impacts. Solid waste recycle can eliminate air, water and land pollution. Ammonia, carbon dioxide, carbon monoxide, methane are produced from solid wastes and creates huge pollution. In Bangladesh maximum rivers and water bodies are polluted by industrial wastes. In same way land and air also become polluted. Therefore practice of recycling should be encouraged that not only will improve the environment but also the recycling practice will generate income for the poor.

Composting

This is a natural process that is completely free of any hazardous by-products. This process involves breaking down the materials into organic compounds that can be used as manure. Using leaves, grass, twigs and add vegetable and fruit peels and skins anyone can practice composting in their backyard. People should encourage composting as a rich nutrients to improve the soil of their garden.

Reduce

It reduces or prevents green house gas emissions, reduce the release of pollutants, conserve resources, save energy and reduce the demand for waste treatment technology and landfill space.

Landfill

Amongst the many waste management methods, using a landfill is probably the most practiced in more cities of the world than any other method. Landfills are often old and abandoned quarries and mining areas, considered as the most cost-effective way of waste disposal. The waste is layered in thin spreads and then compacted, with a layer of clean earth covering the waste material before more layers are added over time.

Reuse

The most important strategy is to reuse, that is when an item is cleaned and the materials are used again. There are two main ways that the concept of reusing can be applied to reduce waste. First, when purchasing a new item, people can look for a product that can be used repeatedly instead of a version that is only used once and thrown away. The second way to reuse is to buy an item secondhand, borrow, or rent an item, instead of buying the product new. There are many ways that an individual can reuse items.

Some common examples include shopping at thrift stores or yard sales for second hand items. People can also donate items that are no longer need to thrift stores so that someone else can use them. Another common method of reuse is to bring own reusable shopping bags instead of using plastic or paper bags provided at the store.

3.4.5 Other Recommendations

Publicity of waste management practices through distributing leaflets, posters and mass media support. Spontaneous participation and involvement of government, households, service holders, students, day labor, businessperson, etc should be ensured to manage and dispose solid wastes properly in order to maintain clean and healthy environment.

4. Conclusion

The solid waste management in Rajshahi City Corporation area appeared to be inadequate and it should be improved. The solid waste should be disposed off scientifically through sanitary landfill and recycle. Segregation of recyclable material would also leads to reduce the quantity for final disposal. Higher priority needs to be assigned to manage municipal solid waste by the local authority. A systematic approach needs to be adopted for optimizing the entire operation of solid waste management encompassing segregation at source, timely and proper collection, transportation and proper operation of sanitary landfill site.

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Qualitative behavior of the discrete equilibrium solutions of the Boltzmann equation with respect to the parameters

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Abstract

The nonlinear Boltzmann equationdescribes theevolution of molecules of rarefied gases in which the mean free-path travelled by a molecule between two subsequent collisions is notnegligible compared tothe structure considered.For $f = f(t, x, v)$, *a non-negative density function depending on the variables,time* $t \in \mathbb{R}$, $t \geq 0$, *the molecularvelocity* $\boldsymbol{v} \in \mathbb{R}^d, d = \{2,3\}$, *and thespace* $x \in \mathbb{R}^m, 1 \leq m \leq d$ *the nonlinear Boltzmann equation is given by* $(\partial t + \boldsymbol{v}.\nabla x)f(t,x,v) = \boldsymbol{J}[\boldsymbol{f},\boldsymbol{f}]$. *Where* $\boldsymbol{J}[\boldsymbol{f},\boldsymbol{f}]$ *is a* $(2d-1)$*-fold integral known as Boltzmann collision operator. The discrete velocity model (DVM) as a deterministic method in which thevelocities of molecules are confined to a finite set vectors has been used for solvingthe Boltzmann equation. The DVM approximates the* $(2d-1)$*-fold collision integral on a discrete lattice inthe velocity space. In this paper, we describe the qualitative behavior of the discrete equilibrium solutions of the Boltzmann equationwith respect to the parametersfor the generalized N-layer hexagonalgrid* $\mathcal{G}_N = \boldsymbol{c} + h \cdot \left(sin\left(\frac{2\pi}{6}(k-0.5)\right),\ cos\left(\frac{2\pi}{6}(k-0.5)\right)\right)_{k=1}^{6}$. *The equilibria* $f \in \mathcal{E}$ *of the discrete Boltzmann equation is described by four parameters characterizing mass, (x,y)-momenta and kinetic energy.*

Keywords: Boltzmann equation,Equilibria(equilibrium Solution), Discrete Velocity Model(DVM) and Numerical Simulations.

1. Introduction:

In this paper**, w**e discuss some aspects of the discrete equilibrium solution of the Boltzmann equation of kinetic theory of gases. The discrete equilibrium solutions(eqilibria) of the Boltzmann equation is based on discrete velocity model(DVM) approximation carried on meshes of hexagonal grids.The equilibria of the discrete Boltzmann equation can be expressed in terms of four parameters characterizing mass, (x,y)-momenta and kinetic energy. We construct necessary algorithm for the computation of the equlibria and perform numerical simulation for a 10-layer hexagonal grid in $\mathbb{R}^2$.we derive the discrete equilibrium solutions (equilibria) for the generalized *N*-layer hexagonal grid by induction method as established in [1.5]. The equilibria $f \in \mathcal{E}$ of the discrete Boltzmann equation is described by four parameters characterizing mass, (*x*,*y*)-momenta and kinetic energy. Subsequently, we present some estimations on the discrete equilibria of the Boltzmann equation with respect to the parameters. We also represent subsequent numerical simulations of the equilibrium solution.

2. Boltzmann equation

We present some estimations on the discrete equilibria of the Boltzmann equation with respect to the parameters.For $f = f(t, x, v)$, a non-negative density function depending on the variable,time $t \in \mathbb{R}$, $t \geq 0$, the molecular velocity $\mathbf{v} \in \mathbb{R}^d, d = \{2,3\}$,and the space $x \in \mathbb{R}^m, 1 \leq m \leq d$ the nonlinear Boltzmann equation is given by $(\partial t + \mathbf{v}.\nabla x)f(t,x,v) = \mathbf{J}[\mathbf{f},\mathbf{f}]$ (2.1)

Where $\mathbf{J}[\mathbf{f},\mathbf{f}] := \int_{\mathbb{R}^{\mathbf{d}}} \int_{S^{d-1}} k(\mathbf{v}-\mathbf{w},\eta)[\mathbf{f}(\mathbf{v}')\mathbf{f}(\mathbf{w}') - \mathbf{f}(\mathbf{v})\mathbf{f}(\mathbf{w})]d^2\eta d^3\mathbf{w}$ (2.2)

is a $(2d-1)$-fold integral known as Boltzmann collision operator. Here $k(.,.)$ is the collision kernel in the operator satisfying some symmetry properties, the post collision velocities $\mathbf{v}', \mathbf{w}'$result from the pre-collision velocities $\mathbf{v}, \mathbf{w}$ satisfying the collision relations, conservation of momentum

$$\mathbf{v} + \mathbf{w} = \mathbf{v}' + \mathbf{w}' \quad (2.3)$$

conservation of kinetic energy $|\mathbf{v}|^2 + |\mathbf{w}|^2 = |\mathbf{v}'|^2 + |\mathbf{w}'|^2$ (2.4)

3. AN-layer hexagonal model

Fig. 1 shows a 54-velocity model (as a regular collision model defined in [2]) constructed by adding two-

layers of regular basic hexagons centering to a central one and thus called a two-layer model. Similarly by adding one more layer of regular basic hexagons, one can obtain a 3-layer model and so on. In general, we may call such models the *N*-layer model which can be divided into six symmetric partition as shown in the Fig. 1.

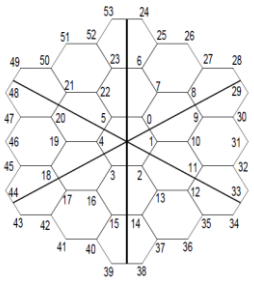

Fig. 1: A 54-velocity model as a two-layer model.

In order to generate a hexagonal mesh for the *N*-layer model, we first collect thecenters of all basic hexagons ordered layer-wise and partition-wise as in algorithm 2.1[3].By algorithm 2.1[3], we obtain the vectors c_x, c_y for the (*x*, *y*)-coordinate of the centers of all regular basic hexagon of the *N*-layer model. The (*x*, *y*)-coordinates of nodes of the model are given by the formula (as stated in (2.1)[3])

$$\mathcal{G}_N = \mathbf{c} + h \cdot \left(sin\left(\frac{2\pi}{6}(k-0.5)\right),\ cos\left(\frac{2\pi}{6}(k-0.5)\right)\right)_{k=1}^{6} \tag{3.1}$$

where *h* is the discretization parameter,
$\mathbf{c} = (c_x, c_y)$ is already obtained by the above algorithm 2.1[8]. By algorithm 2.2[8] we obtain the vectors $G_x, G_y$for the (*x*, *y*)-coordinate of the grid points of the *N*-layer model and plots the hexagonal mesh as seen in Fig. 2.

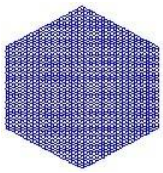

Fig. 2: A 25-layer hexagonal mesh.

Lemma 1:*There are* 6*n number of regular basic hexagons in the n-th*(*n* = 1,…,*N*) *layer of a N-layer grid and the total number of regular basic hexagons of a N-layer grid is given by* 3*N*(*N* + 1) + 1.
Proof: follows lemma 2.11[3].
Lemma 2:*There are* 6(2*n* + 1) *number of nodes in the n-th*(*n* = 0,…,*N*) *layer of a N-layer grid and the total number of nodes in a N-layer grid is given by* $6(N\ +\ 1)^2$.
Proof: follows lemma 2.12[3].

4. Equilibrium solutions

We introduce a function $f : \mathbb{R}^d \to \mathbb{R}$ satisfying $J[f, f] \equiv 0$.It will be evident that the solution of the space homogeneous Boltzmann equation converge to the equilibrium solution.
Definition: Equilibrium solution: A function $f \in L^1(\mathbb{R}^d) \cap C(\mathbb{R}^d)$ with the properties
(a) $f(v) > 0$for all $v \in \mathbb{R}^d$,(b) $\ln(f)J[f, f] \in L^1(\mathbb{R}^d)$
is called equilibrium solution of the Boltzmann equation for which $J[f, f] \equiv 0$.

5. Equilibria for a*N*-layer model

It has been shown in ([2], [3]) that the equilibria$\mathbf{f} \in \mathcal{E}$of the discrete Boltzmann equation can be expressed in terms of four parameters characterizing mass, momenta and energy. In this section we present such equilibrium distribution for a generalized *N*-layer model for any $N \in \mathbb{N}_0$.

Strictly positive density vectors $\mathbf{f} = (f_i)_{i=0}^{\mathbf{6(N+1)^2-1}}$ for which $J[f, f] \equiv 0$ is said to be the equilibrium solutions (equilibria) for a*N*-layer hexagonal model.

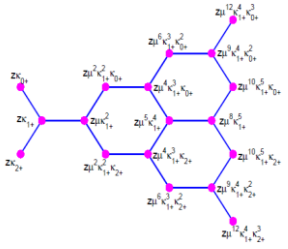

Fig.-3: Equilibria restricted to a partition of a 3-layer model.

The set of equilibria for a *N*-layer hexagonal model is denoted by $\mathcal{E}_N$. Suppose $\mathbf{f} \in \mathcal{E}_N$ be the equilibria of a*N*-layer model and the equilibria at the six nodes of 0-st layer (i.e. at the nodes of the central basic hexagon) is given by$(f_0, f_1, f_2, f_3, f_4, f_5) = z \cdot (k_{0+}, k_{1+}, k_{2+}, k_{0-}, k_{1-}, k_{2-})^{\mathrm{T}}$, where z, $k_{0+}, k_{1+}, k_{2+} > 0$ are arbitrary quantities satisfying $k_{0+}, k_{1-}, k_{2+} = 1$(see prop. 3.3 [2]).For a 3-layer model, the Fig.-3. presents the equilibria for the nodes of the partition corresponding to the triple $z \cdot (k_{0+}, k_{1+}, k_{2+})$.
The values of the equilibria are calculated in a similar way as in Theorem 4.1 in [2], for the layer $n = 1,2,3$respectively as$z(\mu^2 k_{1+}k_{0+}, \mu k_{1+}^2,\ \mu^2 k_{1+}^2 k_{2+}) \in$ 1st layer
$z(\mu^6 k_{1+}^3 k_{0+}^2, \mu^4 k_{1+}^3 k_{0+}, \mu^5 k_{1+}^4, \mu^4 k_{1+}^3 k_{2+}, \mu^6 k_{1+}^3 k_{2+}^2) \in$ 2nd layer
$z(\mu^{12} k_{1+}^4 k_{0+}^3, \mu^9 k_{1+}^4 k_{0+}^2, \mu^{10} k_{1+}^5 k_{0+}, \mu^8 k_{1+}^5, \mu^{10} k_{1+}^5 k_{2+}, \mu^9 k_{1+}^4 k_{2+}^2, \mu^{12} k_{1+}^4 k_{2+}^3) \in 3rd$ layer

wherez parameterizes mass, (k_{0+}, k_{1-}, k_{2+}) characterize non-vanishing bulk-velocity, and μ is responsible for kinetic energy. At each *n*-th layer of a partition we have $(2n + 1)$ nodes and the node numbering is from the top to bottom of at each layer. We generalize these values of equilibria for a partition of a*N*-layer model as in the proposition below.

Proposition 1:*For a partition (of a N-layer model) corresponding to the triple* $z(k_{0+}, k_{1+}, k_{2+})$, *the equilibria is described in-terms of the parameters* $\mu, k_{0+}, k_{1+}, k_{2+}$*as in the following three steps.*

1. Corresponding to the values at the first node (the top one in the figure) of the $(n-1)th$layer (n = 2,…,*N*), there obtained two values of equilibria with increments $k_{1+}\mu^n$ and $k_{0+}k_{1+}\mu^{2n}$ which are assigned respectively to the second and first nodes of the n-st layer. Corresponding to the values at the last node (the bottom one in the figure) of the $(n-1)$-th layer, there obtained also two values of equilibria with increments $k_{1+}\mu^n$ and $k_{0+}k_{1+}\mu^{2n}$ which are assigned respectively to the 2n-th and (2n + 1)-th nodes of the nth layer.
2. Corresponding to the values of each 2m-th (even) node $(\mathrm{m} = 1, \dots, \mathrm{n}-1)$of the$(n-1)$-th layer, there obtained values at the*(2m + 1)*-th node of the *n*-th layer with an increment$k_{1+}^2\mu^{2n}$
3. Corresponding to each (2*l* +1)-*th* (odd) node $(l = 1, \dots, n-2)$of the $(n-1)$-th layer, there obtained values of equilibria with an increment$k_{1+}\mu^n$ which is assigned to the (2*l*+ 2)-th node of the nth layer.

The equilibria for other partitions as well as for the complete *N*-layer model is determined by symmetry.

Let the equilibria at the *i*th$(i = 1, \dots, 2n + 1)$ node of the *n*th $(n = 0, \dots, N)$ layer is given by

$$f(n,i) = z\mu^{m(n,i)}k_{0+}^{k_0(n,i)}k_{1+}^{k_1(n,i)}k_{2+}^{k_2(n,i)};\ (m, k_0, k_1, k_2) \in \mathbb{N}_0 \qquad (5.1)$$

Then following the statement of the proposition 3.1, one can calculate the exponents $m(n,i),\ k_0(n,i),\ k_1(n,i),\ k_2(n,i);\ n = 0, \dots, N;\ i = 1, \dots, 2n + 1$ as shown in Algorithm

INITIALIZE *m*(0,1) = 0, *m*(1,1) = 2, *m*(1,2) = 1, *m*(1, 3) = 2 FOR *n* = 2 TO *N* *m*(*n*, 1) = *m*(*n* −1,1) + 2*n* FOR *i*= 2(2)2*n* *m*(*n*, *i*) = *m*(*n*−1,*i*−1) + *n* END FOR *j* = 3(2)(2*n*−1) *m*(*n*, *j*) = *m*(*n*−1, *j*−1) + 2*n* END *m*(*n*, 2*n* + 1) = *m*(*n*−1,2*n*−1) + 2*n* END	INITIALIZE $k_0(0,1) = 1, k_0(1;\ 1) = 1$ FOR *n* = 2 TO *N* $k_0(n,1) = k_0(n-1,1) + 1$ FOR *i*= 2 TO *n* $k_0(n,i) = k_0(n-1,i-1)$ END FOR *i*= *n* + 1 TO 2*n* + 1 $k_0(n,i) = 0$ END END
INITIALIZE $k_1(1,1) = 2, k_1(1,2) = 2,$ $k_1(1,2) = 2$ FOR *n* = 2 TO *N* $k_1(n,1) = k_1(n-1,1) + 1$ FOR *i*= 2(2)2*n* $k_1(n,i) = k_1(n-1,i-1) + 1$ END FOR *j* = 3(2)(2*n*−1) $k_1(n,j) = k_1(n-1,j-1) + 2$ END $k_1(n,2n+1) = k_1(n-1,2n-1) + 1$ END	INITIALIZE $k_2(0,1) = 1, k_2(1,3) = 1$ FOR *n* = 2 TO *N* $k_2(n,2n+1) = k_2(n-1,2n-1) + 1$ FOR *i*= *n* + 2 TO 2*n* $k_2(n,i) = k_2(n-1,i-1)$ END FOR *i*= 1 TO *n* + 1 $k_2(n,i) = 0$ END END

Algorithm: To calculate the exponents $m(n,i),\ k_0(n,i),\ k_1(n,i),\ k_2(n,i)$.

Theorem 1. Let $\mathbb{N}_o, \mathbb{N}_e$ denote respectively the set of odd and even natural numbers. The *i*-thequilibria in the n-th layer of the partition corresponding to the triple $(k_{0+}, k_{1+}, k_{2+})$is given by

$$f(n,i) = z\mu^{m(n,i)}k_{0+}^{\bar{k}_0(n,i)}k_{2+}^{\bar{k}_2(n,i)};\ n = 0, \dots, N;\ i = 1, \dots, 2n + 1 \qquad (5.2)$$

where for $i = 1, \dots, n + 1,$

$$m(n,i) = n^2 + n - d_i\ for\ i \in \mathbb{N}_o d_{i=2k+1} = nk - k^2,$$

$$k = 0, \dots, \frac{n}{2}\ \ if\ \ \mathrm{n} \in \mathbb{N}_e\ and\ k = 0, \dots, \frac{n-1}{2}\ \ if\ \ \mathrm{n} \in \mathbb{N}_o,$$

$$= n^2 - d_i\ for\ i \in \mathbb{N}_e d_{i=2k+2} = nk - k(k+1),$$

$$k = 0, \dots, \frac{n-2}{2}\ \ if\ \ \mathrm{n} \in \mathbb{N}_e\ and\ k = 0, \dots, \frac{n-1}{2}\ \ if\ \ \mathrm{n} \in \mathbb{N}_o,$$

$$\bar{k}_0(n,i) = 2n + 1 - d_i\ for\ i \in \mathbb{N}_o d_{i=2k+1} = k,$$

$$k = 0,\dots,\frac{n}{2} \quad if \quad n \in \mathbb{N}_e \ and \ k = 0,\dots,\frac{n-1}{2} \quad if \quad n \in \mathbb{N}_o,$$
$$= 2n - d_i \ for\ i \in \mathbb{N}_e d_{i=2k+2} = k,$$
$$k = 0,\dots,\frac{n-2}{2} \quad if \quad n \in \mathbb{N}_e \ and \ k = 0,\dots,\frac{n-1}{2} \quad if \quad n \in \mathbb{N}_o,$$
$$\bar{k}_2(n,i) = n + 1 + d_i \ for\ i \in \mathbb{N}_o d_{i=2k+1} = k,$$
$$k = 0,\dots,\frac{n}{2} \quad if \quad n \in \mathbb{N}_e \ and \ k = 0,\dots,\frac{n-1}{2} \quad if \quad n \in \mathbb{N}_o,$$
$$= n + 1 - d_i \ for\ i \in \mathbb{N}_e d_{i=2k+2} = k,$$
$$k = 0,\dots,\frac{n-2}{2} \quad if \quad n \in \mathbb{N}_e \ and \ k = 0,\dots,\frac{n-1}{2} \quad if \quad n \in \mathbb{N}_o,$$
$$For\ the\ rest\ i = n+2,\dots,2n+1,$$
$$m\left(n, i_{(=n+2,\dots,2n+1)}\right) = m\left(n, i_{(=n,\dots,1)}\right) \ respectively,$$
$$\bar{k}_0\left(n, i_{(=n+2,\dots,2n+1)}\right) = \bar{k}_2\left(n, i_{(=n,\dots,1)}\right) \ respectively,$$
$$\bar{k}_2\left(n, i_{(=n+2,\dots,2n+1)}\right) = \bar{k}_0\left(n, i_{(=n,\dots,1)}\right) \ respectively,$$

Proof: By lemma-3.2 [2],

Corollary 1. For a regular collision model $(\mathcal{H}_b, \gamma)$, let $f \in \mathcal{E}$ be the equilibria. If we denote the *i*-th component equilibria as $f_i := z\mu^m k$ and the corresponding $r_1 := \sqrt{3n+1}$, where $r_1^2 = v_{x,i}^2 + v_{y,i}^2$, then m = n.

6. Some estimations of discrete equilibria

From the geometrical construction of the discrete equilibria(Fig. 1 and Fig. 3), one can easily read the following properties at a glance.

$$k_{0+} = 1, k_{2+} = 1 \Rightarrow \bar{v}_x = 0, \bar{v}_y = 0$$
$$k_{0+} = k_{2+} > 1 \Rightarrow \bar{v}_x > 0, \bar{v}_y = 0$$
$$k_{0+} = k_{2+} < 1 \Rightarrow \bar{v}_x < 0, \bar{v}_y = 0$$
$$k_{2+} = \frac{1}{k_{0+}}, k_{0+} > 1 \Rightarrow \bar{v}_x = 0, \bar{v}_y > 0$$
$$k_{2+} = \frac{1}{k_{0+}}, k_{0+} < 1 \Rightarrow \bar{v}_x = 0, \bar{v}_y < 0$$
$$k_{2+} < \frac{1}{k_{0+}}, k_{0+} > 1 \Rightarrow \bar{v}_x < 0, \bar{v}_y > 0$$
$$k_{2+} < \frac{1}{k_{0+}}, k_{0+} < 1 \Rightarrow \bar{v}_x < 0, \bar{v}_y < 0$$
$$k_{2+} > \frac{1}{k_{0+}}, k_{0+} > 1 \Rightarrow \bar{v}_x > 0, \bar{v}_y > 0$$
$$k_{2+} > \frac{1}{k_{0+}}, k_{0+} < 1 \Rightarrow \bar{v}_x > 0, \bar{v}_y < 0$$

In the following, we analyze some estimations of the equilibria given by theorem 1 for some special cases of the parameters.

Case 1: $k_{0+} = k_{2+} =: k$, First we consider $k_{0+} = k_{2+} =: k$ then the n*th* layer, the equilibria restricted to the second partition is given by $f(n,i) = z\mu^{m(n,i)} k_{0+}^{\overline{k_0}(n,i)+\overline{k_2}(n,i)} := z\mu^{m(n,i)} k^{k(n,i)}$ (6.1)

Where$k(n,i) = 3n+2 \ for\ i = 1,3,\dots.,2n+1 = 3n+1 \ for\ i = 2,4,\dots.,2n$

Choosing $\mu \in (0,1), k > 1$:For $\mu \in (0,1)$and $k > 1$, $\overset{max}{i} f(n,i)$ attains for $\overset{mim}{i} m(n,i)$ and $\overset{max}{i} k(n,i)$. Then it is verified that the $\overset{max}{i} f(n,i)$ attains for $i = n, n+1, n+2$.

Let $\mathbb{N}_o$ and $\mathbb{N}_e$ denote the set of odd and even integers respectively. Then for $n \in \mathbb{N}_e$

$$m(n, i = n) = \frac{3}{4}n^2 + \frac{n}{2} = m(n, i = n+2)$$
$$m(n, i = n+1) = \frac{3}{4}n^2 + n$$

Then $f(n, i = n) = z\mu^{\frac{3}{4}n^2+\frac{n}{2}} k^{3n+1} = f(n, i = n+2)$ (6.2)

$f(n, i = n) = z\mu^{\frac{3}{4}n^2+n} k^{3n+2}$ (6.3)

For $n \in \mathbb{N}_o$, we have $f(n, i = n) = z\mu^{\frac{3}{4}n^2+n+\frac{1}{4}} k^{3n+2} = f(n, i = n+2)$ (5.4)

$f(n, i = n+1) = z\mu^{\frac{3}{4}n^2+\frac{n}{2}-\frac{1}{4}} k^{3n+1}$ (6.5)

We choosing now $\mu = 1/k$, then for $n \in \mathbb{N}_e$, equations (5.2) and (5.3) yields

$$f(n, i = n) = zk^{-\frac{3}{4}n^2+\frac{5}{2}n+1} =: zk^{k_o(n)} = f(n, i = n+2)$$
$$f(n, i = n+1) = zk^{-\frac{3}{4}n^2+2n+2} =: zk^{k_e(n)}$$

Now for $\overset{max}{n} f(n,i)$, both $k_o'(n) = 0$, $k_e'(n) = 0$ yields n = 2, and n = 2, $k_o(n) = k_e(n) = 3$

Thus we have three $\overset{max}{i} f(n,i)$ in the n = 2nd layer.Again for $n \in \mathbb{N}_o$, it follows from equations (5.4) and (5.5) that $f(n, i = n) = zk^{-\frac{3}{4}n^2+2n+\frac{7}{4}} =: zk^{k_o(n)} = f(n, i = n+2)$

$$f(n, i = n+1) = zk^{-\frac{3}{4}n^2+\frac{5}{2}n+\frac{5}{4}} =: zk^{k_e(n)}$$

and both $k_o'(n) = 0$, $k_e'(n) = 0$ yields n = 1, and n = 1, $k_o(n) = k_e(n) = 3$
Thus we have three more $\overset{max}{i} f(n,i)$ in the n = 1st layer which are equal to those in the n = 2nd layer. Therefore, the six maximum values attains at the nodes $\big(5 + (2 \times n(=1) + 1) + i(=1,2,3)\big) = (9,10,11)$ and $\big(5 + (2 \times n(=2) + 1) + i(=1,2,3)\big) = (30,31,32)$, where $(30,31,32,11,10,9)$ are the six-tupel nodes of the 2nd regular basic hexagon in the 2nd partition of the 2nd layer(see Fig. 1). Then the six maxima of the equilibria $\overset{max}{i} f(n,i)$ can be obtained from equation (6.1)
Choosing $\mu \in (0,1), k < 1$:
Similarly, if we choose $k < 1$ then for $\mu = k$ we found maximum at six-tupel nodes $(20,19,18,45,46,47)$ of the regular basic hexagon in the 5th partition (which is just the opposite of the 2nd) and in the 2nd layer.We collect the results of this subsection as follows.

