Shock Wave Excitation in Unmagnetized Multi-Component Plasmas

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Department of Mathematics

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Dedicated to……..

My Beloved Parents, Husband

and

My Loving Son

List of Publications

[1] P. Akter, M. G. Hafez, M. N. Islam, and M. S. Alam, "Ion Acoustic Shock Wave Excitations Around the Critical Values in an Unmagnetized Pair–Ion Plasma," *Brazilian Journal of Physics*, vol. 51, no. 5, pp. 1355–1363, Jul. 2021, doi: 10.1007/s13538-021-00946-z.

[2] P. Akter and M. G. Hafez, "Dust-ion-acoustic shock wave excitations at super-critical points with quartic nonlinearity," *Contributions to Plasma Physics*, vol. 63, no. 8, pp. 1-11, Jun. 2023, doi: 10.1002/ctpp.202300048.

Approval/Declaration by the Supervisor(s)

This is to certify that Parvin Akter has carried out this research work under our supervision, and that she has fulfilled the relevant Academic ordinance of the Chittagong University of Engineering and Technology, so that she is qualified to submit the following thesis in the application for the degree of MASTER OF PHILOSOPHY IN MATHEMATICS. Furthermore, the Thesis complies with the PLAGIARISM and ACADEMIC INTEGRITY regulation of CUET.

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Abstract

This thesis deals with the nonlinear propagation of shock wave excitations by assuming unmagnetized collisionless plasmas. The unmagnetized collisionless plasma is assumed by the mixture of (i) inertial pair-ion and inertial-less (α, q)-distributed electrons and (ii) the (α, q)-distributed electrons, negatively charged distributed inertial heavy ions, positively charged distributed Maxwellian light ions, and negatively charged distributed stationary dusts. Then, the nonlinear propagation of ion acoustic and dust-ion acoustic shock wave excitations is investigated by deriving Burgers equation via the reductive perturbation technique. When Burgers equation is unable to describe the shock wave excitations in the considered plasmas for the critical values of any specific parameters, the modified forms of Burgers equation involving higher-order nonlinearity or composition of nonlinearities are derived by taking the higher-order correction of the reductive perturbation technique. Based on the useful solutions of Burgers equation involving higher-order nonlinearity or the composition of nonlinearities, the effect of plasma parameters is investigated not only around the critical values but also at the composition of critical values. This thesis also deals with the progress in understanding the propagation of shock wave excitations for the super critical values of any specific parameter that accompany an unmagnetized collisionless four-component dusty multi-ion nonextensive plasma. To accomplish this goal, the formation of a modified Burgers-type equation with quartic nonlinearity via the reductive perturbation method and its analytical solution have been obtained. It is found that the compressive electrostatic shocks are supported not only around the super critical values but also at the super critical values of the specific parameters. The outcomes of this thesis are expected to contribute to an in-depth understanding of shock wave excitations in many astrophysical and space environments in general. Moreover, it is expected that the outcomes of this research work will be useful in understanding the nature of shock wave propagation in plasmas and further laboratory verification.

বিমূর্ত

এই থিসিসটি চুম্বকহীন সংঘর্ষহীন প্লাজমা বিবেচনা করে শক ওয়েভ উত্তেজনার অরৈখিক প্রকম্পনের সাথে সম্পর্কিত। চুম্বকহীন সংঘর্ষবিহীন প্লাজমা (i) জড় যুগল–আয়ন এবং জড়-হীন (α, q) বন্টনকৃত ইলেকট্রন এবং (ii) (α, q) -বন্টিত ইলেকট্রন, ঋনাত্মক চার্জ বন্টনকৃত ভারী আয়ন, ধনাত্মক চার্জ বন্টনকৃত ম্যাক্সওয়েলিয়ান আলো আয়ন, এবং ঋনাত্মক চার্জ বন্টনকৃত স্টেশনারী ডাস্ট এর মিশ্রণ বিবেচনা করা হয়। তারপরে, আয়ন অ্যাকোস্টিক শক ওয়েভ উত্তেজনার অরৈখিক প্রকম্পনকে রিডাক্টিভ পর্টাবের্শন কৌশলের মাধ্যমে Burgers সমীকরণ তৈরী করে অনুসন্ধান করা হয়। যখন Burgers সমীকরন কোন নির্দিষ্ট প্যারামিটারের ক্রিটিক্যাল মানের জন্য বিবেচিত প্লাজমাগুলিতে শক ওয়েভ উত্তেজনা বর্ণনা করতে অক্ষম হয়, তখন Burgers সমীকরণের পরিবর্তিত রুপগুলি যার মধ্যে উচ্চ ক্রম অরৈখিকতা বা অরৈখিকতার সংমিশ্রন জড়িত থাকে তা উচ্চ ক্রম সংশোধন করে রিডাক্টিভ পর্টাবের্শন কৌশলের মাধ্যমে অনুসন্ধান করা হয়। Burgers সমীকরনের দরকারী সমাধানগুলির উপর ভিত্তি করে যার মধ্যে উচ্চ ক্রম অরৈখিকতার সংমিশ্রণ জড়িত, প্লাজমা প্যারামিটারের প্রভাব শুধুমাত্র ক্রিটিক্যাল মানগুলির চারপাশে নয় বরং ক্রিটিক্যাল মানগুলির সংমিশ্রণও অনুসন্ধান করা হয়। এই থিসিসটি কোন নির্দিষ্ট প্যারামিটারের সুপার ক্রিটিক্যাল মানগুলির জন্য শক ওয়েভ উত্তেজনার প্রকম্পন বোঝার অগ্রগতি নিয়েও কাজ করে যা একটি চুম্বকবিহীন সংঘর্ষবিহীন চার