Corollary 2 For $k := k_{0+} = k_{2+} > 1$ and $\mu = 1/k$ the equilibria $f(n,i) \in \mathcal{E}_N$ given in theorem 1 has six maximum attained at the six-tiple nodes $(30,31,32,11,10,9)$ of a regular basic hexagon of the *N*-layer model(Fig. 1).

Corollary 3 For $k := k_{0+} = k_{2+} < 1$ and $\mu = k$ the equilibria $f(n,i) \in \mathcal{E}_N$ given in theorem 1 has six maximum attained at the six-tiple nodes $(20,19,18,45,46,47)$ of a regular basic hexagon of the *N*-layer model (Fig. 1).

Case 2: $\boldsymbol{k_{2+} = \frac{1}{k_{0+}}}$,In this case, theorem 1 yields$f(n,i) = z\mu^{m(n,i)} k_{0+}^{-n+i-1}, i = 1, \dots, 2n+1$

$$=: z\mu^{m(n,i)} k_{0+}^{k(n,i)}$$

Then for $\mu \in (0,1)$and $k_{0+} > 1$, $\overset{max}{i} f(n,i)$ attains for the optimal choice of the pair $\big(m(n,i), k(n,i)\big)$.
For $\mu = 1/k_{0+},\ k_{0+} > 1, f(n,i) = z k_{0+}^{-m(n,i)+n-i+1} =: z k_{0+}^{\tilde{k}(n,i)}, i = 1, \dots, 2n+1$
Then for $n \in N_{0,}$ $\overset{max}{i} f(n,i)$ attains for $\overset{max}{i} \tilde{k}(n,i)$ and it is verified that $\overset{max}{i} \tilde{k}(n,i)$ attain for $i = n-1$. Therefore $\overset{max}{i} \tilde{k}(n,i) = -\left(\frac{3}{4}n^2 + \frac{n}{2} + \frac{3}{4}\right) + 2 =: \bar{k}(n), \quad \overset{max}{n} \tilde{k}(n,i) = z k_{0+}^{\bar{k}(n)}$
exists for $\bar{k}'(n) = 0 \Longrightarrow n = -1$ which is impossible. Thus maxima cannot attain in the 2ndpartition in this case.Now we consider the first partition and For$k_{2+} = 1/k_{0+.}$ Then we have from the theorem 1

$$f(n,i) = z\mu^{m(n,i)} k_{2-}^{\bar{k}_0(n,i)} k_{1+}^{\bar{k}_2(n,i)} = z\mu^{m(n,i)} k_{0+}^{\bar{k}_0(n,i)}$$

Now For $\mu = 1/k_{0+},\ k_{0+} > 1, f(n,i) = z k_{0+}^{-m(n,i)+\bar{k}_0(n,i)} =: z k_{0+}^{\bar{k}_0(n,i)}$
Then $\overset{max}{i} f(n,i)$ attains for $\overset{max}{i} \tilde{k}(n,i)$ and it is verified that for $n \in N_{0,}$ $\overset{max}{i} \tilde{k}(n,i)$ attain for $i = n-1, n+1,$ and for both $i = n-1, n+1,$ we find $\tilde{k}(n, i = n-1) = -\frac{3}{4}n^2 + n + \frac{3}{4} =: \bar{k}(n)$
$\overset{max}{n} f(n, i = n-1) = z k_{0+}^{\bar{k}(n)}$ exists for $\bar{k}'(n) = 0 \Longrightarrow n = 1 \Longrightarrow \bar{k}(n) = 1$. But $i = n-1$ doesn't exist for $n = 1$. However, it can be seen that $\tilde{k}(1,1) = 1$, $\tilde{k}(1,2) = 1$, $\tilde{k}(1,0) = 0$. Thus maxima attained for the first two nodes (6,7) in the n =1-st layer of the first partition is this case.Now for $n \in N_{e,}$ $\overset{max}{i} \tilde{k}(n,i)$ attains for $i = n$. Then$\tilde{k}(n, i = n) = -\frac{3}{4}n^2 + n + 1 =: \bar{k}(n)$ and $\overset{max}{n} f(n, i = n) = z k_{0+}^{\bar{k}(n)}$ exists for $\bar{k}'(n) = 0 \Longrightarrow n = 0 \Longrightarrow \bar{k}(n) = 1$, and for $i = 0$ we have only one node and it is first node (0) which belongs to the first partition. As in the both $n \in N_{o,e,} \bar{k}(n,i) = 1$, therefore f has maximum at the three nodes (6, 7, 0). Now if we consider the 6th partition with the similar ansatz and arguments it is verified that f has maximum the three nodes (5, 22, 23) for $k_{2+} = \frac{1}{k_{0+}},\ k_{0+} > 1,\ \mu = 1/k_{0+}$. Thus maximum attained at the six-tupel nodes (6,7, 0,5, 22, 23) of the 2nd hexagon of our model. For $\mu = k_{0+},\ k_{0+} < 1,$ With $k_{2+} = 1/\ k_{0+}$, it is verified that maximum of f attained at the six-tupel nodes (2, 13, 14, 15, 16, 3)of the 5th hexagon of our model which is just in the opposite side of the previous one.We collect the results of this subsection as follows.

Corollary 4 For $\mu = k_{2+} = 1/k_{0+} < 1$ the equilibria $f(n,i) \in \mathcal{E}_N$ given in theorem 1 has six maximum attained at the six-tiple nodes (6, 7, 0, 5, 22, 23) of a regular basic hexagon of the *N*-layer model(Fig. 1).

Corollary 5 For $k_{2+} = 1/k_{0+} > 1$ and $\mu = k_{0+}$ the equilibria $f(n,i) \in \mathcal{E}_N$ given in theorem 1 has six maximum attained at the six-tiple nodes $(2,13,14,15,16,3)$ of a regular basic hexagon of the *N*-layer model(Fig. 1).

Case 3: $\boldsymbol{k_{2+} > \frac{1}{k_{0+}}, k_{0+} > 1}$, We choose the first partition and then $f(n,i) = z\mu^{m(n,i)} k_{2-}^{\bar{k}_0(n,i)} k_{1+}^{\bar{k}_2(n,i)}$
Now for $\mu = \frac{1}{k_{0+}},\ k_{0+} > 1, k_{2+} := \frac{d}{k_{0+}},$ for $d = 1/\mu\ k_{2+} = 1$ and we obtain
$f(n,i) = z\mu^{m(n,i)-\bar{k}_2(n,i)} =: z\mu^{\tilde{k}(n,i)}$. Thus $\overset{max}{i} f(n,i)$ attains for $\overset{mim}{i} \tilde{k}(n,i)$.For $n \in N_{0,}$ it is seen $\overset{min}{i} \tilde{k}(n,i)$ attained for $i = n+1, n+3$. For both $i = n+1, n+3, \tilde{k}(n,i) = \frac{3}{4}n^2 - n - \frac{3}{4} =: \bar{k}_0(n)$,

where $\bar{k}_0'(n) = \frac{3}{2}n - 1 = 0 \Longrightarrow n \equiv 1$ and for $\bar{k}_0(1) = -1$and we see that in the n =1-st layer the two max in this partition are $f(1,2) = f(1,3) = z/\mu$. Thus maxima attain at the nodes (7, 8).
For $n \in N_{e,} \underset{i}{min}\tilde{k}(n,i)$ attained for $i = n + 2$and $\underset{i}{min}\tilde{k}(n,i) = \tilde{k}(n, i = n + 2) = \frac{3}{4}n^2 - n - \frac{3}{4} =: \bar{k}_e(n)$, where $\bar{k}_e'(n) = \frac{3}{2}n - 1 = 0 \Longrightarrow n \equiv 0$ and for $\bar{k}_e(1) = -1$ and in the $n = 0$-st layer the only max is $f(0,1) = zk_{0+} = z/\mu$.Thus maxima attained at the three nodes $(0, 7, 8)$. Choosing the 2nd partition, with the similar ansatz it is verified that maxima attained at $(1, 9, 10)$ and the maximum values is equal to the maximum values at the nodes $(0, 7, 8)$. Thus at the nodes $(8, 9, 10, 1, 0, 7)$ (which are the nodes of the third hexagon) maxima attained in this case.We collect the results of this section as follows.

Corollary 6 For $\mu = 1/k_{0+}$, $k_{0+} > 1, k_{2+} = 1$ the equilibria $f(n,i) \in \mathcal{E}_N$ given in theorem 1 has six maxima attained at the six-tiple nodes $(8, 9, 10, 1, 0, 7)$ of a regular basic hexagon of the *N*-layer model(Fig. 1).

7. Numerical simulation of discrete equilibria

We presents numerical computation of the discrete equilibria (normalized i.e. $\rho = 1$) based on a 10-layer (725-velocity) model. First of all we compute the equilibria for the trivial case $k_{0+} = k_{2+} = 1$with $\mu = 0.25$. In this case we computed zero bulk-velocity$\mathbf{v} = \frac{1}{\rho}\sum_{i=0}^{725} f_i v_i = (0,0)$, Which is just the center of the central regular basic hexagon with six-tupel nodes $(0, 1\ 2, 3, 4, 5)$ and the maxima of the equilibria attains at this six nodes as shown in Figure 4

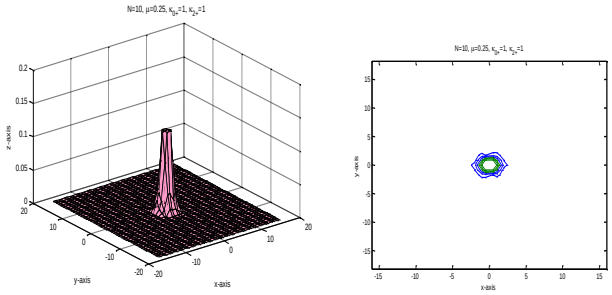

Figure 4: Equilibrium distribution with zero bulk-velocity

We choose case $k_{0+} = k_{2+} = 1$ with $\mu = 0.75$. In this case we computed zero bulk-velocity $\mathbf{v} = \frac{1}{\rho}\sum_{i=0}^{725} f_i v_i = (0,0)$ as shown in the Figure 5.

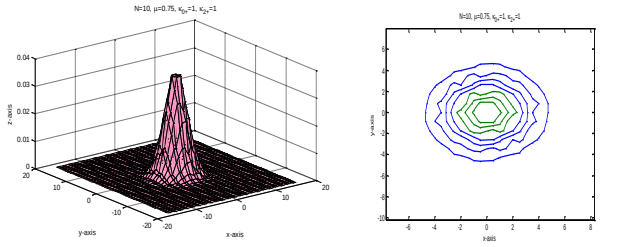

Figure 5: Equilibrium distribution with zero bulk-velocity

8. Conclusion

We have performed numerical simulations of discrete equilibriadistribution of a model Boltzmann Equation based on a hexagonal grid in $\mathbb{R}^2$. The results show the effects of parameters on temperature and bulk-velocity. The analytic results of error estimation and convergence can be investigated as a future work.

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A Study on the Effect of Textile Effluents and Best Effective Effluent Treatment Plant in Bangladesh Textile Industry

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Abstract

Textile Effluents are unavoidable in most of the Textile Production. Despite having proper treatment of these effluents many people are severely affected by cancer, skin diseases, allergy, and respiratory problems. It also has adverse impact on animal, fish, plant growth and soil chemistry. Present study aimed to find more cost effective and eco safe treatments of textile effluents. In this study we report a comparison and analyze the performance between different ETP methods i.e. Physico- chemical, biological activated sludge method, combined physyco-chemical and biological method according to many testing parameters. We find Biological treatment method as best among them in economic, efficiency and ecologic perspective. Biological effluent treatment plant (ETP) optimizes effluent generation and toxicity of dye house. It needs less chemical cost as well as total running cost. We find that only Biological ETP can satisfy discharging standard as owners are very much interested to run it 24 hours in 365 days for additional economic benefits.

Keywords: Textile Effluents, ETP, Testing parameters, Discharging standard, Economic benefits.

"1. Introduction"

Textile Effluents are the stream of excess chemical liquor extracted from textile industry after using in original operations like pretreatments, dyeing and finishing. The risk factors of textile effluents are primarily associated with the wet processes - scouring, desizing, mercerizing, bleaching, dyeing and finishing that produce large quantities of wastewater. The major chemical pollutants present on textiles are dyes containing carcinogenic amines, toxic heavy metals, pentachlorophenol, chlorine bleaching, halogen carriers, free formaldehyde, biocides, fire retardants, and softeners. Heavy metals used as oxidizing agents, as metal complex dyes, dye stripping agents, fastness improvers, and finishers (water repellents, flame retardants, anti-fungal and odor preventive agents) are not only poisonous to humans but also found toxic to aquatic life (WHO, 2002) and they may result in food contamination (Novick, 1999). Dyeing process usually contributes chromium, lead, zinc and copper to wastewater. Copper is toxic to aquatic plants at concentrations below 1.0 mg/l while concentration nears this level can be toxic to some fish (Sawyer and McCarty, 1978). Studies indicate that effluents have harmful effects on a wide variety of aquatic organisms. Table 1 represents the effluent characteristics from textile industry. Experiment shows that the presence of metals and other dye compounds in textile effluents inhibit microbial activity, damages of organs, disorders in the respiratory tract and lung diseases, dysfunction of the heart and blood producing organs, disorders in the nervous system, skin diseases, and abnormalities in fertility and pregnancy are reported. Contaminated air, soil, and water by effluents from the industries are associated with heavy disease burden (WHO, 2002) and this could be part of the reasons for the current shorter life expectancy in the country, (WHO, 2003) when compared to the developed nations.

Effluent can be treated in a number of different ways depending on the level of treatment required. These levels are known as preliminary, primary, secondary and tertiary (or advanced). The mechanisms for treatment can be divided into three broad categories: physical, chemical and biological, which all include a number of different processes like Physico- chemical, biological activated sludge method, combined physico-chemical and biological method.

Several pollutants in Textile effluent can be removed with the help of an effluent treatment plant (ETP). Effluent from textile dyeing industries must meet the national effluent discharge quality standards set by the Government of Bangladesh, including the "Quality Standards for Classified Industries" (Tables 2 and 3), and may also need to meet additional standards set by international textile buyers. Consequently any ETP must be designed and operated in such a way that it treats the wastewater to these standards. Some

others information are needed for planning an ETP. They are volume of the effluent, chemical cost & concentration, any plan to increase production, if any increase to the amount of effluent to be treated, affordability to spend on constructing & running ETP, availability of the land for ETP, ETP expert or designer, best suited plant, capacity in the factory to manage ETP, requirement of hiring and training staff. Present research will help textile industry personnel to select the most suitable, profitable and eco-friendly ETP considering above factors.

Table 1. Effluent characteristics from textile industry

Process	Effluent Composition	Nature of pollution
Sizing	Starch, waxes, Carboxymethyl cellulose (CMC), Polyvinyl alcohol (PVA), wetting agents.	High in BOD, COD
Desizing	Starch, CMC, PVA, fats, waxes, pectin.	High in BOD, COD, SS, dissolved solids (DS)
Bleaching	Sodium hypochlorite, C12, NaOH, H_20_2, acids, Surfactants, $NaSi0_3$, sodium phosphate, short cotton fiber.	High alkalinity, high SS
Mercerizing	Sodium Hydroxide, Cotton wax	High pH, low BOD, high DS
Dyeing	Dyestuffs urea, reducing agents, oxidizing agents, Acetic acid, detergents, wetting agents.	Strongly coloured, high BOD, DS, low SS, heavy Metals,
Printing	Pastes, urea, starches, gums, oils, binders, acids, thickeners, cross-linkers, reducing agents, alkali.	Highly coloured, high BOD, oily appearance, SS slightly alkaline, low BOD

Source: AEPA (Australian Environmental Protection Authority, 1998).

Table 2. National Standards - Waste Discharge Quality Standards for Industrial Units and Projects (quality standard at discharge point)

Parameter	Unit	Inland surface water	Public sewer secondary treatment plant	Irrigated land
Ammoniacal Nitrogen(N molecule)	mg/l	50	75	75
Ammonia(free ammonia)	mg/l	5	5	15
Arsenic	mg/l	.2	.5	.2
$BOD_5$200C	mg/l	50	250	100
Boron(B)	mg/l	2	2	2
Cadmium(Cd)	mg/l	.005	.5	.5
Chloride(Cl^-)	mg/l	600	600	600
Chromium	mg/l	.5	1	1
COD	mg/l	200	400	400
Copper(CU)	mg/l	.5	3	3
Dissolved Oxygen(DO)	mg/l	4.5-8	4.5-8	4.5-8
Electrical Conductivity		1200	1200	1200
Total Dissolved Solids(TDS)	mg/l	2100	2100	2100
Fluoride(F)	mg/l	7	15	10
Sulfide(S)	mg/l	1	2	2
Iron(Fe)	mg/l	2	2	2
Lead(Pb)	mg/l	.1	.1	.1
Manganese(Mn)	mg/l	5	5	5
Mercury(Hg)	mg/l	.001	.001	.001
Nickel(Ni)	mg/l	1	1	1
Nitrate(N molecule)	mg/l	10	undermined	10
Oil and Grease	mg/l	10	20	10
Phenol Compounds(C_2H_5OH)	mg/l	1	5	1
Dissolved Phosphorous(P)	mg/l	8	8	10
Radioactive materials	As determined by Bangladesh Atomic Energy Commission			

P^H	mg/l	6-9	6-9	6-9
Zn	mg/l	5	10	10
Temperature	Centigrade			
Summer	mg/l	40	40	40
Winter	mg/l	45	45	45
Total Suspended Solid(TSS)	mg/l	150	500	100
Cyanide(CN)	mg/l	.1	2	

Table 3. Discharge Quality Standard for Classified Industries - Composite Textile Plant and Large Processing Units (investment over Tk 30,000,000)

Parameter	**Limit (mg/l)**
Total Suspended Solid (TSS)	100
BOD_5 20° C	150*
Oil and Grease	10
Total Dissolved Solid (TDS)	2100
Waste Water Flow	100 l/kg of fabric processing
Ph	6.5-9
Special parameters based on classification of dyes used	
Total Chromium (as Cr molecule)	2
Sulfide (as S molecule)	2
Phenolic compounds as C_6H_5OH	5

* BOD limit of 150 mg/l will be applicable only for physico-chemical processing method.

"2. Methodology"

In this study we have collected information about the harmful effects of textile effluents from several journals, books, publications and investigating from different textile factories and their surrounding inhabitants. In addition, we have also compared and analyzed the performance of various types of effluent treatment plants by collecting information from some textile factory in Bangladesh where ETP is running. Our findings about different types of ETP are presented here which will help us to decide the most suitable type of ETP for Bangladeshi Textile Industry.

"Effluent Treatment Methods"

Table 4. Wastewater Treatment Levels, Mechanism, and Processes

Treatment level	**Description**	**Process**
Preliminary	Removal of large solids such as rags, sticks, grit and grease that may damage equipment or result in operational problems.	Physical
Primary	Removal of floating and settle able materials such as suspended solids or organic matter.	Physical and chemical
Secondary	Removal of biodegradable organic matter and suspended solids	Biological and chemical
Tertiary/advanced	Removal of residual suspended/Dissolved solids	Physical biological and chemical

"Biological Treatment Processes"

The basic units needed for biological treatment are: screening; an equalization unit; a pH control unit; an aeration unit; and a settling unit. A sludge dewatering unit may also be included. Biological treatment plants require the presence of microorganisms that are adapted to degrade the components of the effluent to be treated.

Textile industry waste will not contain suitable microorganisms so these must be added to the ETP when it is set up. Traditionally in Bangladesh cow dung is used as a source of microorganisms. Evidence shows that output quality from biological treatment can satisfy the national standards for most of the required parameters except colour. A properly designed biological ETP can efficiently satisfy BOD, pH, TSS, oil and grease requirements (Metcalf & Eddy, 2003). A sludge recycle line is essential for activated sludge systems but is not needed for fixed film systems. The aeration unit can be either activated sludge or a fixed film reactor.

"Physico-chemical Treatment Plant"

The basic units needed for a stand-alone physico-chemical treatment plant are screening, an equalization unit, a pH control unit, chemical storage tanks, a mixing unit, a flocculation unit, a settling unit and a

sludge dewatering unit. With physico-chemical treatments generally used in Bangladesh (coagulation and flocculation) it is possible to remove much, possibly all of the colours depending on the process used. It is however difficult to reduce BOD and COD to the value needed to meet the national effluent discharge standard, and impossible to remove TDS. The removal rate is dependent on the influent wastewater quality. The removal efficiency of this type of treatment has been found to be 50% and 70% for BOD_5 and COD respectively.

"Physico-chemical and Biological Treatment"

In this type of treatment a combination of physical operations, and physico-chemical and biological processes are used. The basic units needed for a physico-chemical and biological treatment plant are screening, an equalization unit, a pH control unit, chemical storage tanks, mixing units, flocculation units, a primary settling unit, an aeration unit, and a secondary settling unit. The physico-chemical unit always comes before the biological unit. A sludge recycle line is essential for activated sludge systems but is not needed for fixed film systems. The aeration unit can be either activated sludge or a fixed film reactor.

"3. Result and Discussion"

"Cost Comparison"

The installation costs of ETPs can vary greatly depending on such factors as the materials used, including the quality and source of the equipment (e.g. pumps and air blowers), and dimensions for construction, the quality and quantity of wastewater to be treated, and the quality of the required output. In addition, the operating costs of ETPs can also vary greatly depending on quality and quantity of inputs such as chemicals, the efficiency and size of motors and therefore the energy required the method of treatment and the efficiency of ETP management.

Biological plant incurs 12 times less chemical cost than other plants. Generally in combined method-1 chemical treatment (coagulation & flocculation) is done before biological treatment. By modifying (first biological then chemical treatment) i.e. in combined method-2 running cost per m^3 is 28% reduced due to less chemical cost, sludge treatment and disposal cost (Courtesy: Interstoff Apparels Ltd).

Table 5. Chemical Consumption of different ETP

Process	Peak flow m^3/hr	Chemicals	Dosing Rate kg/day	Consump-tion kg/ m^3	Price tk/kg	Cost tk/m^3	Total tk/m^3
Physico-chemical	65	Lime	600-650	0.38-0.42	10-12	3.86-5.04	15.5-22.5
		$FeSO_4$	1000-1200	0.64-0.77	14-16	8.97-12.32	
		Polyelectrolyte	8-10	0.005-0.01	260-280	1.33-2.80	
		H_2SO_4	250-300	0.16-0.19	8-12	1.28-2.28	
Biological	60	H_2SO_4 (98%)	150-200	0.10-0.139	8-12	0.83-1.6	1.5-2.0
		Polyelectrolyte	1.5-2	0.001-0.0013	260-300	0.27-0.36	
		Antifoam	0ccasional	-	200-250	-	
		Decolorant	0ccasional	-	95-100	-	
		Nutrient	0ccasional	-	150-300	-	
Combined chemical-biological -1	55	Lime	650-800	0.49-0.61	10-12	4.92-7.32	17-25
		$FeSO_4$	1000-1300	0.75-0.98	14-16	10.6-15.68	
		Polyelectrolyte	2-3	0.0015-0.002	260-280	0.39-0.64	
		HCl	120-150	0.09-0.11	8-12	0.72-1.32	
		Nutrient	0ccasional	-	150-300	-	
Combined chemical-biological -2	75	Lime	600-700	0.33-0.39	10-12	3.33-4.68	12-17
		$FeSO_4$	1050-1200	0.58-0.67	14-16	8.16-10.72	
		Polyelectrolyte	1.5-2	0.0008-0.001	260-280	0.22-0.3	
		HCl	120-150	0.06-0.08	8-12	0.53-0.96	
		Nutrient	0ccasional	-	150-300	-	

Chemical consumption can fluctuate according to effluent composition and concentration.

Table 6. Man Power Cost:

Process	Peak flow m^3/hr	No of labour	Salary/month Tk	Treatment/month m^3	Cost tk/m^3
Physico-chemical	65	9	60000	46800	1.28

Biological	60	6	45000	43200	1.04
Combined-1	55	9	58000	39600	1.46
Combined-2	75	10	80000	54000	1.48

"Performance Analysis"

In Biological method the average BOD removal efficiency gained the highest value (84%) compared to other methods. In combined bio-chemical method the average COD removal efficiency gained the highest value (70.8%), in biological method 59.1%. Among all methods highest average TSS removal efficiency (81.7%) found in biological method. Before treatment TDS level was under discharging standard (2100 mg/l) in ETPs A1, A2, C1, C2, C3. Physico chemical based ETP A3 cannot maintain discharging standard. Except biological method, TDS value increased after treatment in physico-chemical based ETP A2 and combined bio-chemical ETP C3. Biological treatment reduces TDS significantly and satisfy discharging standard.

Table 7. Performance Analysis of Active ETP

Physico-chemical												
			A1			A2			A3			
Facts	Unit	Standard	BT	AT	RE %	BT	AT	RE%	BT	AT	RE%	Avg. RE%
BOD_5	mg/l	50/150	125	65	48	147	69	53.1	115	56	51.3	50.8
COD	mg/l	200	340	135	60.3	290	110	62.1	295	153	48.1	56.8
TSS	mg/l	150	170	62.9	63	276	80	71	210	53.88	74.3	69.5
TDS	mg/l	2100	1956	1795	8.2	1600	1820	-13.8	3045	2245	26.2	6.9
DO	mg/l	4.5-8	0	4.9		0	5.1		0	4.9		
P^H	-	6-9	11.5	8.6		11.2	7.3		10	7.72		
TEMP	ºC	40	37	29		41	30		40	29		
Biological												
			B1			B2			B3			
Facts	Unit	Standard	BT	AT	RE%	BT	AT	RE%	BT	AT	RE%	Avg. RE%
BOD_5	mg/l	50	110	29	73.6	145	19.45	86.6	281	23	91.8	84
COD	mg/l	200	320	128	60	304	102	66.4	356	174	51.1	59.19
TSS	mg/l	150	130	18	86.2	230	54	76.5	204	36	82.4	81.7
TDS	mg/l	2100	4950	2010	59.4	2492	1135	54.5	3200	1580	50.6	54.8
DO	mg/l	4.5-8	0	4.5		0	4.7		0.1	4.6		
P^H	-	6-9	10.5	8.03		9.76	7.69		10.3	8.1		
TEMP	ºC	40	41	35		43	34		50	35		
Combined chemical & biological												
			C1			C2			C3			
Facts	Unit	Standard	BT	AT	RE %	BT	AT	RE %	BT	AT	RE%	Avg. RE%
BOD_5	mg/l	50	110	43	60.9	144	36	75	112	24	78.6	71.5
COD	mg/l	200	284	110	61.3	372	95	74.5	292	68	76.7	70.8
TSS	mg/l	150	75	52	30.7	192	30	84.4	62	34	45.2	53.401
TDS	mg/l	2100	1960	1610	17.9	1880	1600	14.9	840	1050	-25	2.6
DO	mg/l	4.5-8	0	4.4		0	4.4		0	5.9		
P^H	-	6-9	8	7.5		11	6.4		9.2	7.7		
TEMP	ºC	40	35	29		36	27		38	32		

*BT-before treatment value, AT-after treatment value, RE (removal efficiency)={(BT-AT)/BT}*100

Table 8. Sludge Characterization

Parameters	Physico-chemical	Biological	Combined chemical & biological
Sludge quantity	2-5 kg/m^3	300-400 gm/m^3	2-5 kg/m^3
Sludge toxicity	Highly toxic	Non-toxic	Toxic
Sludge disposal problem	Severe	Slight	Medium
Sludge disposal cost	High	Very low	High
Sludge utilization	Brick	Fertilizer,brick	Brick

"4. Conclusion"

There is wide variation between actual efficiency and typical efficiency except Biological method. Considering chemical consumption, Biological treatment plant needs very less amount of money to do treatment of waste water compared to other treatment plants. Biological treatment plant needs low manpower cost that is 1.04 tk/m^3 whereas more cost is essential for other treatment plants. Physico-

chemical treatment shows average removal efficiency of BOD, COD, TSS from 51% to 70%, Whereas it is between 60 % to 84% for Biological treatment plant. Besides after doing biological treatment we can get non toxic and less amount of sludge(300-400 gm/m^3) form discharging water, while it is toxic and more amount(2-5 kg/m^3) for other treatments. To run the plant regularly and efficiently and to bring business profit, owners prefer Biological ETP. So by considering the economic, ecologic and functionality perspective Biological ETP perform best than any other Effluent Treatment Plants (ETP).

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"Gulshan Lake- an area of concern, probable threats and extreme consequences"- A case study

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Abstract

Water bodies are a vital and prolific resource to our environment. They provide a variety of amenities and services to communities across the world such as drinking water, food, travel, recreation, wildlife, aesthetic appeal, economic development, etc. This case study deals with the importance of the Gulshan lake, areas of problem in the lake water resulting in pollution and shrinkage of lake-water in the rapidly urbanizing city of Dhaka, and how the impact of this problems can be minimized by initiating some controlling measures. Bulk of the problems has been identified throughout the study are owing to accelerated demographic pressure, urbanization, land encroachment and land fill, disposition of increased volume of domestic and industrial waste. The lakes having suffered accelerated eutrophication, eco-system of the lake is tremendously stressed and now it is gradually turning in to a hazardous sewer- the contaminated waters of which are still being consumed. To amend this critical situation, some approaches have been mentioned in the study. Water exchange method, water hyacinth based hydroponics method and proper implementation of DPSIR (drivers, pressure, state, impacts and responses) monitoring model are those pillar which may prove advantageous in the longer run.

Keywords: 3-5 Water bodies, Industrial waste, contaminated water, water exchange method, DPSIR model

1. Introduction [6], [7], [8]

Water bodies and pollution probably go together in Bangladesh. Almost all the major water bodies (lake/khal) particularly in the urban or semi-urban area have been suffering severe pollutions. Wastes from households, sewerage systems and factories find an easy solution for dumping them in the nearby water bodies. Despite strict laws, it remains a challenge for everybody to find a solution. ". According to a study conducted by the Department of Environment (DoE), the water of the Gulshan, Baridhara and Banani Lake is the most polluted among the city's five lakes. It may be noted that the Gulshan, Baridhara and Banani Lake was declared as an ecologically critical area (ECA) in 2001[8]. Hence Gulshan Lake is the primary point of concern for this study. The study of the lake's water revealed that presence of bacteria was 1,200 counts, which researchers say is not even fit to be touched by humans. The tolerable bacteria count in water bodies is 200 or less. The amount of Dissolved Oxygen (DO) in the same water was 0.5. According to a former scientist of the Bangladesh Atomic Energy Commission (BAEC), marine life could not survive in water that had a dissolved oxygen level of less than 5.0. The Biological Oxygen Demand (BOD) in Gulshan Lake was found at 35, indicating serious contamination of the water. The acceptable level is 3 or less. The level of pH in Gulshan Lake showed acidity and alkali of 7.1[7].Various studies have identified that Gulshan, Baridhara and Banani Lake and other water bodies in and around Dhaka city is playing an extremely critical role for sustenance of the city in many different ways. The water bodies acts as the kidney for the city to cleanse its daily waste. According to environmental scientists, a city should have 10% water bodies of its land mass to effectively function as a city and helps reduce sound and air pollutions. The water bodies also help recharge the ground water. As such effective conservation and improvement of water quality of water bodies is an extremely important issue today for the survival of Dhaka.

2. Problem Analysis

Gulshan Lake has a lot of prevailing pollution problems because of rapid urbanization and loss of sociological balance. The lake water is not properly maintained and it has lost its clarity and nutrient balance. Flood water

runoff flows into these water bodies practically turning these into buffer flood control reservoirs, except some pockets of transient water-logging. Drains and sewerage pipes dumping wastes in the Gulshan Lake has been identified as major pollution problem by DWASA. The malodorous wastes tend to spill over when the roads are flooded. People do not come close to the water edge for their refreshment and recreation because of the ill-maintenance and poor treatment by the people themselves. It's a vicious cycle of human intervention in the nature's own state and the extreme consequences of nature bouncing back on the human being.

Fig-1: Present scenario of Gulshan Lake

Major problems observed may be listed as:

- Unplanned commercial growth
- Loss of green space
- Absence of community hub
- Buildings along lake creating wall effect

Accelerated demographic pressure, urbanization, land encroachment and land fill, disposition of increased volume of domestic and industrial waste are major contributing factors behind water pollution of Gulshan Lake. Besides, Continuation of all sorts of banned activities in the ECA has almost turned Gulshan Lake into an ecologically dead lake.[2] This has been manifested recurrently through the death of fishes and for vigorous media coverage each year. The water of the lake is being polluted day by day and during the summer season the level goes down and the pollution becomes worst.

Apart from these existing problems, due to neglegency of taking proper initiative as soon as possible, few more problems might creep in. This problems can be listed as:

- Inefficient use of economic factor
- Lost of ecological banalnce
- Lost of physical and psychological connectivity with the lake

3. Impacts of the problems found [4], [11]

The lakes having suffered accelerated eutrophication, tremendously stress on the lake eco-system and is now gradually turning in to a hazardous sewer- the contaminated waters of which are still being consumed and used by slum dwellers living near the lake. Experts in a recent seminar on "Updating the Existing Groundwater and Land Subsidence Model" organized by Dhaka WASA, expressed concerns of land subsidence ground water in many areas of city contaminated by toxic waste. Ground water in many areas of the city has become contaminated with a number of toxic chemicals discharged by different industries. Statistics presented by the BUET experts showed that ratios of toxic minerals like lead, cadmium, phosphate, ammonia and chromium are much higher than acceptable levels in ground water in Dhaka city most prominently in Hazaribagh and Armanitola areas of the city. The experts felt that the toxic chemicals will ultimately affect the ground water in other parts of the city in phases. According to WASA, about 450 water pumps in Dhaka operate to meet the present demand for water. If the water level goes down to 70 meters due to continuation of the present rate of extraction of ground water, a large number of WASA pumps will become inoperative in next 4 to 5 years. According to Dr. Mozammel Haq of BUET, the maximum subsidence of 2.73 cm in Dhaka city occurred near the New Airport during the period between 1990 and 1999. [11] The maximum land subsidence occurred at such a place where the level of ground water is higher than in other parts of the city. According to the experts, if land subsidence in the capital had any relation with the fall of ground water level, then the maximum land subsidence

would have occurred in Gulshan-Baridhara-Banani and Motijheel area where the level has gone down more than in any other area of the city

Furthermore,

-The laboratory result of Gulshan Lake indicated that the water body is flabby for any kind of use.

-As lake contains high level of BOD (Biochemical Oxygen Demand) and COD (Chemical Oxygen Demand) added with low amount of dissolved oxygen designate poor water quality.

- High concentration of 'Coliform Bacteria' in the lake water is the indicator of water-borne pathogenic diseases includes ear infections, dysentery, typhoid fever, viral and bacterial gastroenteritis and hepatitis 'A'.

- Reducing oxygen level of the lake 'Coliform Bacteria' creates a hazardous condition for aquatic environments

4. Solution of the problems

Considering this grave situation, strategies need to implement the above issues through a community based cost effective biological treatment of polluted water of the lakes. The biological treatment of lake water will be done through water hyacinth based hydroponics, a technology which has been used in other countries of the world.

- The study has wrapped up with some significant and effective recommendations like delineating the boundary according to the design of *Rajdhani Unnayan Kartipakkha (RAJUK)* to identify the encroachers and evict them.

- 'Waste water fed aquaculture' system can be proved as an example of combination of environmental protection and development.

- Lake conservation committee could be formed comprising all stakeholders of the surrounding area of the lake who would be bound to make people aware about the welfare of the lake.

- Household discharge management should be implemented.

- Solid waste management would restrict accumulation of the waste near the bank of the lake free movement of lake water.

- Using of 'Hydroponics Technology'; floating cultivation which could play a significant role to reduce the emission of methane gas.

- Industrial waste water discharge mostly from the Tejgaon, Badda and Mohakhali industrial areas which are the hazardous staffs that pollute the lake water as well as affect aquatic ecosystem.

- DPSIR model may be introduced to monitor the lake management system

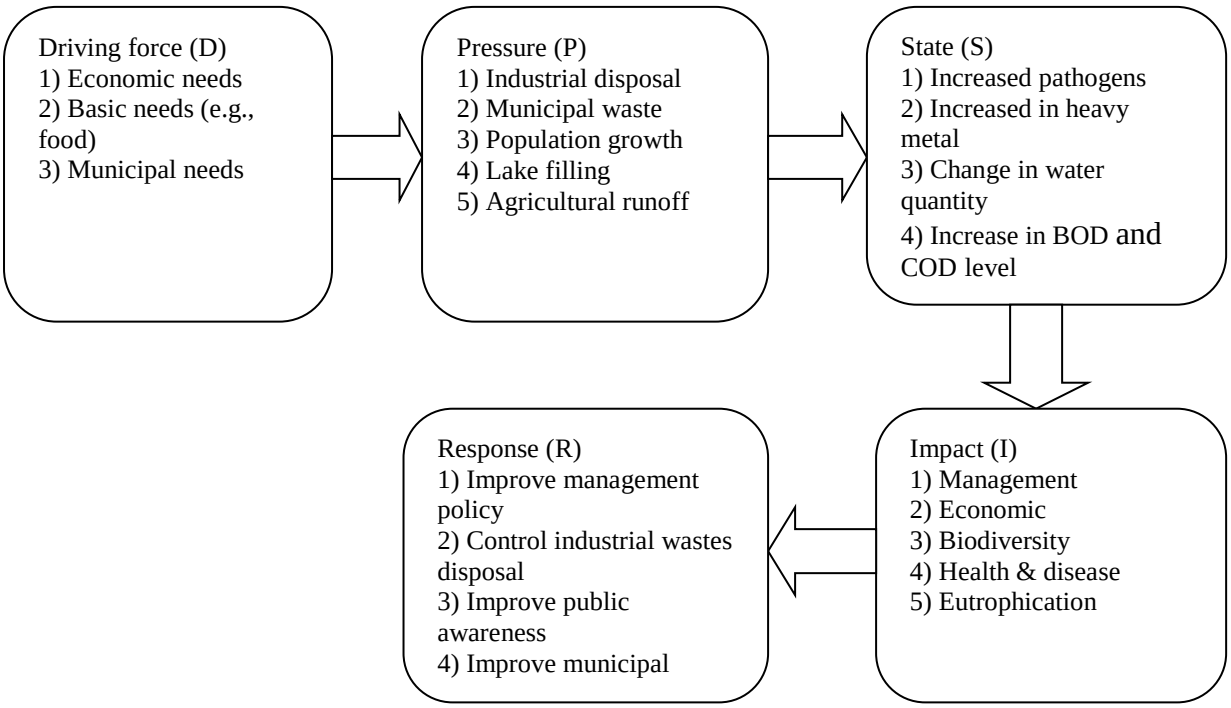

Fig-2 Drivers, Pressure, States, Impacts, Responses (DPSIR) model for lake management in Dhaka

5. Challenges [5], [11]

- The growth of water hyacinth is one of the lake's biggest problems. The proliferation of water hyacinth is currently controlled by removing it from a water body. Using water hyacinth to remove nutrients from water bodies and to produce biogas is another technically feasible option for the control of water hyacinth, but its environmental and economic performances are not well understood. [5]

- Currently, there are several popular methods for preventing the spread of, or eradicating the water hyacinth: biological, chemical and physical control.

- Physical removal method, however, is costly and generally not economically feasible. If it is disposed of in a landfill, water hyacinth can generate methane and carbon dioxide. These gases will eventually enter the atmosphere and can contribute to climate change.

- The use of water hyacinth to produce biogas has two major advantages. For one, the biomass of water hyacinth is used rather than disposed of as a waste. For another, the emission of landfill gas is avoided.

6. Discussions [12]

Water-related and water-based systems and particularly the mutual interrelationships and their interactions between the individual water systems and urban water bodies (including groundwater) are extremely important from the context of urban environment. The scientists today, all over the world, are increasingly emphasizing the importance of integrated approaches to solve the numerous pressing problems conservation and improvement of quality of urban water engineering. The urban wetland is today considered as an entity with all its internal and external interactions. The scientists are focusing on the water-based infrastructure in the city - potable water, sewerage, drainage, discharge and recharge technologies (e.g. recycling, source controls) and their interrelations in the networks. The impact of the networks on the water environment (water resources, groundwater, and watercourses) and vice versa is increasingly discussed for proper functioning of a city. As a result, it's imperative that the various water bodies are conserved and the quality of water of these water bodies be improved systematically and in a sustainable way. In this regard, issues of groundwater aquifers in urban areas and water planning (including the interactions of urban water amenities with city planning and landscaping), design, analysis, information support (GIS/Satellite-Imaging), operational management and associated economics are critically associated with social and economic as well as resource management. Development and dissemination appropriate data relevant to water management, up-to-date and reliable digital information, tools and methodologies as well as software on urban water and water bodies should be made available to the scientific, educational and professional community, specialists in civil, water and environmental engineering, other relevant engineering disciplines and urban planning. Other disciplines include environmental chemistry, biology and toxicology and many others.

7. Conclusion

The challenges rises in terms of lake management in Dhaka, Bangladesh, are immense. This study illustrated a continued decline in water quality across six water bodies in Dhaka, highlighting the centralization of water resources near more affluent neighborhoods. If these patterns continue unchecked into the future, water access for the most vulnerable citizens in Dhaka will become increasingly challenging. This is of special concern in regard to safe drinking water. Researchers and policymakers are not blind to these problems and continue to recommend short-term and long-term lake management strategies and policies. Without public participation, and inadequate means for enforcing mandates, these recommendations will gain little traction. High illiteracy and poverty rates pose furthermore problems, serving to limit educational efforts and preventing individuals (e.g. farmers) from pursuing alternative and more ecologically friendly strategies.

8. Acknowledgement

The authors gratefully acknowledge the contributions, suggestion and advices by Mr. Syed Abu Shoaib, PhD scholar at Civil and Environmental Engineering, University of New South Wales, Sydney, Australia.

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A Feasibility Study of Synthesizing Zeolitic Material from Locally Available Waste Materials

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Abstract

The increasing ingestion of rice and use of coal in coal based power plant in Bangladesh have been producing rice husk ash (RHA), rice straw ash (RSA) and coal fly ash (FA) as byproducts in a massive amount. The current energy scenario of our country is resulting in a sustainable increase in the FA generation, whose proper management has become a cause of concern. Similarly, Bangladesh annually produces millions of tons of rice husk (RH) and rice straw (RS) after milling of the paddy, which produces ash in high amount after being used as a fuel in rural areas. All these ashes currently have limited or no use. Producing a micro porous solid acid adsorbent catalyst (Zeolite) from these locally available alternative sources would be a smart approach of waste utilization. This study is focused on the suitability of synthesizing certain types of low cost zeolite from these waste ash materials. The Si/Al ratio of FA, RHA and RSA is 1.96, 200.54 and 156.35 respectively, which is suitable for Zeolite synthesis. FA having higher (27.33%) alumina content needs only NaOH but RSA, RHA and FSA (0.35%, 0.37%, 6.5% alumina) needs also $NaAlO_2$ for zeolite preparation. This study also suggested a probable breakthrough in finding the feasibility of fecal sludge (human waste) ash (FSA) as a raw material in the synthesis of zeolites.

Keywords: Zeolite, Synthesis, Feasibility, Coal fly ash, Rice husk ash, Fecal Sludge ash, Bangladesh.

1. Introduction

The rapidly growing energy sector in Bangladesh has prompted increased consumption of coal to meet the demand. Due to the current energy policy [1], more coal based power plant will emerge in the near future. Fly ash (FA) is a byproduct of power generation from pulverized coal combustion. Barapukuria coal fired thermal power plant is now the only FA producing facility in the country. At present, there are few more public and private sector coal-fired power plants in the planning, implementation and construction phases. The projected annual production of FA will rise to 3.77 million tonnes per annum from 2018 onwards occupying nearly 0.27 million m^3 under loose condition [1]. FA is mostly used is cement industries. The other usage of FA is in landfilling, roads and embankments, brick manufacturing etc. Only a small portion of the current FA production is utilized and hence management of FA is going to be a challenge for the coal-based power plants. As alumina and silica are the major components of FA, a highly potential use of it is to produce zeolite, a valuable solid acid catalyst.

Bangladesh also has a vast amount of accessible and non-accessible biomass energy sources i.e. crop residues. The total supply of biomass fuel was 236 PJ in 1980 and has increased over the next 20 years to 360 PJ (1.73% growth) [2]. RH contributes the biggest share of biomass energy. In 1991, the production of RH energy was 76PJ and it increased to 106PJ in 2004 [2]. The RH is generally used as fuel in rural areas and a small quantity is used as animal feed. Large quantity of rice husk ash (RHA) is generated in Bangladesh during per-boiling of rice in rice mills. This ash, treated as a waste material, is usually dumped freely causing unanticipated environmental and health hazards. In Bangladesh, about 39.3 million ton of rice is produced annually which generate about 9.83 million tonnes of RH after milling of the paddy [3]. Assuming an ash to husk ratio of 18%, 1151.1 thousand tonnes of RHA is produced per annum in Bangladesh [4]. No planned efforts have yet been made for proper management of RH/RS or RHA/RSA on a commercial basis.

Recent experiments on fecal sludge (FS) have revealed its usefulness as a fuel source [5]. Ash from FS (FSA) has also been proved a good candidate as a raw material for synthesizing zeolite because of having nearly same chemical composition like FA/RHA/RSA.

As proper management of the mentioned waste ash materials has become an issue, an attempt has been made in this study to find out the suitability of these locally available waste materials in synthesis of valuable zeolitic materials in perspective of Bangladesh.

2. Materials and Method

FA was collected from the ESP (Electrostatic Precipitator) of 250 MW Barapukuria coal-fired thermal power plant of BPDB (Bangladesh Power Development Board) located at Dudhipur, Dinajpur. The power plant used coal from Barapukuria Coal Mining Company Limited (BCMCL). RH and RS were collected from rural areas of Comilla district. Treated FS was collected from a research team under IRC Water and Sanitation Centre and BRAC WASH program focusing on feasible and sustainable approach of FS management in Bangladesh, which was pre-treated by drying (sand-bed drying/bio drying) and post-treated by co-composting method [5].

Proximate analysis of all the raw materials was done for their characterization. RH was taken in a porcelain crucible and heated in an electric muffle furnace at 550°C for ~4 hours to get RHA. From the difference of the weight of RH and RHA, the ash% of RH was measured. RSA was produced similarly by combustion at 600°C and came out along with some black unburnt particles. So, it was further heated at higher temperature (~700-750°C) to ensure the complete conversion of ash. Ash% of RS was calculated similarly. For making FSA from treated FS, method similar to RHA was followed (5 hour at 550°C and ash% was calculated. Moisture content of RH, RS was measured by finding the weight differences of the samples before and after keeping them in a dryer at 105-110°C for 4-5 hours in Petri dishes in single layer. For measurement of %volatile matter (in wet basis), RH/RS was taken in four identical (but somewhat different in weight) fused silica crucibles and burnt at 900°C for 7 minutes and then the weight difference before and after burning was measured. Volatile % in dry basis was calculated for each sample by subtracting the % moisture from volatile% in dry basis.

Chemical compositions of the ash samples was analysed by sequential X-ray fluorescence (XRF) spectrometer-1800. A high energy incident X-ray is collided with the atom of sample in a XRF and it produces energy as electron moves between levels and from this, the chemical composition of a sample is analysed. For XRF analysis, the ash samples were first grinded, mixed with a binder and pelletized for convenience.

3. Results and Discussion

The proximate analyses of coal, RH, RS and treated FS, are shown in Table 1. As shown in the table, as the primary raw material FS would be a good choice rather than others because it has the highest ash content (69.34%) where RH/RS/Coal has similar ash content (~13-14%). Coal has the lowest volatile content% (30.69%) where RH and RS have much higher values (65.85% and 68.29% respectively). Moisture content is low in amount for RH and RS and is absent in coal in dry basis. Also, it is evident that coal has the highest FC% (54.57%) whereas RH/RS has very lower values (14.30% and 14.45% respectively).

Table 1. Proximate analysis Coal, Rice Husk, Rice Straw and Treated Fecal Sludge (dry basis) sample

Component	Ash Content (%)	Volatile Content (%)	Moisture Content (%)	Fixed Carbon (%)
Coal	14.74	30.69	-	54.57
Rice Husk	14.08	65.85	5.77	14.30
Rice Straw	13.41	68.29	3.85	14.45
Treated Fecal Sludge	69.34	*ND*	*ND*	*ND*

* *ND= Not determined*

Table 2. Chemical Composition of Ash Samples (FA, RHA, RSA, FSA)

Constituents	% composition by weight			
	FA	RHA	RSA	FSA
SiO_2	61.04	79.50	65.33	69.35
P_2O_5	0.69	10.19	2.27	4.01
K_2O	1.34	4.37	16.83	3.03
MgO	0.31	3.76	4.31	1.62
CaO	0.83	1.00	5.08	3.77
Al_2O_3	27.33	0.35	0.37	6.50
Fe_2O_3	2.93	0.32	0.64	3.35
SO_3	0.55	0.26	1.87	6.40
Na_2O	0.21	0.18	3.22	1.29
TiO_2	4.78	0.07	0.09	0.67

Table 2 shows the compositions of ash samples. From the XRF analysis, FA has the highest Al_2O_3% (27.33%) yielding the lowest Si/Al of all the samples (1.96). On the other hand, RHA, RSA and FSA has much higher Si/Al ratio (0.35%, 0.37% and 6.5% Al_2O_3 respectively), which can be attributed to lower amount of Al_2O_3% in these samples. There are impurities in the ash samples (oxides other than silica and alumina) which need to be removed. Removal of impurities, prior to zeolite synthesis, can be done by acid treating the samples, as oxides other than silica and alumina dissolves in acid. These impurities are low in FA (11.64 wt. %) and are higher in RSA (34.31%) whereas FSA and RHA have 24.154% and 20.15 % impurities, respectively. Also, it is evident that FSA contains oxides similar to FA/RHA/RSA with some slight exceptions in mass fractions of certain oxides. Other than this, FSA is nothing but a waste material which is rich in silica (also has a favourable high ash content) and could be a potential candidate as a raw material for synthesis of zeolites.

Zeolites are actually crystalline aluminosilicates that consist of a tetrahedral framework of $[SiO_4]^{4-}$ and $[AlO_4]^{5-}$, linked each other with the corners by sharing oxygen atoms.

Composition of zeolite:

$$MxDy\ [Alx+2ySin-(x+2y)\ O_2n].\ mH2O$$

Where,

M = monovalent cation, D = divalent cations, m = number of water molecules per unit cell, x = numbers of cations (Monovalent), y = number of cations (Divalent). [6]

Zeolites have a wide range of industrial applications, e.g. - in ion exchange, as molecular sieves, catalysts, and adsorbents [7]. FA, RHA, RSA all have the potential to be converted into zeolites. The most important use of zeolites is catalysis, for several important reactions such as cracking, isomerization and hydrocarbon synthesis. The other importance of zeolites include gas separation by adsorption (the porous structure of zeolites can be used to 'sieve' molecules having certain dimensions and allow them to enter the pores; also known as "molecular sieve") and ion exchange (applications in water softening devices, in detergents and soaps, also it is possible to remove radioactive ions from contaminated water) [8]. Depending on the chemical composition (e.g. - final Si/Al ratio) and crystal structure, zeolites can be of many different types.

From the study it can be seen that the initial Si/Al ratio is 1.96 for the given coal FA sample which is somewhat close (1.57) to the FA used for preparing zeolite described by Kondru *et al.* and could be used to make Na-Y type zeolite [9]. The composition of RHA and RSA is also compatible for synthesizing Na-Y zeolite by the method described by Jan-Jezreel and Rizalinda, 2011 [10] and the initial Si/Al ratio (200.54 for RHA, 156.35 for RSA) of the samples of this study also supports this fact. As the alumina content is in an appreciable amount (27.33%) in FA, zeolites can be produced through fusing the ash only with NaOH followed by hydrothermal crystallization. But due to lower initial Si/Al ratio in RHA/RSA/FSA (0.35%, 0.37%, 6.5% alumina respectively) samples, $NaAlO_2$ solution is needed in these cases to balance the final Si/Al ratio in the synthesized zeolite. FSA sample, which contains initial Si/Al ratio of 9.4, would need $NaAlO_2$ much less than RHA/RSA.

3.1 Proposed preparation techniques

Ash samples were pre-treated before synthesizing zeolites. FA was first sieved and calcined at 800°C for 2 hours before acid treatment. Ashes was treated with 10% H_2SO_4 and kept for 24 hours before washing with excessive water and filtering by vacuum filtration. Filtered ashes were dried at 105°C for ~4 hours and later, stored in a desiccator.

Figure 1 shows the general procedure for zeolite synthesis from waste ash materials.

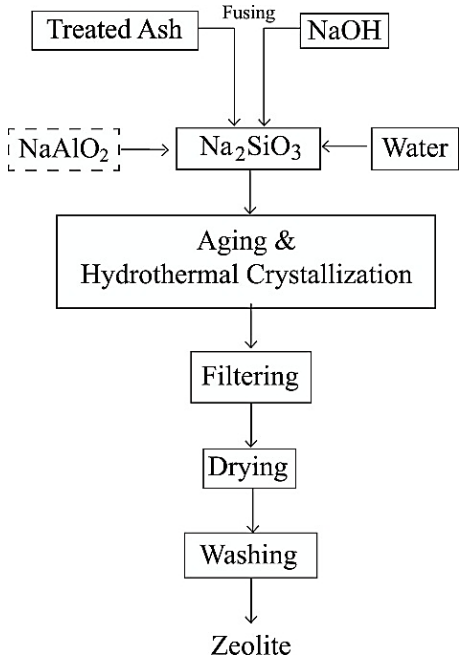

Fig. 1. General preparation technique for Zeolite synthesis ($NaAlO_2$ have to be added for RHA/RSA/FSA)

For zeolite synthesis FA would have to be added, mixed, grinded and fused with NaOH (alkali source) and then mixed with distilled water. Then the slurry sodium aluminate solution would have to be aged and hydrothermally crystallized (without disturbance) which would yield the final product after filtering, washing and drying.

For preparing zeolite from RHA/RSA, first, extraction of silica from the ashes would have to be done by refluxing with acid solution, washing, drying and finally through calcination. Then rice husk ash silica (RHAS) or rice straw ash silica (RSAS) would have to be dissolved in NaOH solution to produce sodium silicate (Na_2SiO_3) solution, which would be added with $NaAlO_2$ solution to produce a seed gel solution by stirring and aging. The feedstock gel, prepared in the same way as seed gel would have to be instantly mixed into the seed gel under vigorous stirring followed by aging and crystallization to get the final product. Lastly, filtering, washing and drying would have to be done to get the final product.

Zeolite could be synthesized from FSA same as the procedure of RHA/RSA but the ratio in which NaOH and $NaAlO_2$ have to be mixed with FSA would be slight different.

4. Concluding Remarks

FA/RHA has been proved previously by many researchers as a suitable source for synthesizing various types of zeolites. Correspondingly, in this research, locally available FA/RHA has successfully been converted to certain type of zeolite. Synthesizing zeolite from FSA could a major breakthrough. FSA has been found to be a good primary source as it has higher ash content than FA/RHA/RSA. RHA/RSA/FA has similar ash content but the first two has more volatile% with similar moisture%. FA could be the best source for synthesizing certain type zeolite for its lower Si/Al ratio due to higher Al_2O_3%. The other ashes, being the opposite, therefore, would need $NaAlO_2$ for preparing zeolite to gain a certain Si/Al ratio at the end product. The impurities of the ashes are similar in nature but different in mass fractions, which have been suggested to be removed by pre-treatment. Successful conversion of these wastes not only gives a feasible and sustainable approach to waste management of Bangladesh but also brings an economic value by making a valuable adsorbent solid catalyst. Future work by the authors involves characterization and performance evaluation of the produced zeolites.

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Effect of Material Model on the FEM Based Stability Analysis of Slope

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Abstract

Due to rapid development of computer technology, Finite element method (FEM) has gained increasing popularity over traditional methods in geotechnical engineering. FEM based numerical analysis for geotechnical engineering problems such as the stability analysis of slope requires the incorporation of any material model. In this paper, different material models such as the Mohr-Coulomb, Modified Mohr-Coulomb, and Drucker-Prager models are used to numerically investigate their effects on the computation of the factor of safety with water and without water. A model slope is considered and the material properties are assigned. FEM based software GEO5 is used to calculate the factor of safety. From the analysis, it is found that Modified Mohr-Coulomb model yields higher factor of safety compared to that of Mohr-Coulomb model and Drucker-Prager mode while Mohr-Coulomb material model yields the lowest factor of safety. Incorporation of water lowers the factor of safety of slopes regardless of the material model as expected. Material models appear to have insignificant effect on the evolution pattern of the equivalent plastic strain.

Keywords: Material model, FEM, Slope stability, Factor of safety.

1. Introduction

Slope, made of soil, is often observed in different engineering structures such as the road and railway pavements, dams, etc. The proper design of slope is important for the stability of such engineering structures. The accurate approximation of the factor of safety of the slopes often requires the adaption of the precise methods. Conventional methods such as Bishop [1], Fellenius [2], Morgenstern-Price [3], Spencer [4] and Janbu [5] are often used due to the ease in calculation. Consequently, many research works have been reported in the literature that considered the conventional limit equilibrium methods [6-8]. However, these methods have several limitations. It requires many prior assumptions. Moreover, only simple models can be considered by using the limit equilibrium method. With the tremendous advances of computational power, the numerical approach such as the finite element method (FEM) has been implemented in the stability analysis of slope [9-10]. The important point of FEM based stability analysis is that it requires no prior assumption. The failure surface of slope is not predetermined or pre-set in FEM. Moreover, any complex shape of slope can easily be considered and complex multi-material slopes can be analyzed using FEM. The FEM based model analysis of slope requires the incorporation of any material model. In this paper, the effect of different material models on the factor of safety of slope by FEM is studied using GEO5 [11] with water and without water. Mohr-Coulomb material model, Modified Mohr-Coulomb material model and Drucker-Prager material model are used in this study. The consequences of using different models in the FEM based study have been investigated and the numerical results have been reported.

2. Finite element method in slope stability analysis

Finite element method (FEM) is a very widely used and powerful tool for solving boundary value problems. It is used for appropriate solution of boundary value problems in stress analysis, fluid flow, heat transfer etc. In FEM, the actual continuum is divided into small and regular subdivisions referred to as finite elements which are interconnected at specified joints referred to as nodes. The solution is approximated over each finite element and the governing equations for each finite element are assembled to analyze the behavior of the externally loaded structures through the boundary. The stability of slopes are traditionally solved by the conventional limit equilibrium method. But, limit equilibrium method is limited by assumptions regarding the analysis method itself. By contrast, the most difficult part of the use of FEM in slope stability analysis is the calculation of factor of safety. Such difficulties can be overcome by the introduction of the strength reduction technique proposed by

Zienkiewicz et al. [12]. Griffiths, and Lane [9] also used FEM for the calculation of the factor of safety by following the strength reduction technique.

3. Geometry of the numerical model

The geometry of the numerical model considered in the present study is depicted in Fig. 1. All the dimensions in the model are given in meter. In the numerical model, a slope angle of 45° is used. The geometric boundaries are horizontally constrained on the left and right sides and completely fixed at the bottom of the geometry. The geometric model is incorporated in GEO5 [11] and the material properties are assigned. The desired material model is selected and the whole geometry is divided into small subdomains known as mesh for FEM based analysis. In this study, only triangular shape mesh is generated. After generation of mesh and assigning related properties, the stability analysis is performed by strength reduction methods [9]. The generated mesh (size=0.5 m) within the boundary of the numerical model is depicted in Fig. 2.

4. Material properties

In the present study, a single material (homogeneous soil) is used, the properties of which are shown in Table 1. The properties of material are selected with the aid of the previous studies from the literature. This is done because the aim of present study is to focus the importance and the effect of the material model in the calculation of the factor of safety of a numerical model slope and not a real slope.

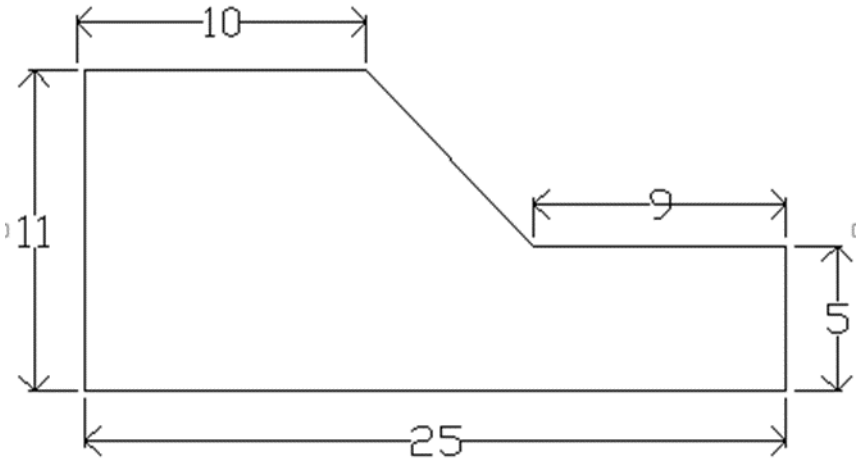

Fig. 1. Geometry of the numerical model of slope considered in the present study

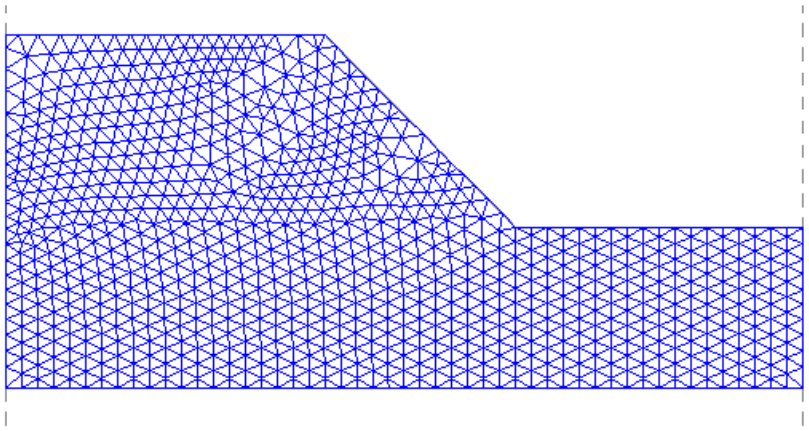

Fig. 2. Model slope with mesh (mesh size=0.5 m) without considering any water

Table 1. Material properties used in the present study

Material (soil) parameters	Value
Cohesion, c (kN/m²)	10
Frictional angel, ϕ (deg)	20
Unit weight, γ (kN/m³)	18
Modulus of elasticity, E (MN/m²)	8
Poisson's ratio, ν	0.3
Dilation angle, ψ (deg)	0.0

5. Effect of material models in slope stability analysis

In the present FEM based analysis, three different material models are considered. They are: Mohr-Coulomb material model, Modified Mohr-Coulomb material model and Drucker-Prager material mode. Three material models are varied to find the factor of safety for different element numbers and mesh sizes. In the following sections the effect of different material models on the stability analysis of slopes are reported.

Mohr-Coulomb Model

The FEM analysis using Mohr-Coulomb material model requires parameters such as modulus of elasticity, poison's ratio, angle of internal friction and cohesion. The latter two parameters serve to define the yield condition. The angle of dilation must also be specified. The failure surface of Mohr-Coulomb model can be expressed as follows:

$$\tau = \sigma \tan\phi + c \tag{1}$$

where, τ represents the shear stress, σ represents the normal stress, ϕ represents the angle of internal friction (slope of the failure envelope) and c represents the cohesion (the intercept of the failure envelope with the τ axis). The Mohr-Coulomb yield surface is represented as a non-uniform hexagonal cone in the principal stress space.

The factor of safety considering Mohr-Coulomb material model with 6-node triangle element for eight different mesh configurations is presented in Table 2. The analysis is carried out with water and without any water. Note that the factor of safety varies up to an approximate element number of 2300 (mesh size 0.5 m) for this material model considering the water and without considering the water. Beyond that, the factor of safety remains almost constant even though the size of mesh decreases and the number of element increases.

Modified Mohr-Coulomb Model

The Modified Mohr-Coulomb model unlike the Mohr-Coulomb model smoothes out the corners of the Mohr-Coulomb yield surface. A slightly stiffer response of the material can be expected with the Modified Mohr-Coulomb plasticity model when compared to the Mohr-Coulomb and Drucker-Prager models [11]. The factor of safety using the Modified Mohr-Coulomb material model for 6-node triangle element for eight different mesh configurations considering water and without considering water is shown in Table 3. Note that the factor of safety varies up to an approximate element number of 2300 (mesh size 0.5 m) for this material model. Beyond that, the factor of safety remains almost constant even though the size of mesh decreases and the number of element increases. It is noted that Modified Mohr-Coulomb material model yields elevated factor of safety compared to Mohr-Coulomb model.

Drucker-Prager Model

Drucker-Prager yield surface is smooth and appears as a cylindrical cone in the principal stress space. The Drucker-Prager model modifies the Mohr-Coulomb yield function to avoid singularities associated with corners [11]. The Drucker-Prager yield criterion [13] can be given by

$$f = \sqrt{J}_2 - \alpha I_1 - k = 0 \tag{2}$$

where, J_2 is the second invariant of the deviatoric stress tensor, I_1 is the first invariant of the stress tensor, α and k are the material constants determined from the experiments.

The factor of safety using the Drucker-Prager material model for 6-node triangle element for eight different mesh configurations considering water and without considering water is shown in Table 4. Note that the factor of safety varies up to an approximate element number of 2300 (mesh size 0.5 m) for this material model with water and without water. Beyond that, the factor of safety remains almost constant even though the size of mesh decreases and the number of element increases. It is noted that Drucker-Prager material model yields a bit higher factor of safety compared to Mohr-Coulomb model but less than the Modified Mohr-Coulomb model whether water is used or not.

Table 2. Factor of safety for Mohr coulomb material model

Mesh size (m)	Elements	Factor of safety without water	Factor of safety with water
1	703	1.25	1.23
0.9	834	1.23	1.21
0.8	996	1.23	1.21
0.7	1256	1.23	1.20
0.6	1662	1.21	1.20
0.5	2291	1.20	1.17
0.4	3407	1.20	1.17

Table 3. Factor of safety for Modified Mohr Coulomb material model

Mesh size (m)	Elements	Factor of safety without water	Factor of safety with water
1.0	703	1.42	1.41
0.9	834	1.41	1.39
0.8	996	1.42	1.39
0.7	1256	1.41	1.41
0.6	1662	1.41	1.37
0.5	2291	1.39	1.37
0.4	3407	1.37	1.35

Table 4. Factor of safety for Drucker-Prager material model

Mesh size (m)	Elements	Factor of safety without water	Factor of safety with water
1.0	703	1.33	1.32
0.9	834	1.32	1.30
0.8	996	1.33	1.30
0.7	1256	1.33	1.30
0.6	1662	1.32	1.28
0.5	2291	1.30	1.27
0.4	3407	1.28	1.27

\

6. Effect of material models on the contour of equivalent plastic strain

The effect of the material model on the contour of the equivalent plastic strain for dry sample (without any water) is depicted in Fig. 3. The red color indicates the maximum equivalent plastic strain while the green color indicates the minimum equivalent plastic strain. Note that, the highest equivalent plastic strain is observed near the slope. Note also that the material model appears to have minimum effect on the evolution of the equivalent plastic strain.

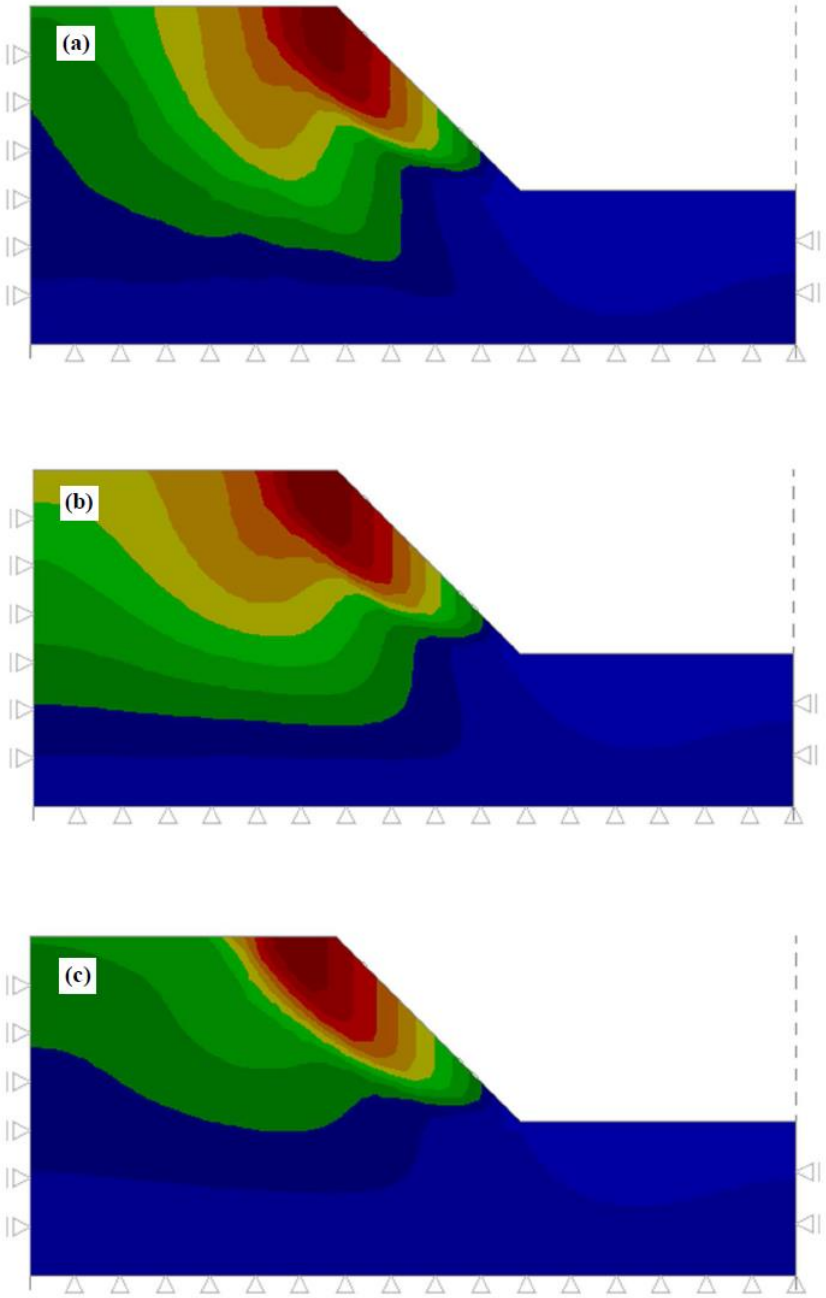

Fig. 3. Contour of the equivalent plastic strain for mesh size 0.5 m without considering water: (a) Mohr-Coulomb Model; (b) Modified Mohr-Coulomb Model; (c) Drucker-Prager Model

7. Conclusions

The effect of the material model on the FEM based stability analysis of slope has been investigated considering water and without considering water. Different material models such as the Mohr-coulomb, Modified Mohr-Coulomb and Drucker-Prager models are used to investigate their effects on the computation of the factor of safety. Some of the important points of the study are summarized as follows:

(i) Modified Mohr-Coulomb model yields higher factor of safety compared to that of Mohr-Coulomb and Drucker-Prager models. Mohr-Coulomb material model yields the lowest factor of safety.
(ii) Incorporation of water lowers the factor of safety of slopes regardless of the material model.
(iii) Material models appear to have negligible effect on the evolution pattern of the equivalent plastic strain.

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Investigation of Bodyweight Distribution on Metatarsophalangeal Joint of Human Foot for Ladies High Heel Footwear

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Abstract

This study aimed to investigate the bodyweight distribution on metatarsophalangeal (MTP) joint of human foot for ladies high heel footwear. Distribution of percentage of bodyweight from rear foot to MTP joint of forefoot observed with varying heel heights. Young healthy women participated in the test and force on MTP joint & leg muscle activities have been recorded with data recording systems. The results of this experiment showed that the increased heel height shifted the force and peak pressure from the rear foot to the MTP joint of the forefoot region. Around 21.37%, 31.45%, 32.67%, 34.05%, and 36.59% of bodyweight is imposed on MTP joint for the heel height of 0cm, 2.54cm, 5.08cm, 6.35cm, and 8.89cm respectively. Semi-high heel results significant imposition of bodyweight on MTP joint only whereas high heel creates more unstable footsteps and imposes less significant additional compressive forces on MTP joint that may increase discomfort, muscle fatigue & pain.

Keywords: high heel, metatarsophalangeal joint, electromyography, bodyweight distribution.

1. Introduction

Footwear is considered one of the basic requirements of a complete costume. There are various types' footwear worn by people but high heels and stilettos are very popular and fashionable among women especially in today's modern society. Since the ancient period high heel shoes have been one of the cornerstones of a woman's wardrobe, most women like wearing high-heeled shoes for the benefit of sensuous attractiveness and self-esteem. Footwear purchase is dictated by fashion and not a sense of comfort; for many people fashion surpasses the need of comfort [1]-[2]. In today's society, fashionable footwear designs are now becoming increasingly complex and incorporating high heels [3]. High heeled shoes not only does it made female look taller but also attractive and prettier when match with favorite dress or gown, hence always become a popular choice among women. Although this fashion is intended to be uncomfortable, few women plan to give up their high heels [4]. According to surveys on footwear, between 39% and 69% of women wear high heels on a daily basis, representing a huge proportion of the female population [5]-[6]. High-heeled shoes (HHSs) are defined as shoes in which the heel is higher than the forepart of the shoe. HHSs often also include a narrow toe box, rigid heel cap and curved plantar region, all of which interfere with natural foot motion [7]. High heeled shoes have come under much speculation as one of the causative factors for forefoot pain and discomfort. A prevalence proportion of foot problems in women were associated with wearing high heeled shoes [8]. A study in Netherlands found that 60% of women suffered foot problems directly caused by shoes. Studies on high heel shoes effect have become a hot issue in biomechanical field [9]. Biomechanical studies showed that walking in high heeled shoes may alter lower-extremity joint function, raise the peak pressure in the fore foot and alter the body weight load distribution on the media foot region [10]-[11]. According to several studies, walking with high heels leads to changes in load distribution beneath the feet, increase of the foot bone internal stress [12]-[13], decrease of the contact area and transfer of the center of pressure from midfoot to forefoot [13]-[14]. Another distinct difference reported is that peak pressure with positive correlation to the heel height shifting from the lateral forefoot to medial forefoot: from the area of 3rd, 4th, and 5th metatarsal head to 1st and 2nd metatarsal head [14]-[16]. These previous findings could be correlated to high heel wearing induced foot pain and uncomfortableness. However, most previous researches focused on the effect of the body weight/pressure on the different regions of the foot but only limited research has been performed regarding the analysis of bodyweight distribution specifically on metatarsophalangeal (MTP) joint of the human foot. Therefore, the aim

of the study is to experimentally characterize the body weight distribution on MTP joint of the human foot especially on ladies footwear varying heel heights.

2. Methodology

The methodology of this work can be explained clearly by the following sub-sections:

Subject's specification

In this study, a young healthy female subject free from any physiological disease was enrolled. Subject wasn't smoker and refrained from alcohol and caffeine containing drinks before the study. Food intake was totally restricted to a light meal prior to the test. After being informed about the summary of the study design, the subject gave her written consent. The subject had an initial visit to the experimental laboratory, for a physical examination and a medical history assessment. The age, weight, height, and body mass index (BMI) are listed in Table 1.

Sample development

Correct designs of ladies high heel footwears with various heel height such as 0cm, 2.54cm, 5.08cm, 6.35cm, and 8.89cm were developed and constructed. The design of ladies footwears were developed in such way so that it could compatible with the subject foot and body anatomy. The footwears provided normal walking facilities to the subject. The uppers of the high heel footwear were constructed in such way so that it couldn't bring any bad impact to the subject during the experiment. After identifying the position of MTP joint on the footwear, a channel was made on that position with exact depth and width compared to the dynamometer by which measurment of force or load on footwear was done as shown in Fig. 1(a). Five pairs of footwear with different heel height were constructed in such a way so that it could make exact fitting to the subject's feet. It should be mentioned that footwear was validated with the subject's and experimental setup requirements by minor necessary correction in footwear constructions. The designed and constructed ladies footwears were quite similar to the conventional ladies footwears.

Experimental setup

The study was performed in a quiet room with the temperature kept constant. After putting footwear, at least five minutes were allowed for acclimatization before the measurements were performed on MTP joint and skeletal muscles of leg. Dynamometer was placed just below MTP joint for force measurement as shown in Fig. 1(c). Dynamometer was interconnected to data acquisition unit and computer. For electromyogram (EMG) recording, electrodes were placed on skeletal muscles of leg as shown in Fig. 1(b). EMG recording was performed simultaneously with force measurement on MTP joint to investigate the affects of heel height increment on muscular activity of leg. Using necessary software force and EMG were recorded in computer.

Experimental procedures

Using proper experimental setup and calibration, data recording was done with normal walking of the subject. At first, subject was allowed to make few steps of walking by putting footwear to analyze that the foot was correctly adjusted or not. If foot and footwear is correctly adjusted then subject was allowed to walk freely for few steps and then data was recorded for the load imposed on MTP joint. Besides, EMG signal recording in leg muscle was simultaneously done. Data recording was done with varying heel heights of ladies high heel footwear. The subject was allowed to be rested several times so that the subject can walk smoothly without being tired for next heel height episodes. Results were noted down separately for each heel height to investigate the bodyweight imposed on MTP joint.

Table 1. Subject's specification

Parameters	Value
Age (yr)	21
Weight (kg)	60
Height (cm)	167.64
BMI (kg/m^2)	21.35

3. Data collection and analysis

In this study, force on MTP joint and EMG signal were recorded with few natural footsteps with varying heel heights of ladies high heel footwear. Typical recordings for flat footwear (heel height: 0cm) and semi-high heel

footwear (heel height: 5.08cm) are shown in Fig. 2 and Fig. 3 respectively. In Fig. 2, it is observed that the forces on MTP joint for each footstep are nearly constant and the mean force of these footsteps is around 12.82 kg on MTP joint. EMG recording is shown in Fig. 2 also that shows the activity of skeletal muscles in leg for flat footwear. EMG shows irregular muscle contraction and constant muscular contraction is not found due to the poor electrode-skin contact. Poor electrode-skin contact might be resulted for strong muscle tension or electrode slippage.

(a) (b) (c)

Fig. 1. Experimental setup for (a) sample development (b) EMG recording and (c) force recording

In Fig. 3, it is observed that the forces on MTP joint for each footstep are nearly constant and the mean force of these footsteps is around 19.60 kg on MTP joint. EMG recording is shown also in Fig. 3 that shows the irregular contraction of leg muscles for 5.08cm heel height. Analysis of irregular muscle contraction according to EMG is very difficult. The above analysis results increased force on MTP joint with the increase of heel height.

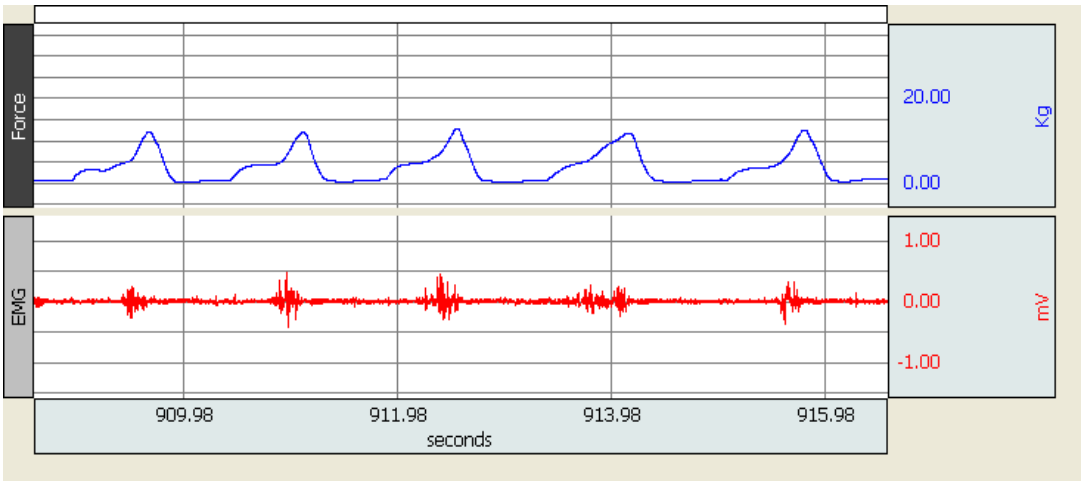

Fig. 2. Force and EMG recording for flat footwear (heel height: 0 cm)

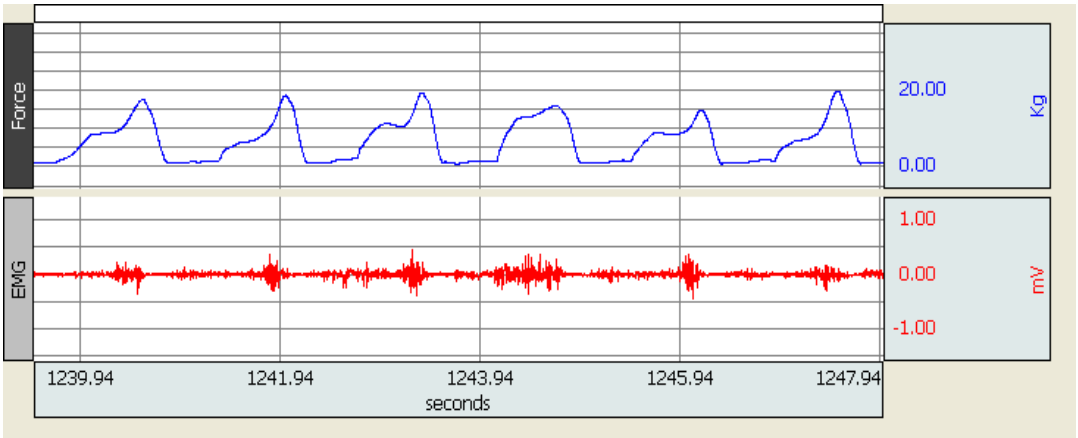

Fig. 3. Force and EMG recording for high heel footwear (heel height: 5.08 cm)

4. Results and discussion

Body weight imposed on MTP joint is investigated by varying heel heights of ladies footwear. Different heel heights and their corresponding forces on MTP joint are listed in Table 2. As the heel height increases the pressure beneath the forefoot surface increases. This phenomenon occurs due to the hips and spine of the body is out of alignment as a result the high heels push the body weight to the forward direction of the foot. As the height of the heel increases the bodyweight concentrates more on MTP joint and the bases of proximal phalanges of foot. From Table 2, it is seen that around 12.82 kg bodyweight is imposed on MTP joint for flat footwear (heel height: 0cm). For 2.54cm heel height the imposed bodyweight on MTP joint is 18.87 kg. As the heel height is increased more, the more bodyweight is imposed on MTP joint. 19.60 kg, 20.43 kg, and 21.95 kg bodyweight is imposed on MTP joint for 5.08cm, 6.35cm, and 8.89cm heel height respectively. Around 6.05 kg bodyweight is imposed more on MTP joint due to the transition from flat footwear (heel height: 0cm) to semi-high heel footwear (heel height: 2.54cm). The largest portion of bodyweight is more imposed for above transition and other transitions (heel height: 2.54cm to 5.08cm, 5.08cm to 6.35cm, and 6.35cm to 8.89cm) result significantly less increment in bodyweight imposition on MTP joint.

Table 2. Body weight imposed on MTP joint with varying heel height

Subject's weight (kg)	Heel height (cm)	Average body weight imposed on MTP joint (kg)*
60	0	12.82±0.584
	2.54	18.87±0.610
	5.08	19.60±0.432
	6.35	20.43±0.545
	8.89	21.95±0.795

*Values are Mean±Standard Deviation

The percentage of bodyweight imposed on MTP joint with varying heel height is shown in Fig. 4. It is found that about 21.37%, 31.45%, 32.67%, 34.05%, and 36.59% of bodyweight is imposed on MTP joint for the heel height of 0cm, 2.54cm, 5.08cm, 6.35cm, and 8.89cm respectively. Around 10.08% of bodyweight is imposed more on MTP joint due to the transition from flat footwear (heel height: 0cm) to semi-high heel footwear (heel height: 2.54cm). The largest percentage of bodyweight is more imposed for above transition and other transitions (heel height: 2.54cm to 5.08cm, 5.08cm to 6.35cm, and 6.35cm to 8.89cm) result significantly less increment about 1.22%, 1.38%, and 2.54% respectively. Analysis shows that 2.54cm heel height results significant imposition of bodyweight on MTP joint but later increment of heel height fails to do so. As the heel height increases more from 2.54cm, the stress on the phalanges of the foot increase that hinders proper functioning and results uncomfortable situations. Long term high heel footwear wearing may create structural damage in foot that would feel severely in old ages.

The limitation of the study is that only one subject is used for this experiment to collect data due to unavailability of the subject. Further research should be conducted to determine the effects of ladies high heel on human foot for different age groups.

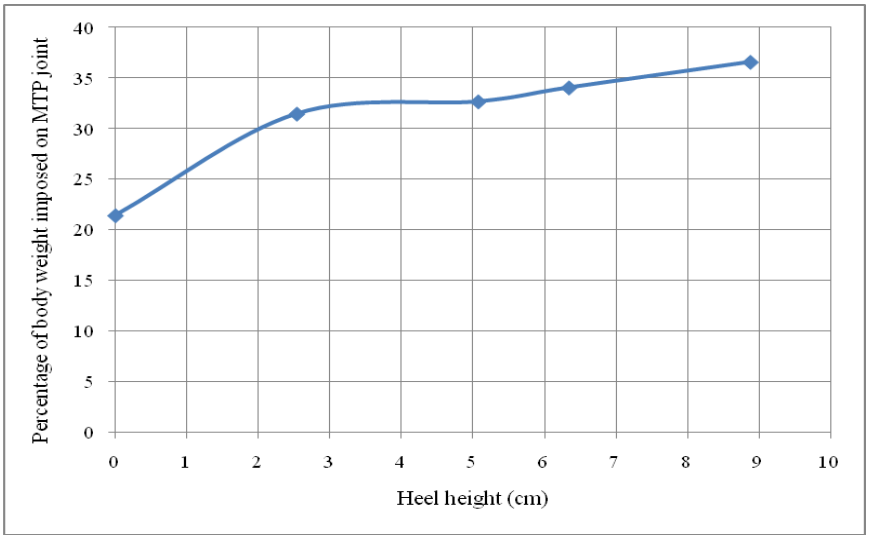

Fig. 4. Percentage of body weight on MTP joint with varying heel heigh

5. Conclusion

The effects of ladies high heel footwear on MTP joint are investigated in this study by bodyweight distribution. The percentage bodyweight imposed on MTP joint increases with the increase of heel height. Semi-high heel results significant percent of bodyweight imposition on MTP joint but high heel can't. High heels not only create a more unstable posture but also impose additional compressive forces on MTP joint that significantly increase discomfort, fatigue level, and foot pain by long term wearing.

6. Acknowledgement

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Characterization of textile ETP sludge to assess heavy metals and their environmental impact assessment

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Abstract

Worldwide industrial solid management is a big issue. Discharging of effluent treatment plant (ETP) sludge is one of the major concerns due to containing toxic heavy metals. In this study, textile effluent treatment plant (ETP) sludge was characterized to assess the heavy metals and their impact assessment. The ETP sludge sample was collected from a renowned textile industry. The sludge was air dried, grinded and sieved. Grinded sludge was acid digested and the aliquot was analyzed by atomic absorption spectroscopy (AAS). The heavy metals content of sludge chromium (Cr), lead (Pb), and arsenic (As) were 31.8±1.2 mg/kg, 3.8±0.3 mg/kg, and 1.2±0.1 mg/kg respectively. Cadmium (Cd) was in the sludge below the detection limit. Disposing of sludge containing heavy metals could be leached in groundwater that could be a great threat for the forthcoming generation.

Keywords: Textile ETP sludge, Heavy metals, acid digestion, Environmental impact

1. Introduction

Increasing industrialization disposing of industrial solid or liquid waste including mine tailings metallurgical slags, and municipal sewage sludge are also increased [1– 4]. Growing awareness and the impact of human activities on the environment is threatening both to the nature and the forthcoming generation. Contamination of sediment, soil or water with heavy metals is a big threat to the environment. Hence, heavy metal mobilization and subsequently leaching into groundwater or surface water or enter the human food chain through various chemical and biological processes.

Textile and clothing industries are very important as they fulfill the second basic requirements of a human being for their living and Bangladesh is the second-largest garment exporter in the world [5]. The Textile and leather industries are rapidly growing industry in Bangladesh and these industries get popularity due to easy availability of labor and low labor cost. Both the industry contributes a lot to the economy of the country. Most industries are located near or on the bank of the rivers or lakes where they dump effluents directly without proper management. In the fiscal year 2013-14 total exporting was $30.18 billion where textile was $24.50 billion [6].

Although textile is a very important sector for our economy but it is one of the most polluting industries in our country. During manufacturing process it consumes huge amount of water, dyestuffs and synthetic chemicals, which mainly based on heavy metals. Wastewater from textile requires a complicated treatment like physicochemical or biological treatment due to containing high concentration of pollutants, complex composition and high concentration of dyes [7].

Unfortunately, most of the textile and leather industries have no effluent treatment plant (ETP). After production, process residues of industries are discharged directly to the rivers, canals or lakes. Wastewater for irrigation, solid waste disposal, sludge applications, vehicular exhaust and industrial activities are the major sources of soil contamination with heavy metals. Few of the industries have ETP and the produced sludge is used for land filling which contain heavy metals e.g. chromium (Cr), lead (Pb), arsenic (As) cadmium (Cd) etc. Heavy metals from the sludge subsequently leach into groundwater, which causes groundwater contamination. Small amount of heavy metals is beneficial for the metabolism of our body and beyond the limit is dangerous to our body and can cause different disease [8].

In this study, textile effluent treatment plant (ETP) sludge was characterized to assess the heavy metals and their impact assessment. The textile ETP sludge was collected from a textile industry. After sieving, grinded sludge was acid digested and aliquot was analyzed by the atomic absorption spectroscopy (AAS).

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2. Materials and Methodology

2.1 Sample collection

The textile ETP sludge was collected in polyethylene bag from a textile industry, Dhaka. The sample was air dried, grinded with mortar and sieved with 80-mesh. After sieving, the sample was homogenously mixed (Fig. 1).

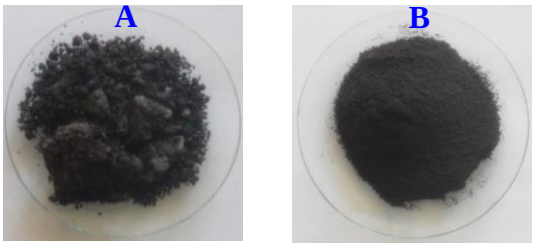

Fig. 1. Textile ETP sludge **a**) after sun dried and **b**) after grinding with sieved

2.2 Reagents

All the stock solutions were prepared from the analytical grade (AR). The sludge was acid digested with nitric (Merck KGaA, Germany) and hydrogen peroxide (Merck, India) and was collected from the Khulna scientific store. Freshly prepared double deionized water, from a quartz still, was used in all experiments. Chromium (Cr), arsenic (As), lead (Pb) and cadmium (Cd) standard solution were from the Fluka-Analytical, Switzerland. To reduce As(V) to As(III) 20% potassium iodide (Sigma-Aldrich, USA) solution was used. Arsenic trihydride (AsH_3) generation was performed with 5M HCl (Sigma-Aldrich, USA), 0.6% sodium borohydride solution (Sigma-Aldrich, USA).

2.3 Acid digestion

The homogeneously mixed sample was acid digested following the EPA Method 3050B. About 2.0 g sample was acid digested with nitric acid (HNO_3 65%, Merck KGaA, Germany). The acid mixed samples were heated, refluxed on hot plate for several hours and occasionally nitric acid was added until no brown fumes was given off. Then the mixture was cooled and hydrogen peroxide (H_2O_2 30%, Merck, India) was added. The mixture was then heated, refluxed on hot plate and hydrogen peroxide was added until the effervescence was minimal or the mixture appearance was unchanged. The mixture was heated continuing until the volume had become 5 mL. Then 50 mL deionized water was added and again heated for another one hour. The mixture was then cooled, filtrate through filter paper (Whatman No.1) and the solution was made up 100 mL with deionized water. The filtrate (aliquot) was preserved in high-density polyethylene (HDPE) bottle at 4°C until to complete metals analysis.

2.4 Analysis heavy metals with AAS

Acid digested aliquot was analyzed by the atomic absorption spectroscopy (SpectrAA-220, VARIAN, Australia) for the quantitative measurement of chromium (Cr), arsenic (As), lead (Pb) and cadmium (Cd). Arsenic was measured by the hydride vapor generation method using sodium borohydride as a reducing agent, carrier gas argon (Ar) at the wavelength of 193.7 nm. Chromium (Cr), lead (Pb) and cadmium (Cd) were measured direct flame (air-acetylene) at the wavelength of 357.9 nm, 217.0 nm and 228.8 nm, respectively.

3. Results and Discussion

3.1 Physiochemical properties of sludge

The color of the sludge was dark brown due to having dyestuffs. The pH of the sludge was 6.7±0.01.

3.2 Heavy metals content in sludge

Heavy metal content in the sludge is shown in the Table 1. The heavy metal Cr, Pb and As were 31.8, 3.8, and 1.2 mg/kg respectively. The Cd was below the detection limit. The amounts of Cr were higher among the metals. It may be the reason is that metals complex dyestuffs contain Cr were used in textile dyeing process. Generally in textile and leather dyeing metal complex dyestuffs are used because it form coordination bond,

which offer high fastness properties. The most commonly used metal in dyestuffs production is Cr. However, the source of arsenic is from the source of reagents or chemicals used for the textile process.

Table 1. Heavy metals (mg/kg)

Chromium	Lead	Arsenic	Cadmium
31.8±1.2	3.8±0.3	1.2±0.06	BDL*

*BDL→ Below Detection Limit

3.3 Comparison of metals content in soil with present study

Different countries have set their standards for heavy metals content in ETP sludge or agricultural soil. The comparison of the metals content in soil is presented in Table 2

Table 2. Comparison of heavy metals content in soil with present study

	Permissible limits of heavy metals (mg/kg)				
Metals	This study	USEPA [9]	India [10]	SEPA [10]	Bangladesh [11]
Cr	31.8 ± 1.2	3000	-	250	Max. 50
Pb	3.8 ± 0.3	300	250-500	350	Max. 30
As	1.2 ± 0.06	41	-	-	-
Cd	BDL	39	3-6	0.6	Max. 5

It is clear from the Table 2 that Cr content in ETP sludge was lower than the standard level of Bangladesh. The Cr content in the sludge was 31.8±1.2 mg/kg where Bangladesh was 50 mg/kg. According to SEPA limit sets by China it is 250 mg/kg. The amount of Pb was in the sample 3.8±0.3 mg/kg, which was lower than the standard level of Bangladesh. The amount of As in the sample was determined 1.2±0.06 mg/kg and it is also below the standard limit set by the USEPA.

3.4 Heavy metals leaching from sediment/soil

Many factors are responsible for leaching of heavy metals from the sediment/soil. pH is one of the most important factors for leaching of heavy metals from the sediment/soil into groundwater. At low pH, the solubility of Pb and Cd are increased whereas Cr and As forms different compound or complexes at different pH. Higher the pH, solubility of Cr(VI) and As(III) are increased. The mobility of As(III) compounds is 4-10 times higher than the As(V) compounds [12]. Cr(III) and As(V) are the least mobile. Soluble and un-adsorbed chromium complexes can leach from the sediment/soil into groundwater. The pH of the sludge was around neutral (6.7). If the pH changes for any causes, it will be horrible for our mankind as well as for the ecology. Other different factors e.g. temperature, amount of organic matter in soil, residual time of the sludge and soil texture and pore structure plays an important role in leaching the heavy metals [13].

3.5 Effect of Cr, As, Pb and Cd

Chromium is associated with allergic dermatitis in humans [13]. Cr(III) has a toxic effect upon daphnia, thus disrupting the food chain for fish life and possibly inhibiting the photosynthesis even in low concentration. Dichromate is toxic to fish life since they swiftly penetrate into cell wall.

Arsenic compounds are adsorbed strongly to sediment/soil. It transports only over short distances into groundwater as well as surface water. It is associated with skin damage, increased risk of cancer, and problems with circulatory system [14]. Children are exposed to arsenic show impaired learning and memory, sleep disturbances, abnormality and hearing problem [15]. Patients are exposed with arsenic may disorder of brain, impairments of higher neurological functions including learning, memory and attentiveness [16].

Inhalation and ingestion are the two routes of Pb exposure and effects from the both are same. Pb accumulates in the body organs (brain), which may lead to poisoning or even death. Children exposed to lead are at risk for impaired development; lower IQ, shortened attention span, hyperactivity, and mental deterioration etc. with children under the age of six being at a more substantial risk. Cadmium is poisonous metal and it is very bio-persistent but has few toxicological properties [17].

4. Conclusion

The study reveals that the textile ETP sludge was mostly contained with chromium. The heavy metals like arsenic and lead were in low concentration. The cadmium was too low that was below detection limit. Although

metals contents were low; landfilling with this sludge could enhance leaching of chromium from sludge to groundwater in the near future. It could be better to develop a process to remove the heavy metals from the sludge so that it could be favorable to the environment and also for the future generation.

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A Finite Element Analysis to Design and Simulation of Effective Shank Piece in Ladies High Heel Shoe

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Abstract

High heel shoes are a type of footwear typically worn by ladies. In ladies high heel shoe, shank is a very important component. It mainly made of steel, metal, wood, plastic but mostly steel shank piece are used. Before using a steel shank piece for high heel ladies shoe, it is important to know its effectiveness. For this purpose several prototype shank pieces are tested in biomechanics lab over and over. It is uneconomical and time consuming. To reduce the cost and time consuming trial and error cycle, a finite element based method can be used to simulate the performance test of the shank piece. In this study a shank piece with high heel was designed in Solid Works Software 2013. After that by assigning the material properties of cast carbon steel shank piece, the simulation was carried out to get the analyzed results. The simulation results reveal that in static analysis the maximum von messes stress is 914216704 N/m^2 and minimum von messes stress is 3.49019 N/m^2. The maximum strain is 0.00279003 and minimum strain is 2.77075e-011. The shank piece will be withstand up to maximum 10000 cycles and minimum 6000 cycles. Finally the simulated results were verified for the model correctness based on the comparative study with the standard requirements of the shank piece .The study represent that the Finite Element Analysis is an emerging approach in footwear industry for reducing time and cost in prototype making.

Keywords: High Heel Shoe, Shank Piece, Performance Test, Finite Element Analysis

1. Introduction

High heel shoes are a type of footwear typically worn by ladies where the heel of the foot is raised significantly higher than the ball of the foot and the toes [1]. It tend to give the aesthetic illusion of longer, more slender legs. High heel infuse the wearer with a sense of power, more importantly feminine power [2]. It come in a wide variety of styles, and the heels are found in many different shapes, including stiletto, pump (court shoe), block, tapered, blade, and wedge.

"High heels" covers heel height ranging from 2 to 5 inches (5.1 to 12.7 cm) or more. Extremely high-heeled shoes, such as those exceeding 6 inches (15 cm), strictly speaking, are no longer considered apparel but rather something akin to "jewelry for the feet". They are worn for display or the enjoyment of the wearer.

In ladies high heel shoe, shank is one of the vital component. The shoe's shank is the center section or bridge between the ball line sole and the front of the heel [3]. The shank bridges between the heel breast and the ball tread. The shank spring lies within the bridge or waist of the shoe, i.e. between heel and ball corresponding to the medial and lateral arches [4]. The shankpiece reinforces the waist of the shoe and prevents it from collapsing or distorting in wear. It provides essential support for the arch, maintains accurate alignment of forepart and heel, helps to retain the shoe shape, maintains structural stability in high heeled shoes and gives strength to the waist of footwear, absorbs shocks, resists stresses and strains, during weight bearing of the body [5]. It extending downwardly and forwardly from the first portion of the arch support, and a toe support region which extends at an inclination upwardly and forwardly from the shank where upon the first metatarsal of the wearer is

buttressed by the phalanges of the wearer to prevent forward sliding of the foot of the wearer relative to the shoe, thereby preventing jamming of the human digits into the toe portion of the shoe [6].
The higher the heel height of the shoe the stronger the shank piece material must be. It mainly made of steel, wood, plastic, metal and mostly steel shank piece are used [7]. Before using a steel shank piece for high heel ladies shoe it is important to know its suitability, strength, flexibility, deformation scale, fatigue life, stiffness for the specific heel height of shoe .For this purpose several prototype shank pieces are tested in biomechanics lab over and over to obtain optimum properties for production [8]. The process is time consuming and costly. To reduce the cost and time consuming trial as well as error cycle a finite element analysis (FEA) based method could be used to analyze the performance test of the shank piece.

Amongst various computer aided design (CAD) simulation techniques; Finite Element Method (FEM) is becoming more and more popular because of its versatile and controllable accuracy [9] in modeling irregular geometric structure, complex material properties and ease of simulating complicated boundary and loading conditions in both static and dynamic simulation [10].

Numerical simulation technique such as the finite element method (FEM) could allow the realistic simulations of foot as well as footwear interface which could offer in-depth biomechanical information including internal stress and strain distributions of modeled structure [11]. Finite element analysis is a computerized method for predicting how a product reacts to real-world forces, vibration, heat, fluid flow, and other physical effects [12].

Simulation is a tool that extensively used in manufacturing system design and analysis for more than 40 years [13]. Computer aided engineering technique allow rapid change of input parameters to analyze their subsequent effects in virtual simulation environment without needing to conduct actual experiment.

In this study, an attempt has been taken to design a solid model of shank piece in order to evaluate the physical properties of a cast carbon steel shank piece such as von misses stress, strain, displacement, fatigue life using Finite Element Analysis (FEA). Moreover, most previous researches focused on the finite element analysis of biomechanics of the human foot, foot-footwear interaction, reduction of planter heel pain through insole but no research has been performed regarding the analysis of effective and durable shankpiece development which plays a vital role for the sustainability of a high heel shoe.

2. Methodology

2.1 Creating Solid Geometry of High Heel Shank Piece

An accurate solid geometry of a shank piece with high heel was designed by SolidWorks 2013 software (SolidWorks Corporation, Massachusetts) for analyzing the performance properties of a shank piece. In **Fig. 1** shows the physical view of ladies high heel shoe and **Fig. 2** represents the solid geometry of shank piece.

Fig. 1. Ladies High heel shoe

Fig. 2. Shank Piece with heel shoe

2.2 Assigning Material Properties

The selected material properties were assigned to evaluate actual physical properties of cast carbon steel shank piece such as elastic modulus, Poisson's ratio, mass density, tensile strength, yield strength, thermal co-efficient. Materials properties often vary to some degree according to the direction of material in which they were measured. Material properties that relate two different physical phenomena often behave linearly in a given operating range, and may then be modeled as a constant for that range. The material properties of cast carbon steel are given below-

Table 1. Material Properties of Cast Carbon Steel Shank piece

Material Property	Value	Unit
Yield strength	2.48168e+008	N/m^2
Tensile strength	4.82549e+008	N/m^2
Elastic modulus	2e+011	N/m^2

Poisson's ratio	0.32	
Mass density	7800	kg/m^3
Shear modulus	7.6e+010	N/m^2
Thermal expansion coefficient	1.2e-005	/Kelvin

2.3 Loads and Boundary Condition

In case of finite element anlysis (FEA) an important step is the application of load and boundary condion. The force is applied on the top surface of the shank piece vertically depending on the body weight where the heel portion of the shank piece which is connected to the high heel was subjected to fix geometry. In this study 588 N force that was equal to the 60 kg weight of human body was applied on the shank surface (Fig. 3).

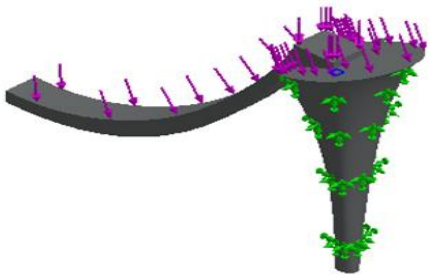

Fig. 3. Loads and Fix Geometry of the Shank Piece and High heel

2.4 Meshing the Solid Geometry of Shank Piece

Meshing is a crucial step in FEA. The automatic mesher in the software generates a mesh based on a global element size, tolerance and local mesh control specifications. A mesh consists of one type of elements unless the mixed mesh type is specified. The size of the generated mesh depends on the geometry and dimensions of the model, element size, mesh tolerance, mesh control, and contact specifications. The accuracy of the simulation mainly depends on the meshing. The solid model of the shank piece was finely meshed (Fig. 4).

Fig. 4. Meshed Model of Shank Piece

2.5 Running the Simulation Analysis

The simulation analysis of the shank piece was carried out for static (stress, displacement, equivalent strain), fatigue life analysis.

2.5.1 Von Misses Stress Analysis

The von misses yield criterion suggests that the yielding of materials begins when the second deviatory stress invariant reaches a critical value. For this reason, it is sometimes named as the -plasticity or flow theory. It is part of a plasticity theory that applies best to ductile materials, such as metals. Von misses stress is widely used by designers to check whether their design will withstand in a given load condition. It is considered to be a safe haven for design engineers. The force was applied on the top surface of the shank piece, then the von-misses stress was found, maximum and minimum values were noted.

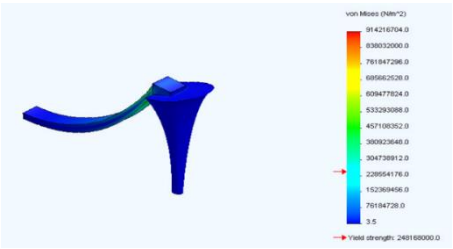

Fig. 5. Von Misses Stress Analysis

Table 2. Von Misses Stress Analysis

Name	Type	Min	Max
Static	Von Misses Stress	3.49019 N/m^2	9.14217e+008 N/m^2

2.5.3 Equivalent Strain Analysis

This analysis was carried out to know where the strain was occurred in shank piece in case of stress. It is the relative change in shape of an object due to applied force. It is an actual physical quantity related to the change in dimensions. In most cases strain is known as equivalent of strain.

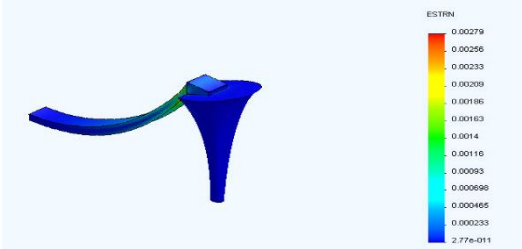

Fig. 6. Equivalent Strain Analysis

Table 3. Equivalent Strain Analysis

Name	Type	Min	Max
Static	ESTRN: Equivalent Strain	2.77075e-011	0.00279003

2.5.3 Displacement Analysis

This analysis was carried out to know how much the cast carbon steel shank piece is displaced by load from its original condition mainly in the fore part which is very essential portion to maintain the shape of the shoe and human foot**.**

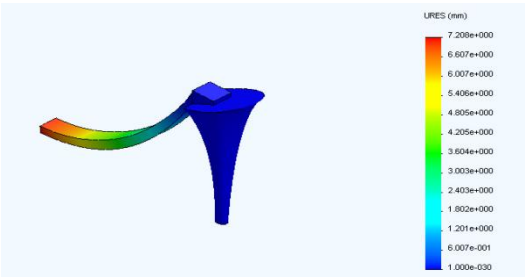

Fig. 7. Displacement Analysis

Table 4. Displacement Analysis

Name	Type	Min	Max
Static	URES: Resultant Displacement	0 mm	7.20802 mm

2.5.4 Fatigue Analysis

Fatigue is the progressive and localized structural damage that occurs when a material is subjected to cyclic loading. This analysis was carried out to ensure how long the shank piece will be survived. Prior to fatigue analysis, the stress and strain analysis were performed. The fatigue analysis includes the damage plot (Fig. 8)

and the S-N curve (Fig. 9).

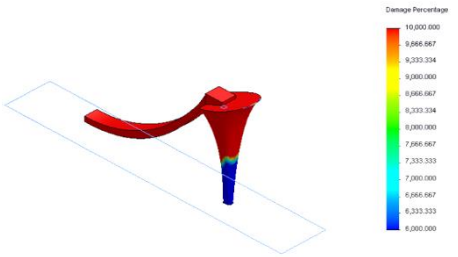

Fig. 8. Fatigue Life of Shank Piece

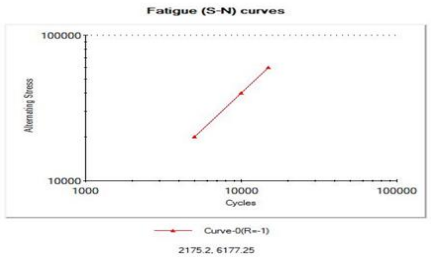

Fig. 9. S-N Curve

Table 5. Fatigue Life Analysis

Name	Type	Min	Max
Fatigue Life	Damage plot	6000	10000

3. Results and Discussion

The static simulation results reveal that in the back part of the shank piece (fig.5) the maximum von misses stress is 914216704 N/m^2 and minimum von misses stress is 3.49019 N/m^2. The red marked portion (fig.5) which is located in the back part of the shank piece where mainly maximum stress occur and the blue marked portion shows that there is minimum stress .Von misses stress is widely used by designers to check whether their design will withstand a given load condition. Bearing this information an engineer could state his/her design will fail, if the maximum value of von misses stress induced in the material is more than strength of the material [14]. Also in the back part of the shank piece (fig.6) the maximum strain is 0.00279003 and minimum strain is 2.77075e-011. Practical measurements of the strains on the shank have also shown that the major strain occur on the shank back part. From stress and strain analysis, it was found that the maximum stress and strain was occurred in the back part of the shank piece. Therefore, material density should be high in this portion to prevent crack development.

The displacement result reveals (Fig.7) that in the forepart of the shank piece, the maximum displacement occur which is 7.20802 mm and in the back part the minimum displacement occur which is 0 mm .The fatigue analysis (Fig.8) shows that the cast carbon steel shank piece will be sustained maximum 1000 cycles and minimum 6000 cycles. According to the fatigue life measurement done by SATRA, the shank piece is near to the grade 3 at the maximum cycle stage. The life cycle of the cast carbon steel can easily be extended by increasing the proportion of elastic modulus, mass density, tensile strength, yield strength.

The validation of the finite element analysis results mainly depends on the accurate materials properties selection. In this study, this factor was maintained properly. Knowing about the effectiveness of the shank piece is very essential before using the in high heel design construction which can easily be done by finite element analysis.

4. Conclusion

In this research paper, complete solution of virtual prototype making of shank piece and evaluation of its physical performance was presented. In industry, vast production of a product mainly depends on prototype model that has to pass through time consuming trial and error cycle for its performance testing. In this case finite element analysis can reduce time consuming trial and error cycle. The effectiveness of the shank piece under loading condition during walking can be easily predicted by finite element analysis. The simulated results which were found on finite element analysis facilitates to take decision on which portion more attention should be given for making stable shank piece for using as well as for protecting the foot on sudden failure.

Besides computer aided design (CAD) provides extra facility to footwear industry for product design and development and its sustainability during wearing life.

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Design and Optimization of a Long Range Jet Aircraft

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Abstract

The design of a long range jet aircraft is presented in this paper. Design requirements were selected from market analysis. The minimum requirements for aircraft design, extracted from market analysis were: - range: 5200 nm, maximum Mach number: 0.8, ceiling: 43000 ft., passenger: 280, load factor: ranging from +3.5 to -1.5. The aircraft should be capable of carrying two crew members. The aircraft had to materialize a certain mission profile. This mission profile contains the flight segments like taxi, take-off, climb, cruise, loiter, descend, climb, cruise to alternate destination, descend and landing. In addition to, the complete aircraft design was accomplished through three basic phases like conceptual, preliminary and detail design. Initially in conceptual design phase, configuration of the basic components of aircraft were selected through figure of merit analysis. After that preliminary design, detail design has been done. At last weight of different components was calculated.

Keywords: Aircraft design, Aerodynamics, Thrust, Figure of merit analysis, Mission Profile.

1. Introduction

From first flight of the Wright Flyer I, December 17, 1903 to the first flight of the Boeing 787 Dreamliner December 15, 2009 a lot has changed in the history of aircraft design, construction and maintenance. In between this period so many different methodologies and approaches have been explored by different designers. And a revolutionary advancement has been observed. But in reality, a lot is yet to come. So many are trying to solve different complication related to the design methods standing for a long time. Old complications are being solved and new are coming. Nowadays, aircraft designing is one of the most challenging sectors due to its huge manufacturing costs and sensitivity. A single paper is not enough for understanding all complications and other challenges related to this field. But in this work, the design of long range jet liner is presented optimizing all those complications and challenges. It will be really helpful for a beginner who wants to design a passenger aircraft.

2. Design Requirements

Here the aim is to design and optimize a Long Range Jet Aircraft following these given requirements.

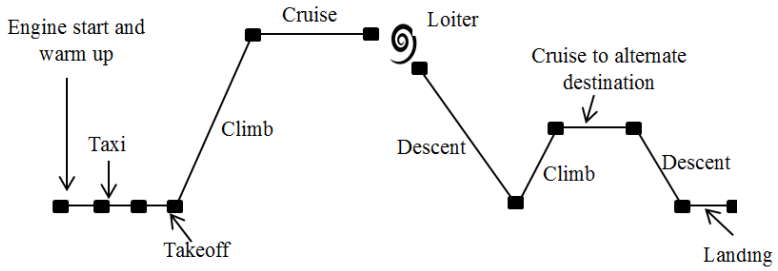

Fig. 1. The mission profile

Parameters	Min. Requirements
Range	9600 km, 5200 nm
Max. Mac	0.8
Ceilling	43000
Pax.	280
Load Factor	+3.5 ; -1.5
Crew	2

Table 1. Design Requirements

3. Conceptual design

Following the design requirements and using figure of merit (FOM) analysis, the following configuration was optimized for the design.

Fig. 2. Conceptual sketch of side view

Fig. 3. Conceptual sketch of top view

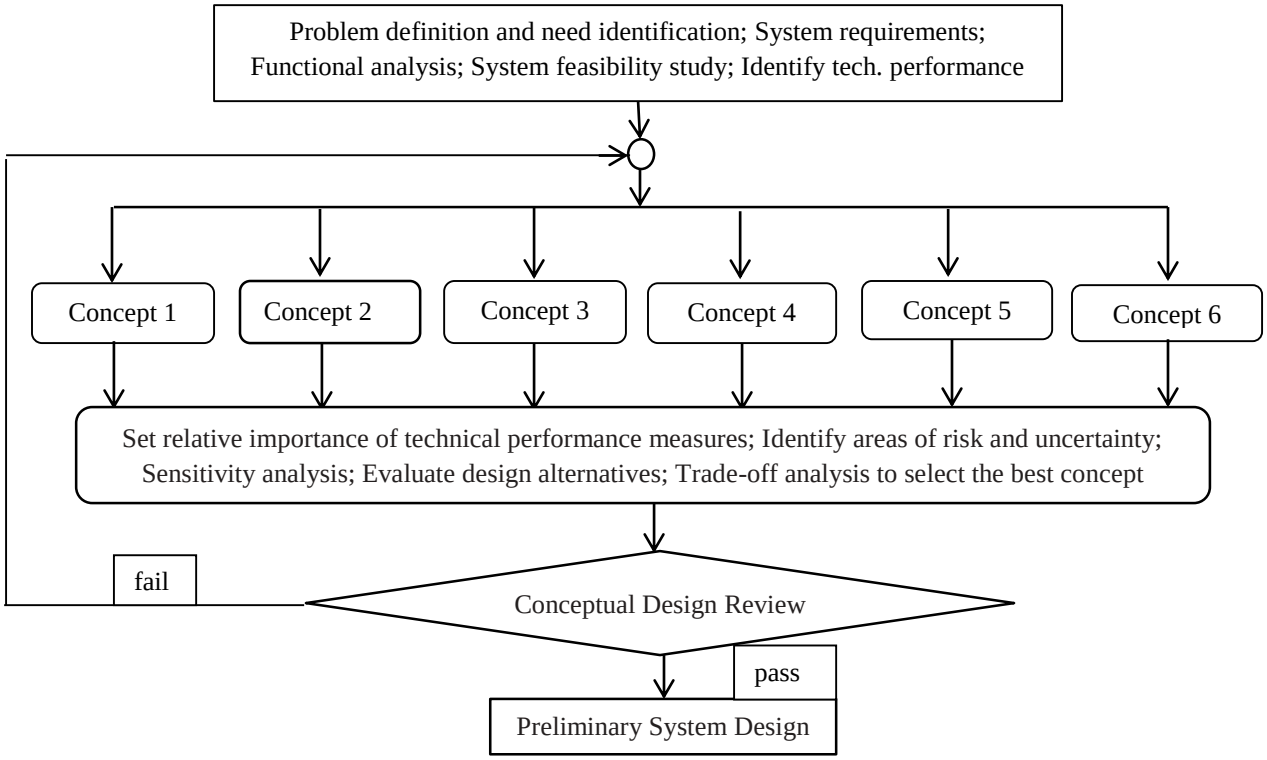

Fig. 4. Conceptual design flowchart

4. Preliminary design

Based on the requirements first the empty weight and the takeoff weight of the aircraft were calculated. Here- Weight of the crew, W_c= 1100 lb; Weight of payload, W_{pl}= 78400 lb; Fuel weight ratio for 1st cruise segment = .834; Fuel weight ratio for loitering = .99 ; Fuel weight ratio for 2nd cruise section = .884 ; Overall fuel weight ratio=.664 ; By calculating, Empty weight, W_e= 342921.82 lb ; And , Takeoff weight, W_{to}= 6.527×10^5 lb ;

The next step is to determine the wing area, S_w and engine size. Here, the matching plot was used for final results.

Five basic equations used for the matching plot are-

(1) Stall speed: $(\frac{W}{S}) = 110.682\ lb/ft^2$; (2) Maximum speed: $(\frac{T}{W})_{max} = (\frac{30.66}{\frac{W}{S}}) + (3.067 \times 10^{-4})(\frac{W}{S})$; (3)

Takeoff Run: $(\frac{T}{W})_{Sto} = \frac{.04 - .05448[exp(\frac{13.949}{\frac{W}{S}})]}{1 - exp(\frac{13.949}{\frac{W}{S}})}$; (4) Rate of climb: $(\frac{T}{W})_{ROC} = \frac{1.31}{2\sqrt{(W/S)}} + .04762$; (5)

Service ceiling: $(\frac{T}{W})_{ROCc} = 0.1133\frac{1}{2\sqrt{\frac{W}{S}}} + 0.2226$

Combining these equations and using Matlab, following matching plot is obtained-

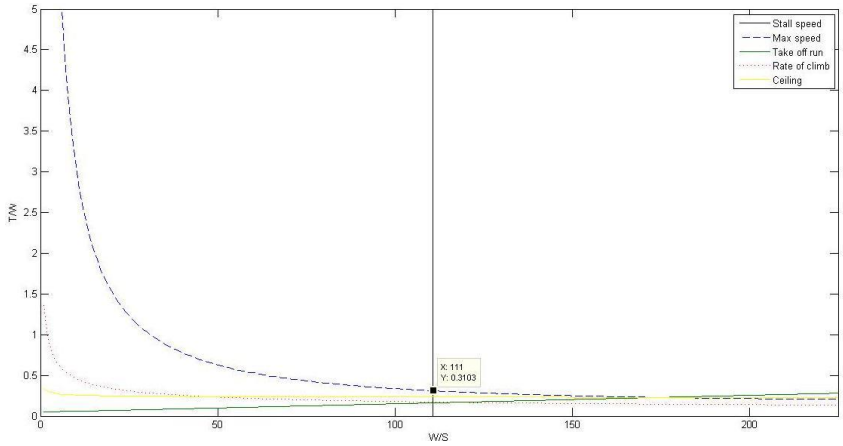

Fig. 5. Matching plot

From matching plot the final results are shown below-
Wing Area, S_w= 5880.18 ft^2 ; Engine thrust, T= 202532.81 lb

5. Detail design

5.1. Wing Design

By using FOM analysis, selected configuration is monoplane, low wing, swept back, tapered configuration for aircraft. Then the aerofoil was selected based on the following results:

Aircraft average weight at cruise, W_{avg} = 2082521.7 N ; Required A/C cruise lift coefficient, C_{lc} = 0.223; Required Takeoff lift coefficient, $C_{l\ t/o}$ = 1.9;

"Trailing edge single slotted flap" was used as high lift device. Now the geometry of wing and HLD is-

AR of the wing, AR_w=11 ; Span of the wing =254.32 ft ; Span of the flap, b_f = 152.592 ft ;
Mean chord of the wing,C_w=23.12 ft ; Mean chord of the flap, C_f = 11.56 ft; Sections ideal lift coefficient, C_{li}=0.26; Sections max lift coefficient,$C_{l\ max\ gross}$= 2.22; Lift coefficient of the high lift devices, $C_{l\ HLD}$= .65;

Then, Maximum lift coefficient, $C_{l\ max}$ = ($C_{l\ max\ gross}$ - $C_{l\ HLD}$) = ***1.57***

Based on this result, ***"NACA 64$_1$-212"*** was selected from the NACA aerofoil chart [1] for the aircraft.

The characteristic curves [2] of the aerofoil are given below-

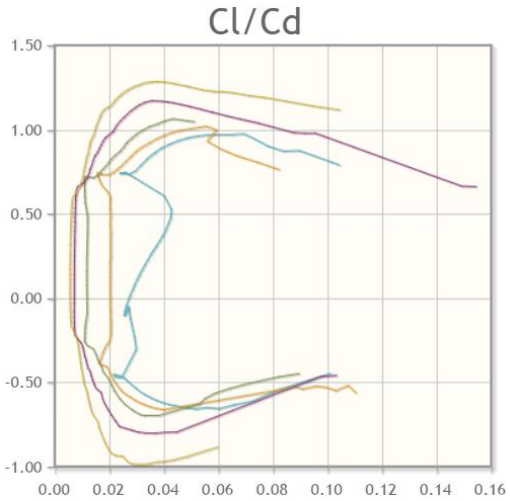

Fig. 6. lift vs drag coefficient curve for diff. reynolds no

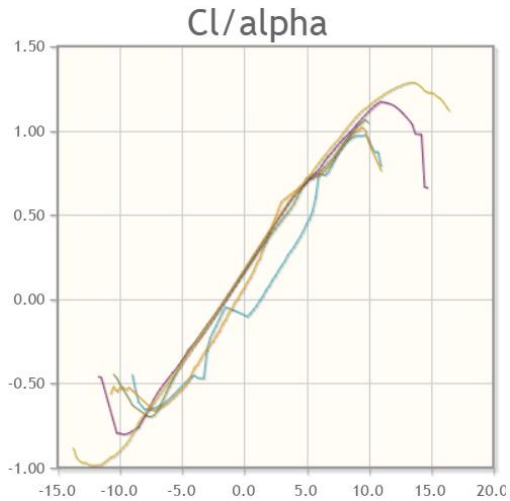

Fig. 7. lift coefficient vs AOA curve

Other findings are,
wing incident angle, i_w= 2° ; wing sweep back angle at mid chord= 30°; twist angle= 1.5 ; tapper ratio= 0.2;

From following result, using Matlab it was checked that if the lift distribution over the wing is elliptical or not. And a positive result was found. The lift distribution curve is shown below,

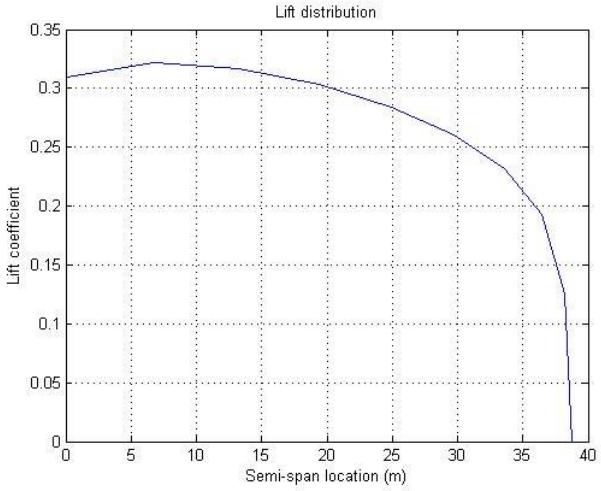

Fig. 8. Elliptical lift distribution over a wing

Other important findings were, α_{flap}= -20° ; wing drag, D_w= 308789 N ;
wing pitching moment, M_{0w}= -9166458.64 ; Root chord,C_r = 10.23m ; Tip chord, C_t= 2.042 m .

Other important results are :
Flap angle, α_{flap}=-20° ; wing drag, D_w=308789 N ; wing pitching moment, M_{ow}= - 916645.64 N-m ;
root chord, C_{root} =10.23 m ; tip chord, C_{tip}= 2.042 m.

5.2. Tail design

5.2.1. By using FOM analysis conventional Aft tail was selected.

5.2.2. Horizontal tail Design: Important finding in the tail sections were, HT volume coefficient, V_h=1.1 [3] ; Optimum tail moment arm, L_{opt}=38.59m ; HT planform area,$S_h = \frac{VhCs}{Lopt}$ = 109.74 m^2; Wing fuselage Aerodynamic pitching moment coefficient, C_{mowf} = -0.0389 Horizontal tail lift coefficient at cruise, C_{lh}= -0.051
Based on these results and other general considerations, **NACA 0009** was selected for the tail section.

Other findings for Horizontal tail are-
Sweep angle, $\wedge_{.5c} = 30^0$; Dihedral angle = 6^0 ; Aspect ratio, A_{HT} =7.3 ; Tapper ratio, λ= 0.2; Span, b_h = 92.8 ft ; AR_{HT}= 7.3; Chord, C_h=12.72 ft; Root Chord, C_{root}=18.47 ft; Tip Chord, C_{tip}=3.69ft; Lift curve slope, $C_{l\alpha\text{-}h}$=4.79 ; AOA at Cruise, $\alpha_h = -0.613^0$; Downwash Angle = 1.5^0; Incident angle $I_h = -1.56^0$; $C_{m\alpha}$ =-4.51 rad^{-1} ; As the value of $C_{m\alpha}$ is negative so it has static longitudinal stability.
Finally iteration, optimization, stall checking as well as dynamic longitudinal stability was checked.

5.2.3. Vertical Tail Design:
The important findings were,
VT volume co-efficient, V_v= .009 [4] ; VT Moment Arm, L_v=126.6ft ; VT Planform area, S_v=1063 ;
Aerofoil NACA0009; Aspect ratio,AR_{VT}=2 ; Tapper ratio, λ= 0.3; Incident angle,i= 0°; Sweep angle, $\Lambda_{c/2} = 30^0$;
Dihedral angle = 0^0; Tip chord,C_t= 9.7 ft; Root chord, C_r=32.33 ft

Finally directional trim, Directional Stability and Optimization has been done.

5.3.Propulsion system design

In case of propulsion system design part, a/c performance, Engine manufacturing cost, Engine operating cost, Flight safety, Engine efficiency, maintainability, manufacturability, maximum thrust needed are the most important criteria what were needed to be considered.

Considering those the result and using FOM analysis the following were obtained,

> Turbofan (High bypass ratio) is more suitable for this a/c.

> Required thrust is more so four engines can be taken.

> It was decided to mount the engine under wing.

> We used “GE engine” for this a/c.

> Determined thrust for each engine $= \frac{953.66}{4}$ =238.4kN

> Based on thrust required, “GE CF6-50E2” engine was selected.

5.4. Fuselage design

Based on A/C type, payload, range, cruise ceiling, crew and passengers, fuelling, wingspan, mechanical and electrical system landing gear and other considerations the fuselage was designed and following were obtained.

Instrument for cockpit suggested by FAR part 25 section 25.1303 ; Optimum length to diameter ratio, L_f/D_f=12.21 ; Length of cabin, L_c=44.2 m; Total luggage volume , V_l=287*2*0.146 =83.8 m^3; No of LD1 container =17.1 17 cont;

Volume of fuel V_f=109.97 m^3; Fuselage available space, V_{avl} =62.848 m^3 ; Wing box volume, V_{wbox} = 45.53 m^3 ; Fuselage maximum diameter D_f=3.62 m ; Fuselage length will be L_f=188 ft ;Final fuselage length to diameter ratio, L_f/D_f=15.8; Upsweep angle, α_{us} =13.25°; Overall length $L_{overall}$=57.32 m ;

Finally lofting, Iteration and optimization are done.

5.5. Landing gear design

The main Considerations are ground clearance, tip back angle, take off rotation, overall angle, structural integrity, A/C ground stability, ground controllability, low cost maintainability and manufacturability. Based on the requirements and design considerations by FOM analysis we selected 'Tricycle landing gear", Retractable landing gear is selected. Height of the landing gear is H_{lg}= 4.17 m ; Distance between the main gear and the A/C forward CG is, X_{mg}= 7.8 ft ; Tip back angle is α_{tb}=22.27 ° which satisfy The clearance angle which is α_c=10 °>α_{to} ; wheel base, B=45.88 ; wheel track ,T=26.68 ft ; Overturn angle φ_{ot}=30.16 °>25 °;

5.6. weight of Component

1. Wing weight, W_w=3672074.254 N
2. Weight of HT, W_{ht}=355639.5 N
3. Weight of VT, W_{vt} =238841N
4. Weight of fuselage, W_f=766058.99 N
5. Weight of landing gear, W_{lg}=9676.97 N
6. Installed engine weight, W_{engine}=856883.7 N
7. Fuel system weight, W_{fs}=745.55 N

6. Conclusion

As the historical values are used for so many calculations, the results found sometimes were deviated from the exact result. In the preliminary design step, for determining required engine thrust and wing area we used matching plot as we are designing in the premature stage. More precise methods is used by the designer in the higher level. Scope is there to improve lift distribution over the wing as there is no limitation of betterment in this section. In the case of tail design, deviation of larger portion of air flow due to the disturbance created by the wing and fuselage, was not considered. That definitely affected the result. In case of propulsion system design, improvement can be done in case of thrust required due to change in altitude. Finally, it can be concluded that, though there are scopes of improvement in this design, but it represents the entire design and optimization of a long range jetliner, which will serve as a useful resource for the design enthusiasts.

7. Nomenclature

T- Engine thrust; S- Wing Area; W- Empty weight; C- Engine specific fuel consumption; Vc- Cruise speed; ROC- Rate of climb; HLD- High lift device; AR- Aspect ratio; μ- frictional coefficient; ʌ- sweep angle; λ- Tapper ratio; b- Wing span; n- Load factor.

8. References

[1] Mohammad H. Sadraey, AIRCRAFT DESIGN A Systems Engineering Approach, Page-192, Figure 5.23.

[2] UIUC Airfoil Coordinates Database Source dat file, http://airfoiltools.com/airfoil/details?airfoil=n64212-il.

[3] Mohammad H. Sadraey, AIRCRAFT DESIGN A Systems Engineering Approach, Page-303, Table 6.4.

[4] Mohammad H. Sadraey, AIRCRAFT DESIGN A Systems Engineering Approach, Page-303, Table 6.4.

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Consumption behavior of water: A case study of Chittagong City

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Abstract

Chittagong is second largest city in Bangladesh with huge population and heavy industrial areas. Due to high population and industrial activities water requirement is increasing day by day. Sufficient quantity and required quality of water is a major concern for drinking and other purposes for the city dwellers. Inadequate quantity and poor quality of water is a major problem in this city. The present water supply system of Chittagong City Water Supply and Sewerage Authority (CWASA) is not appropriate for the city dwellers. There is neither supply of water for all the people from CWASA nor have sufficient number of safe, salt-free and reliable sources of water in Chittagong. This study investigates the present water consumption behaviour in Chittagong city. There have 41 wards divided by Karnafuli river. One part is busy city area and other part is heavy industrial area. A questionnaire survey was conducted in the study area of Chittagong city for collecting necessary data. The questionnaire survey form was prepared based on some important characteristics of the city dwellers. About 9 major wards and a total number of 400 persons were interviewed in Chittagong City Corporation (CCC) area. The main objectives of this study are to investigate different available sources of water, survey the trend of the city dwellers for selection of water sources for different household chores, investigate the diseases and inconvenience they face while using the available water in Chittagong. The arranged and sorted out data were analysed by Service Product for Statistical Solution (SPSS) software to establish relationships among dependent and independent variables by linear regression, cross tabulation and correlation analysis. Finally, a bunch of suggestions was provided to fulfil the water supply necessity of the city.

Keywords: Water Consumption, questionnaire survey, linear regression, cross tabulation

1. Introduction

'Water is life' this is known to us. Water only can save us in the world. For human welfare and economical activities, domestic fresh water is a fundamental requirement. Human survival and welfare generally depend on regular availability and control of water. The paradox of community water supply in developing countries is that everyone has access to water supply, however in fact many people do not. They may access to water, but only a large walking distance, in too little volume or of poor quality. A lot of effort is made in the world to change this situation; however is this situation really changing?

It is very important to study the consumption behavior of water. The water consumption pattern is a very important parameter generally used in estimation of water consumption in a certain periphery. The amount of consuming water needed daily, depends on various factors such as temperature, gender, region, community pattern, stage of production, health, etc. Information on the water consumption pattern of a population is essential in order to estimate the risk of adverse health effects attributable to any water contaminant. It is also crucial in developing a safe water supply program for a population. As a common characteristic of the urban areas of Bangladesh, accessibility to the suitable quantity and quality of water is a major problem in the city areas of Chittagong. Unplanned and improper withdrawal of water is the main reason of the shortage of water. These shortages become more acute during the dry seasons. The ground water withdrawal and recharge can be increased, approaching the potential limits by creating additional storage through increased during dry season

(Ahmed and Rahman, 2000). People in these areas are in such condition that they neither have a supply of water from Chittagong Water Supply and Sewerage Authority (CWASA), nor have sufficient number of safe and reliable sources of water in their surroundings. There is also less availability of sources that provides salt-free water in Khulna city areas (Mohsin, 2007).

The greater Chittagong region is very close to the Bay of Bengal. Due to the geological position of Chittagong, the water in the ground exhibits high salinity. There is a scarcity of water even for drinking and in the city areas of Chittagong people are sometimes bound to use the saline water for drinking and household purposes as the non-saline water they get from surrounding sources is not sufficient to meet their demand. The consumptive use of saline water causes lots of inconvenience to the users. The augmentation of salt water in the surface waters increases the abstraction of groundwater, which then becomes vulnerable to a risk that salt water will be drawn into the aquifer (Md. M. Rahman & A. K. Bhattacharya, 2006)[3]. There are some people who are in better economic condition and well educated are using water from alternative sources for drinking, whereas a lot of people are beneath the poverty line and some people are not well aware of health and hygiene are using the water they can collect from the nearby water sources. It is a matter of great concern that there is no water supply in the city areas of Chittagong. That is why the water consumption pattern analysis is necessary to find the quality and quantity of water people are using for different household purposes.

2. Objective of the study

The social status, economic conditions, level of education, age and experience of people are the factors affecting the selection of sources for water. Availability of fresh water is another factor that has significant influence on it. Factors affecting higher probabilities of a respondent being primarily a bottle water drinker included: higher income, unpleasant taste experiences with tap water, non French-speaking and being a male with children in one's household (Dupont et al., 2010). The objectives of the study are:

1. To investigate of different available sources of water in the city areas of Chittagong.
2. To survey the local people about their selection of water sources for different household uses.
3. To investigate about the disease and the inconvenience they face while using the available water.
4. To find out the correlation between the awareness of people while choosing a source of water and the occupation, experience, authority, maturity, gender, socioeconomic condition, level of education of local people using the SPSS (Statistical Package for the Social Sciences)
5. To analyze the change of trends in the selection of water sources with the changes of the influencing factors.

3. Methodology

The methodology of this study includes:

1. Collection of Information: Information has been collected about the existing condition of sources of water in the areas of CCC, the required information was provided by Chittagong WASA.
2. Preparation of a Questionnaire Survey Form: A questionnaire survey form has been prepared, including all the inquiries required for the analysis of this study.
3. Questionnaire Survey: Surveys of local people have been performed to collect the required data. The survey was performed based on the factors influencing people's selection of water sources. 200 participants were surveyed from the study areas.
4. Data Sorting: The collected data have been arranged and sorted out for the analysis.
5. Analysis: The analysis for determining the correlations have been done by SPSS 16.0 software through the linear regression analysis method. The correlation matrix is formed by a Bivariate correlation method with Pearson correlation coefficients.

4. Study area

The study area is very important in this study. This study is not based on a particular area in Chittagong city. The areas are selected from the different wards, so that it focuses on the whole city area. The research work is to be carried out in Chittagong city as shown in Table 1.

Table 1: Study area of Chittagong city

No	Ward No	Name of the Area
01	4	Chandgaon
02	5	Mohora
03	6	Shulashohor (North side)
04	33	Firingi Bazar

05	34	Pathorghata
06	35	Boxirhat
07	39	South Halishahar
08	40	North potenga
09	41	South potenga

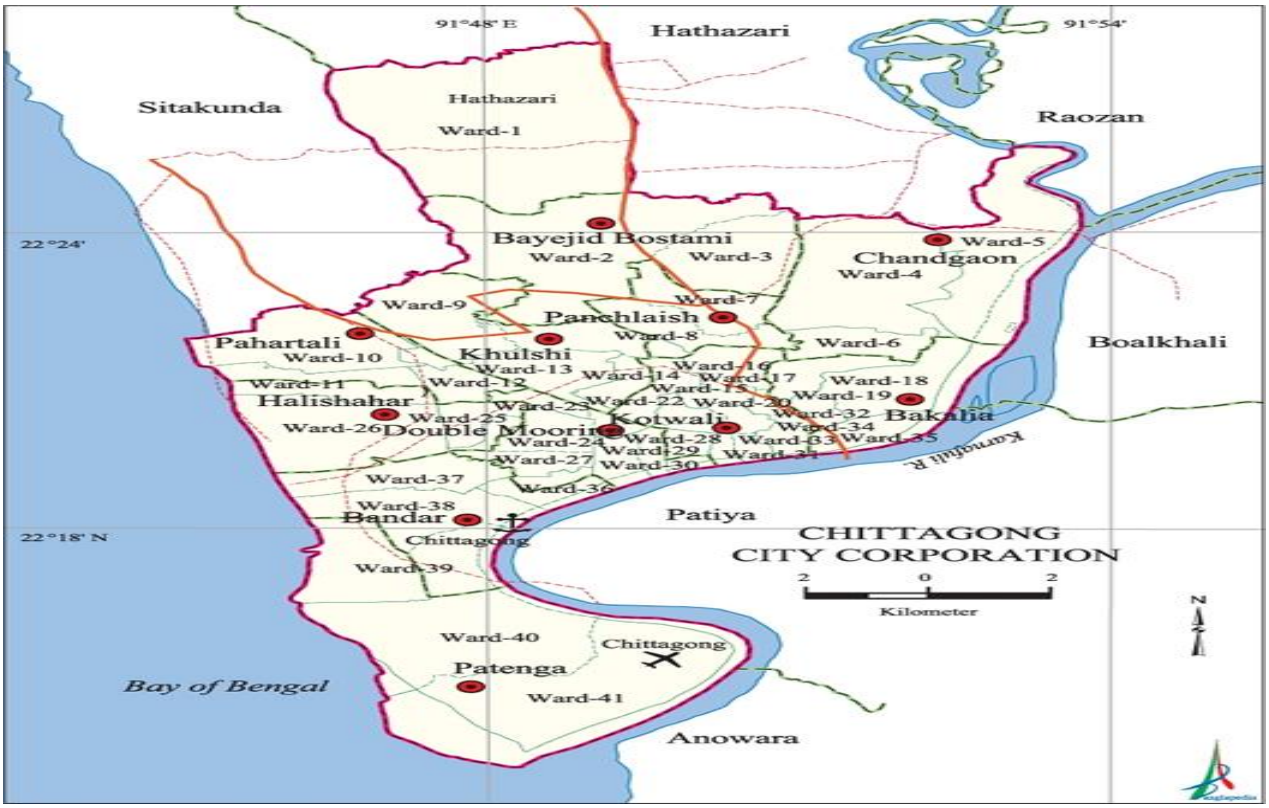

Fig.1: Ward plan of Chittagong city

5.Factors influencing people's selection of water soures

Although the human mind and the choice are unique, there must be some factors that have an influence on the selection of water sources for different uses. Availability of proper water source come in the list. The factors are age, gender, occupation, economic condition, authority, family members, level of education.

6. Result and Discussions:

6.1 Regression analysis in drinking water sources

The output of the model (SPSS software) for drinking water sources are shown in Table 2 and Table 3.

Table 2: Model summary of drinking water sources

Model	r	r^2	Adjusted R square	Std. Error of the Estimate
1	0.523[a]	0.274	0.210	0.7191

a. predictors: (constant), physician cost (PC), detergent (D), color (C), family members (FM), education(E)

Coefficient of Correlation, r = 0.523

Coefficient of determination, $\mathbf{r^2}$ = 0.247

Table 3: Coefficients of drinking water sources

Model		Unstandardized Coefficient		t	Sig.
		B	Std. Error		
1	(constant)	2.216	0.324	5.702	0.000
	Family members	-0.319	0.082	-4.341	0.000
	Detergent	0.254	0.113	2.342	0.025
	Color	0.523	0.123	5.291	0.000
	Education	-0.156	0.045	-3.097	0.004
	Physician cost	0.172	0.072	2.456	0.034

a.dependent variable: drinking

The equation formed by the co-efficient is given by

Drinking water source = 2.16-0.327*(FM) +0.244* (D) +0.536* (C) -0.150*(E)+0.165*(PC) (1)

The results from the Eq. (1) will near 2. In the most cases, as 1 represents the water source is a tap water and 2 denotes the water source is a tube well. For a particular household for which the participant's family members is 2-4 (1), detergent (1), color (1), education graduate(6), physician cost 100-500(1); the result of drinking water source becomes 1.978 which means the household uses a tube-well water for drinking.

6.2 Regression analysis in main water sources

The output for main water sources are shown in Table 4 and Table 5.

Table 4: Model summary of main water sources

Model	**R**	**r^2**	**Adjusted R square**	**Std. Error of the Estimate**
1	0.601[a]	0.361	0.382	0.37432

a. Predictors: (constant), Detergent(D), Color(C), Family members(FM), Diseases(Di), Red(R)

Coefficient of Correlation, r = 0.601

Coefficient of determination, $\mathbf{r^2} = 0.361$

Table 5: Coefficients of main water sources

Model		Unstandardized Coefficient		t	Sig.
		B	Std. Error		
1	(constant)	2.112	0.157	12.657	0.000
	Family members	0.154	0.034	3.918	0.000
	Detergent	-0.411	0.051	-7.967	0.000
	Red	0.079	0.017	3.975	0.000
	Diseases	-0.062	0.015	-3.250	0.001
	Color	-0.156	0.051	-2.292	0.023

a. Dependent Variable: main sources

The equation formed by the coefficient is given by

Main water source = 1.952+-0.134* (FM) -0.410*(D)+0.69*(R)-0.048* (Di) +0.116*(C) (2)

The results from the Eq. (2) will represent whether the household selects a tube-well water source or tap water as their main source. If the value of the equation is closes to 1, it will indicate the tube-well and, if the value is close to 2 it will indicate tap water as their main sources. For a particular household for which the participant's family members 2-4 (1), detergent (1), red (1), diseases (1), color (1); the result for main water source becomes 1.241 that is closer to 1 which means the main water source is tube-well.

6.3 Percentile selection about consuming water

The selection of different water sources is shown in Table 6.

Table 6: Selection of water sources

Water Sources	Drinking		Cooking		Bathing and Washing	
	Frequency, f	Percentage,%	Frequency,f	Percentage,%	Frequency,f	Percentage,%
Tube-well	159	79.5	120	60	138	69

Filtrate water	40	20	0	0	0	0
Bottle water	1	0.5	0	0	0	0
Pond/river	0	0	3	1.5	9	4.5
Tap water	0	0	77	38.5	53	26.5

The selection of medicine and physician cost per year for different diseases are shown in Table 7.

Table 7: Selection of medicine and physician cost per year for suffering diseases

Diseases	Frequency, f	Percentages, %	Average cost per year (TK)
No diseases	95	47.5	
Dysentery	100	50	1100
Diarrhea	5	2.5	

7.Education and drinking cross tabulation

Table 8: Variation of drinking water source with education level

Count		Drinking			Total
		Tap Water	Tube well Water	Filtrated water	
Education	Only signature	4	5	0	9
	Primary	1	5	3	9
	SSC	0	28	10	38
	HSC	0	41	9	50
	Graduate	0	79	15	94
Total		5	158	37	200

The variation of drinking water sources with education level is shown in Table 8. Some survey participants below SSC use tap water for drinking purposes. But the educated people (HSC and graduate) use tube well water and filtrated water for the drinking purposes. The more education level is increased, the more people are interested in using filtrated water as shown in Table 8.

8. Correlation matrix of dependent and independent variables

The relationship between dependent and independent variables are shown in Table 9. The Correlation between two variables represents how the variables are related to each other and in which way one variable will respond to any change of the other. The optimistic value of correlation is 1. If correlation is greater than 0.5 then the relation between two variables is considered better.

Table 9: Correlation matrix

Variable	1	2	3	4	5
1. Age	.081				
2. Gender	.41	1			
3. Occupation	-.574	.402	1		
4. Economic condition	.176	.193	-.236	1	
5. Family members	.071	-.051	-.10	-.193	1
6. Education	.151	-.076	.089	-.222	-.308
7. Drinking water source	-.115	.209	.164	.033	-.292
8. Cooking water source	-.058	.163	.019	-.097	-.145
9. washing water source	.11	.177	.044	-.104	-.081
10. Amount of drinking water	.037	.059	-.207	.052	.137
11. Taste	-.254	.100	.079	-.002	.251
12. Regularity	.041	-.017	-.030	.067	.065
13. All seasons	-.074	-.132	.161	-.11	-.023
14. Diseases	-.074	-.025	-.270	.195	.198
15. Satisfaction	.087	-.158	.105	-.038	-.008

6								.018	.004	.159	.111	-.216	.054	.065	-.175	.150
7								1	.18	.149	-.202	-.146	-.114	-.097	-.110	.008
8									1	.687	.038	-.173	-.098	-.002	.054	.054
9										1	-.01	-.19	-.04	-.01	.14	.05
10											1	.121	-.21	.032	.116	-.11
11												1	.394	.103	-.10	.20
12													1	.068	-.16	.083
13														1	-.37	.149
14															1	-.40
15																1

Conclusions:

The major conclusions and recommendations drawn from the study are as follows:

- For drinking purposes, 76.5% people use tube-well water, 23% use filtrate water, 1.5% people use bottle water and for cooking, 62% people use tube-well water, 37% people use tap water.
- For bathing and washing purposes, 69% people use tube-well water, 31% use tap water.
- By using water, 50% people suffer from dysentery, 2.5% people from diarrhea and others are safe. Each family spends average 1300 TK per year for medicine and physician cost.
- It was found that, the survey participant's level of education, age and economic condition significantly influences the selection of water sources than other factors.
- The deep aquifers should be the primary water source to be used by CWASA in the peripheral areas of Chittagong city. In addition, uncontaminated surface water sources may also be a source of supply after applying proper treatment.
- During rainy season, the rain water should be harvested as much as possible. Use of rain water ensures rapid fulfillment of ground water aquifers and slow depletion rate of Ground Water Table when the dry season comes. As salinity is the governance problem in the study area, solar desalination of water may be useful.

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Conceptual Study and Analysis of Neutron Diffusion and Moderation in Nuclear Reactor

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Abstract

Conceptual study on diffusion and moderation of neutron in nonmultiplying and multiplying medium in nuclear reactor have been conducted by using mathematical explanation, experimental data and graphical representation. The corresponding governing equations have been explained. A brief discussion on neutron and criticality of a reactor is carried on. Necessity of a moderator medium in slow reactors has been discussed. Comparative analysis of usefulness and drawbacks as moderator for different medium (Graphite, Beryllium, light water, heavy water and Lithium Fluoride) have been studied in aspects of different features. Mathematical example of calculating numbers of collision required for thermalization process is shown. A primary starter source is introduced in the reactor to start the initial fission reaction. Once critical condition is reached chain reaction is established. Neutron is diffused from high to low concentration area according to Fick's law of diffusion. While going through the moderator medium fast neutron are slowed down. It is found that Fermi age and thermal diffusion length of neutron is smaller in light and heavy water than graphite or beryllium. Thus light and heavy water are effective moderator and are used conveniently in nuclear reactors.

Keywords: Diffusion, Moderation, Nonmultiplying Medium, Multiplying Medium, Fermi Age Equation.

1. Introduction

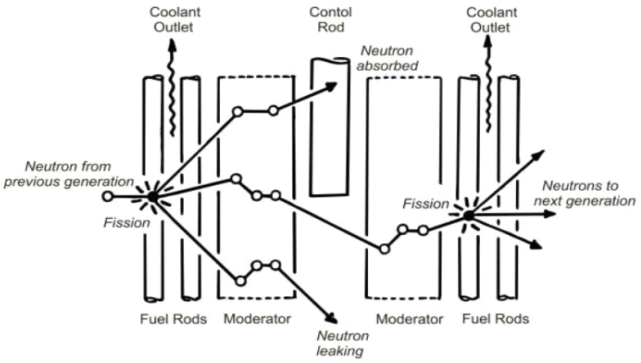

Fig. 1. Neutron life cycle [5]

Neutron generation is the prerequisite condition for a nuclear energy production. Different generation processes are used for different type of power plant. Relative advantages and disadvantages are needed to be analyzed to make sure the continuity of neutron generation. In a nuclear reactor fission of uranium-235 is induced by the absorption of neutrons by the uranium nucleus. Each nucleus may be divided into two new nucleus and on an average 2 or 3 neutrons are released per incident neutron. The neutrons strikes uranium and kinetic energy converts into heat energy. If one of the prompt neutron again strikes another U-235 nucleus a nuclear chain reaction will be established. In a controlled chain reaction in nuclear reactor, after achieving criticality multiple parallel fission reactions go on.

Neutrons produced from fission process have high energy and consequently high velocity. These are called fast neutrons. To increase the possibility of absorption of these neutrons in U-235 nucleus a moderation system is a must. Because fast neutrons do not affect the U-235, meanwhile it strikes the non-fissile U-238 with inelastic collision. The moderation process consists a suitable medium where many scattering collision with nucleus will reduce their energy to a state of thermal equilibrium within the medium. Commonly used moderator include

light water, beryllium, graphite, heavy water etc. But different type of moderation process have different degrees of advantages in various physical properties such as neutron age, slowing down length etc. Meanwhile cost is another crucial phenomena.
After getting thermally balanced, neutrons are called thermally balanced neutrons. At this lower velocity there is much more high probability of being absorbed by the fuel (U-235) to cause another fission. In design of the reactor leakage neutron must be considered. Neutron reducing phenomena can be controlled by employing a refractor around the reactor. This allows the neutron to diffuse in all direction.

2. Neutron diffusion

The net passage of neutrons from regions of higher neutron densities is called neutron diffusion. However diffusion of neutrons is a consequence of non-uniform density in the reactor assembly. Unfortunately determining the neutron distribution is a difficult problem. An approximation can be arrived at by solving the diffusion equation. This differential equation describes how high concentration solute diffuses to regions of lower concentration. This is similar to how neutrons behave in a reactor. Diffusion equation can be found out by the combination of two laws of physics:
1. Fick's law
2. Equation of continuity
Diffusion equation can be written as:

(1)

Where,
D = the diffusion co-efficient (in units of meter or centimeter)
Φ = neutron flux (in units of $cm^{-2}sec^{-1}$)
n - The number density of neutron at any point
s - The rate at which neutrons are emitted from sources per cm^3 in V
Σ_a - Macroscopic absorption cross-section
In the case of time independent problem the equation may be considered as:

(2)

This steady state diffusion equation is applicable to both multiplying i.e. system containing fissile materials that can produce additional neutrons as a result of neutron absorption and nonmultiplying systems, in which neutrons are introduced from an independent source(s=0).
In a real power reactor, there is no independent or extraneous source. In this case a production of neutron by fission is equal to neutron lost by absorption or by escape in different energy range in a given time. One of the basic properties to explain such case in a nuclear reactor is infinite multiplication factor, K_{eff} which is the ratio of rate of neutron production to rate of neutron absorption plus leakage.
If the value of K_{eff} is equal to 1; the power reactor is said to be critical which is desired; if less than 1 than said to be sub-critical and said to be supercritical if it is greater than 1. In the case of supercriticality, the condition may be out of control and accident may happen. On that case control rods which has a large absorption cross-section. K_{eff} can be mathematically explained by 6 factor formula that is discussed below:

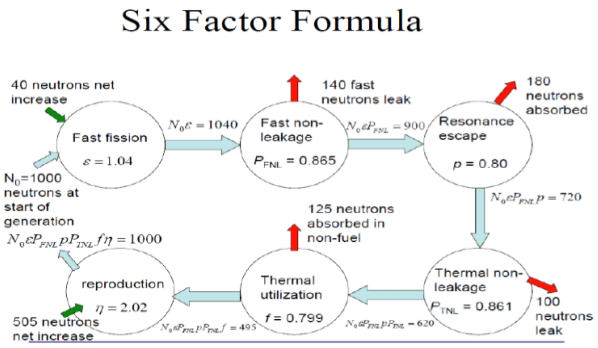

Fig. 2. Six factor formula

3. Neutron Moderation

The importance of a moderator:

The problem lies in the fact that whenever a thermal neutron causes fission it also leads to the release of fast neutrons. Now these fast neutrons have to be slowed down and brought to lower energy levels if they have to cause successful fission in turn. It is here that the concept of a moderator comes in the picture.

With high kinetic energy of fast neutron fission of U-235 nucleus is not likely to happen. A moderator is a medium which is used to absorb a portion of the kinetic energy of fast neutrons so that they come in the category of thermal neutrons which help to sustain a controlled chain reaction. To moderate a neutron kinetic energy is needed to be reduced from 2MeV to 0.025eV. Although moderators are necessary in most nuclear reactors this does not mean to say that all reactors require moderators. There is a special class of reactors known as fast reactors which do not use moderators [12] but depend on the use of fast moving neutrons for causing fission. Even otherwise it must be remembered that fast moving neutrons have lesser probability of getting absorbed and causing fission but it does not mean that they are incapable of causing the fission reaction. A fast moving neutron travels with a speed which is nearly in the region of 10% of the speed of light, while a thermal neutron travels with a speed which is typically of the order of a few kilometers per second. The loss of kinetic energy and slowing down of neutron can be expressed by Fermi age equation.

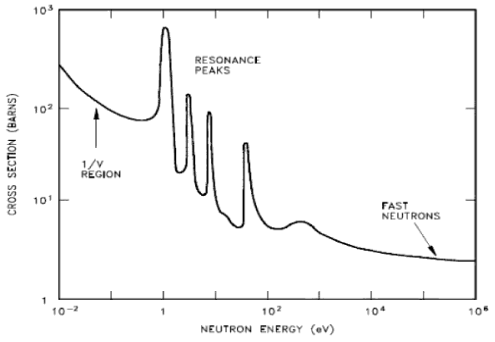

Fig. 3. [1]Fission cross section (in unit of [2]barns) of U-235 with different energy of neutron

1. Fission cross section is the probability of an incident neutron to cause fission in the target nucleus.
2. 1 barns = 10^{-24} cm^2.

Fermi age equation:

The complete slowing down diffusion equation is known as Fermi age equation. A detailed reactor theory combine Fermi age theory for non-thermal neutrons with diffusion theory for thermal neutrons. During the slowing down stage, neutrons cannot be treated like a collection of monoenergetic neutrons. Because they undergo considerable energy changes while diffusing. For Fermi age equation, in order to simplify calculations, it is assumed:

1. A continuous loss of energy for a slowing-down neutron.
2. Free path between collisions, λ_a considered constant and varies slightly with neutron energy.

Fermi age equation represents the spatial distribution of the slowing down density in the moderator. After some certain mathematical calculation with assumption, the Fermi age equation can be written as:

(3)

Where,

q - Slowing down density, the number of neutrons whose energy drops below a given energy E per second in a unit volume element of the medium.

τ - Fermi age or neutron age, slowing down area for neutrons calculated from Fermi age theory. Slowing down area is one sixth of the mean square distance from the source of a neutron in an infinite, homogeneous medium to the point at which the neutron reaches a given energy. It should be remembered that the Fermi age equation does not contain the time variable explicitly and that is therefore a time-independent or steady-state equation. Fermi age equation contains a complete description of neutron density distribution in both energy and space co-ordinates for neutrons undergoing moderation. In equation (3) the new variable τ is introduced as [8]:

(4)

If λ_s and λ_{tr} can be taken as constant over the slowing-down energy range or if they are replaced by suitable average values over the energy range integration of equation (4) in the process of having its energy reduced from E_0 to E is given by:

(5)

where,

(6)

Here, is the mean logarithmic reduction of neutron energy per collision. In this expression $C\lambda_s$ represents the total zigzag length of a neutron between the moment of its creation or the beginning of its slowing down and the moment of its arrival at energy E. If $\Lambda_s = C\lambda_s$; where, Λ_s is quite analogous to λ_a and the equation (5) may be written as:

(7)

Where L is called thermal diffusion length or slowing down length. Physically, it is a measure of the distance a fission neutron has traveled away from its point of creation by the time it reaches thermal energies.
Here,

(8); (9); (10)

Consider a mathematical analysis based on neutron age, τ_0 and slowing down length, L for fission neutrons of 2 MeV average energy to different thermal energy in graphite, beryllium, heavy water and light water by plotting graph. To solve this problem equations (6), (8), (9), (10) are substituted in equation (5) to get the following equation.

(11)

Table 1. Atomic or molecular weight (A) and Macroscopic cross-section (Σ_s) of some elements [4]

	Beryllium (Be)	Graphite (C)
A	9.013	12.011
Σ_s (cm^{-1})	0.865	0.385

Using these data in equation (11) a graph of Fermi age for varying energy of neutron for these four type of moderators may be produced. Now using equation , another graph of neutron energy versus slowing down length can be produced

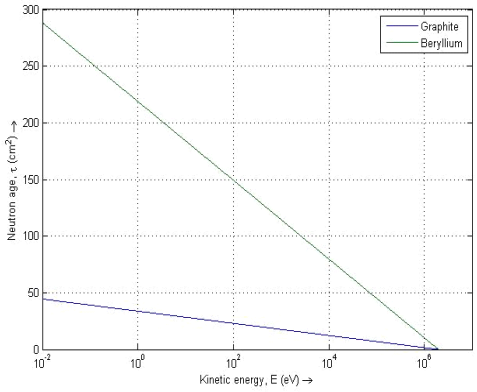

Fig. 4. Fermi age vs. Kinetic energy graph

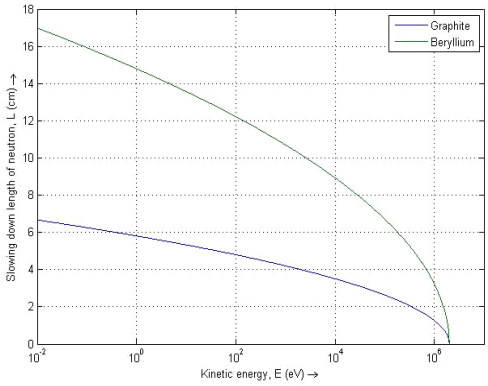

Fig. 5. Slowing down length vs. Kinetic energy

In moderation process, fast neutrons are collided with other particles in a suitable moderator medium. As more collision happens, kinetic energy is passed from fast neutron to other particles. There are some factors to denote a moderator whether it is good or bad. A good moderator should not absorb neutrons itself. The probability of absorbing neutron from flux is called absorption cross section of the moderator, denoted by Σ_a. The probability of neutron being scattered by moderator nucleus is denoted by scattering cross section, Σ_s. So, less the

absorption cross section and larger the scattering cross section is better moderator. The moderator should slow down the neutron efficiently. A good moderator should thermalize the neutron in a few collisions. The least collision needed is the best. The first couple of collisions with a nucleus will be inelastic. It will transform some of the kinetic energy of neutron to potential energy of the moderator nucleus. The nucleus will be in an excited state. As kinetic energy of neutron is removed further collisions will be elastic. As in gas molecules' model, nucleus of closer mass will remove more energy in collision. So, nucleus of closer mass to neutron is more favored. Also a head-on collision will take more energy than a glancing collision. Both the energy reduction in a head-on and glancing collisions should be considered. Reduction in kinetic energy of fast neutron due to collision is expressed by mean logarithmic reduction of neutron energy per collision, and is given by:

(12)

where,
E_0 and E - kinetic energy of the neutron before and after one collision.
A - atomic mass of the moderator nucleus.
For a moderating medium consisting of more than one atom it is needed to consider the moderating and absorbing effect of all the atoms to calculate . The n umber of collisions to thermalize a neutron from the fission energy E_0 to the thermal energy E_{th} can be calculated from:

(13)

For example the number of collision needed to thermalize fast neutron with hydrogen and graphite moderator is calculated.
Assume, for fast neutron E_0 = 2 MeV; for thermal neutron E_{th} = 0.04 eV.
For hydrogen moderator:
A = 1, , collisions.
For graphite or carbon moderator:
A = 12, , n collisions.
It becomes apparent that a hydrogenous material is a better neutron moderator than graphite.The efficiency of a moderator is expressed as moderating ratio given by:

(14)

Table 2: Number of Collisions, on Average, to Moderate a Neutron from 2 MeV to 1 eV [3]

Moderator	ξ	Number of Collisions	$\xi \Sigma_s / \Sigma_a$
H	1.0	14	–
D	0.725	20	–
H_2O	0.920	16	71
D_2O	0.509	29	5670
He	0.425	43	83
Be	0.209	69	143
C	0.158	91	192
Na	0.084	171	1134
Fe	0.035	411	35
^{238}U	0.008	1730	0.0092

The ideal moderator is of low mass, high scattering cross section, and low absorption cross section. More neutron is absorbed with more collision. So a moderator which can reduce more kinetic energy in less collision is preferred. With a large density of moderator nucleus probability of collision is higher. In gas moderator medium density of gas nucleus is lower and so number of collisions is not enough to eliminate enough kinetic energy from the fast neutron. Thus, gas cannot be used as moderator.

4. Summary and Closing Remarks

Understanding the neutron distribution in space and energy space is necessary to design a nuclear reactor properly. Therefore, study of neutron generation, diffusion and moderation is important. For nuclear reactor fast neutrons can be moderated or thermalized using any of the described moderator medium. But the choice of most suitable moderator depends on type, construction, working principle of that specific nuclear reactor; cost, availability, pros and cons of different moderators.
1. Moderator medium must be solid or liquid. Gaseous medium must not be used as moderator.

2. Better moderation occurs with the moderators that is associated with smaller values of Fermi age and thermal diffusion length. So beryllium is better moderator than graphite according to Fermi age equation.
3. Moderating ratio of heavy water is more than light water. This makes heavy water a more efficient moderating medium than light water. But light water is much cheaper and easily available than heavy water. Light water is used in most reactors.
4. In the early age solid, pure graphite was generally used as moderator medium. But with presence of a small amount of boron poison or due to the unstable Wigner energy graphite is not reliable as a good moderator. Hence recently the usage of graphite as moderator medium is substantially reduced.
5. Beryllium has a good moderating effect in nuclear reactor. But it is a metal, expensive and toxic. Also it has a risk of sudden structural failure due to brittleness.
6. Lithium fluoride salt, typically in conjunction with beryllium fluoride salt is commonly used in molten salt reactors.
7. In fast reactors, where U-238 is used as fuel, no moderator medium is required.
8. The safety of a nuclear reactor is greatly dependent on the right choice of suitable moderator medium. A slight mistake or impurity in moderator can lead to devastating result or failure of operation due to not self-sustaining chain reaction.

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Survey on Socioeconomic Impact of Battery Operated Auto Rickshaw in Jessore City

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Abstract

In Bangladesh, unemployed problem is considered one of the major problems in recent time. Battery operated auto rickshaw helps in reducing this unemployment problem. In addition, environmental hazards including air pollution and sound pollution are another threat for the human being. Although, battery disposal is a concern for the manufacturer, these do not produce any harmful gasses like CO_2, CO, SO_2 and NO_2 etc. and run at a less vibration and noise. On the other hand, battery operated auto rickshaw causes serious traffic jam and consumes a lot of electricity which are considered the reason of serious headache for our country. This paper covers the social, economic and environmental impacts of battery operated auto rickshaw in Jessore city. This study also includes economic analysis of battery operated auto rickshaw and the comparison with other vehicle available in the city.

Keywords: Auto rickshaw, Jessore, payback period, socio-economic, environment.

1. Introduction

Battery operated auto rickshaw is the updated form of rickshaw which is locally known as easy bike. It has normally four to six seats for passenger along with the driver. Auto rickshaw is a three wheel vehicle which is suitable for transportation of people and goods in a small distance due to its small and narrow structure. The operators of this vehicle come from the poor society and belong the vehicle privately due to its low investment and operating cost [1]. Battery operated auto rickshaw was first introduced in Bangladesh in 2008 [2]. Recently, battery operated auto rickshaw has become one of the most environment friendly, time saving and the cheapest mode of transportation. However, it causes the increase of traffic congestion, high electricity consumption and small collisions with other vehicles in high ways which led to the banning of auto rickshaws in the major streets. Generally, it uses four or five 12 volt batteries with fully charged for running all the day. As it is operated by battery, it usually makes no pollution in the environment but leads to the huge amount of electricity consumption for charging the battery. The number of unemployed people in Bangladesh is near about 7 lacks where battery operated auto rickshaw reduces this problem in 2%. The aim of this paper is to analyze the role of battery operated auto rickshaw in socio-economic and environmental development in Jessore city.

2. Methodology

In this work we frequently asked various types of questions to the passengers, drivers, workers, businessmen who are related to the battery operated auto rickshaw that clarified the socio-economic condition of these people. For this survey, a set of questionnaires were developed and all the data related to the social, economic and environmental aspects were collected from different locations in Jessore City such as Palbari Mor, Daratana, Monihar bus stand, Dharmatala Mor, Arabpur Mor, New market bus stand, High court Mor. The payback period is calculated according to the equation 1 [3] for estimating quickly the initial investment would be recovered.

$$\text{Payback period} = I/(R - E) \quad (1)$$

Where, I: Investment, R: Return, E: Expenses.

3. Purposes of battery operated auto rickshaw

There are various types of vehicle normally people use for their transportation mode in Jessore town. In general, battery operated auto rickshaw is used for going to work, making social trip, shopping trip and going to school and college. From the survey it has been found that, almost 40% of total passengers use battery operated auto rickshaw for the purpose of going to school as presented in Figure 1.

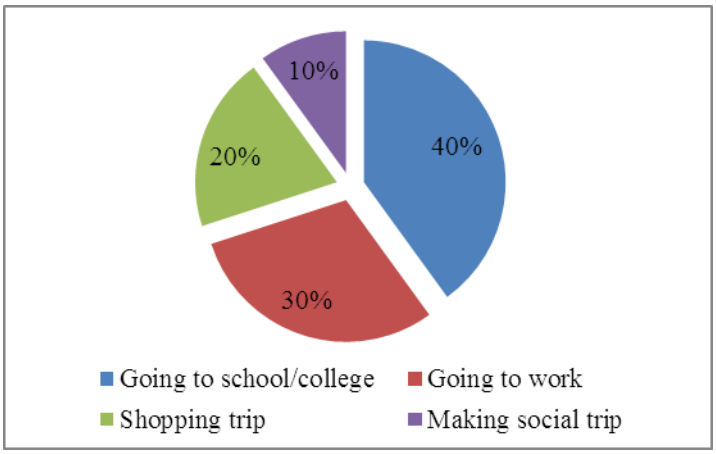

Figure 1: Purposes served by auto rickshaw

4. Positive aspects of auto rickshaw

4.1. Preferential choice of auto rickshaw

Distance, road condition, financial ability, availability of transport, time etc. make an impact on choosing the transportation mode for their movement. Several random passengers were asked to know about their preference of the transportation mode for movement. Figure 2 illustrates that approximately 53% of total passengers choose battery operated auto rickshaw for their movement in the city. On the other hand, 33% of total passengers use diesel operated vehicle and only 14% use paddle rickshaw.

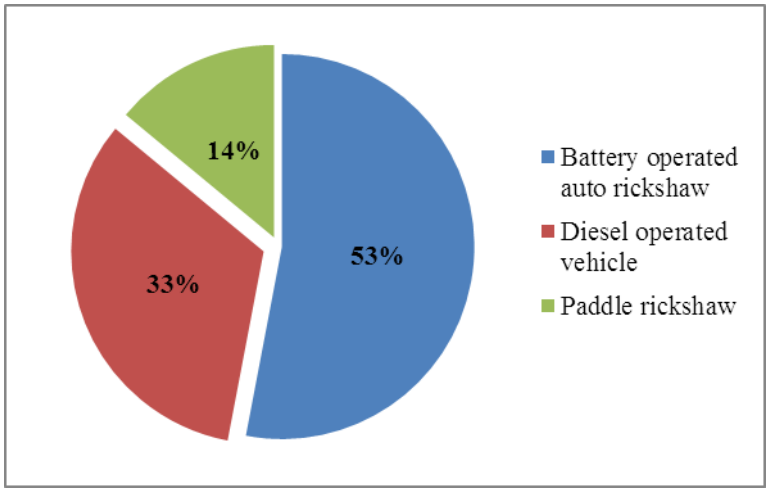

Figure 2: Preferential choice of different vehicle in Jessore city

It is clearly depicted from the survey that, the transportation cost per person in case of auto rickshaw is less than the other vehicle. Although, the fare for diesel operated vehicle is the same of auto rickshaw (Table 1), passengers have to wait for long time for this vehicle due to its limited number.

Table 1: Comparison of fare of different vehicle

Type of vehicle	Distance	Cost/person
Auto rickshaw	Palbari to Daratana (3 km appr.)	10 taka
Paddle rickshaw	Palbari to Daratana (3 km appr.)	20 taka
Diesel operated vehicle	Palbari to Daratana (3 km appr.)	10 taka

Therefore, passengers prefer battery operated auto rickshaw among the three wheeler vehicle available in Jessore city due its less traveling cost, time saving, easy going and comfortable feature. Almost 90% of passengers choose because of less travelling cost, while 60% choose for comfortable journey as presented in Figure 3.

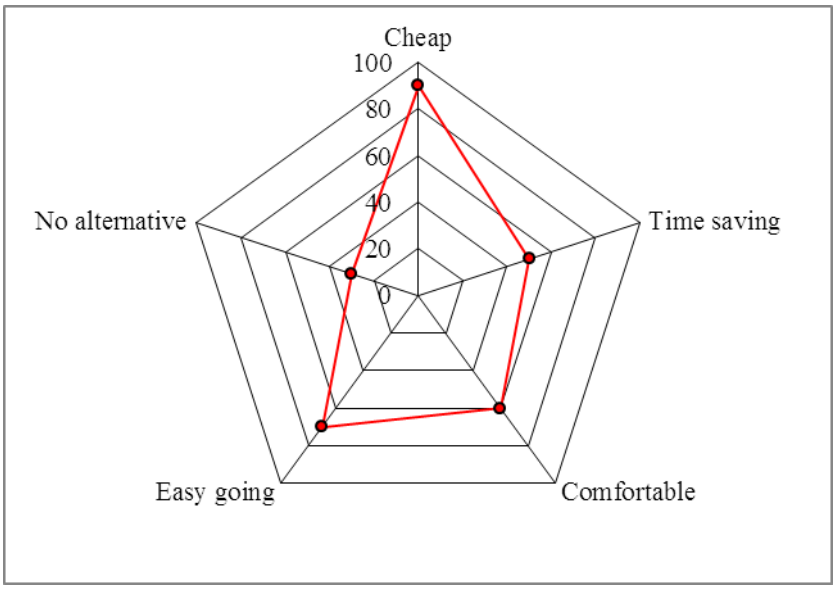

Figure 3: Reasons of choosing the auto rickshaw

4.2. Employment opportunity

4.2.1. Earning of the driver

Usually, the driver of the auto rickshaw drives his own auto rickshaw or hires it from the owner in daily basis. Normally, he earns BDT 800-1000 per day which depends on the size of the auto rickshaw and the charging of the battery. He has to pay BDT 300-400 to the owner according to the number of batteries in the auto rickshaw. Finally, he has BDT 400-600 in hand. Accordingly, a driver earns about BDT 12000-16000 per month which is good enough to run a small family in the rural area. But the problem is that the amount that does not remain constant every month for a driver of the auto rickshaw.

4.2.2. Earning of the owner

The owner of the auto rickshaw normally gets about BDT 300-400 per day for per auto rickshaw from the driver who hired. Accordingly, the owner earns BDT 9000-12000 per month for one auto rickshaw. On the other hand, he has to spend almost BDT 90 per day which is BDT 2700 in a month and BDT 500 taka for maintenance and repairing cost of the auto rickshaw. In addition, he has to spend according to the Table 2 for replacement of battery, tire and motor for long lifecycle of the vehicle. Therefore, considering all the profits and costs, the payback period is found about 3.48 years indicating the recovery time of investment.

Table 2: Replacement cost of accessories

Accessories	No. of unit	Per unit cost (BDT)	Total cost (BDT/year)
Battery	5	10,000	50,000
Tire	3	2200	6600
Motor	1	6000	6000

4.3. Social status of related family

From the survey it is found that most of the drivers of the auto rickshaw were unemployed before. On the other hand, some drivers are students and farmers who are taking this occupation as a part-time job to maintain his tuition fees and pocket money and in off season. Some hawkers have also come to this occupation. Approximately 20% drivers who were related to hawking or other similar occupation strongly agree that this occupation increase their social status and prestige, while, more than 46% drivers are strongly disagree. Besides, around 30% of the people say that their economic conditions have not been changed at all, whereas, about 70% of people say that their economic condition are better than before.

4.4. Pollution

There are various types of pollution caused by the vehicles running with diesel, petrol etc. in the streets. Typically a motor vehicle is responsible for over 70% of the total emissions of air pollutants [4] as well as sound

pollution. On the contrary, battery operated auto rickshaw does not emit any harmful pollutants and creates less sound comparing to other vehicles which is considered to be negotiable. Although, only lead from waste battery material pollutes the water and soil, it will overcome by proper management and disposal.

4.5. Accident

Battery operated auto rickshaw is commonly run inside the city. This transportation mode is low speed and lightweight and hence it does not produce fatal accidents.

5. Negative aspects of auto rickshaw

5.1. Electricity consumption

The auto rickshaw collects its power for the batteries from the electric supply line while charging. The charging time of the auto rickshaw is normally 10 pm to 8 am. It takes 8 to 10 hours for full charging of the batteries. It has been estimated that, battery operated auto rickshaw consumes approximately 300 MW electricity every day for recharging their batteries. As Bangladesh is electric power crisis country and this huge consumption for this purpose creates daily load shedding.

5.2. Traffic Jam

In recent days, traffic jam is one of the severe problems in many large cities in Bangladesh. Battery operated auto rickshaws are commonly parked in the busy road as well as foot path. In Jessore city, there are more than 7,000 battery operated auto rickshaw running in the street every day. Therefore, this creates traffic jam in a narrow road in Jessore city.

5.3. Unsuitability for heavy load

Battery operated auto rickshaw has low power capacity. Therefore, this mode of vehicles is not suitable for carrying heavy load and for hilly road or bad weather conditions.

6. Technical information and issues on auto rickshaw

6.1. Longevity of Battery

Usually each battery operated auto rickshaw consists of 4 to 6 batteries of different companies. Usually it runs about 120-140 km with full charge when it is in a good condition. But for the time being the performance tends to reduce. Normally, after 12 months, the battery condition seems to be very poor as it runs only 20-40 km with full charge. Then there arises a need to change the battery of the auto rickshaw to get the desired outcome.

6.2. Cost of auto rickshaw

6.2.1. Investment cost

Generally auto rickshaws available in the street of Jessore city are mostly imported from china and few from India. But China manufactured auto rickshaw is most popular for the buyer for their service and longevity. Now a day, some local workers are making the auto rickshaw in their workshop which is not as good as the product of china but still some people prefer this because of their low cost. Normally, a China imported auto rickshaw cost about BDT 1,50,000 to 2,00,000 with the battery according to the size and the battery capacity of the auto rickshaw where locally made auto rickshaw cost about BDT 80,000 to 1,00,000 with the battery.

6.2.2. Operating and maintenance cost

Every auto rickshaw has 4 to 6 batteries but most of them have 5 batteries. If it takes 10 hours to fully charge the 5 batteries and each consume 2 units for full charging then 5 batteries consume 10 (5×2=10) units. The unit cost of electricity for commercial electric supply line is about BDT 9 taka. So, the cost of charging of an auto rickshaw with 5 batteries is approximately 90 (10×9= 90) daily.

6.3. Road condition

The condition of road in Jessore city is so poor that it is almost impossible for run not only the auto rickshaw but also all type of vehicles used for transportation. This road condition is considered one of the main reasons of low performance of the vehicle and low lifecycle.

6.4. License of driver and auto rickshaw

The field survey shows a very strange matter that, there is no license or registration of auto rickshaw and driver at all. Therefore, to take proper actions against the driver and owner in case of accident, robbery, misuse etc. is impossible as there is no number in the auto rickshaw.

6.5. Auto rickshaw driver union

During the survey many auto rickshaw driver complain that they have to pay BDT 10 in several auto stand to the so called auto rickshaw driver union. But the reality is that, there is no government approved auto rickshaw driver union in the Jessore city. The drivers and owners of the vehicle are disturbed by this union.

7. Recommendations

- There should be a registration of the battery operated auto rickshaw and the driver should have the driving license.
- As the auto rickshaw is restricted in the highway there should be an individual lane for slow moving vehicle like battery operated auto rickshaw.
- There should be a government approved auto rickshaw driver union which works for the welfare of the drivers.
- Charging duration of the battery should be reduced as much as possible. This will reduce the charging cost as well as the load shedding. In the recent time, scientists of Purdue University invented a new alternative of graphite which is used as anode in Li-ion battery. This alternative is actually a material that made from tin-oxide nanoparticles. If it is used as anode in the Li-ion battery the charging duration will be reduced from multiple hours to minutes.
- Alternative sources such as solar panel or wind turbine should be introduced for charging the batteries.

8. Conclusion

The study reveals that battery operated auto rickshaw plays a vital role in socio-economic and environmental development of the Jessore city. The people related to this occupation not only the drivers and the owner but also the worker who work in the auto rickshaw garage, workshop and the seller who sells the auto rickshaw accessories are getting economic benefit. The economic analysis shows that the payback period of the vehicle is 3.48 years which indicates that his investment is risk free. The major issue of energy consumption can be reduced by incorporating renewable energy sources for battery charging or using tin-oxide nanoparticles as anode in Li-ion battery.

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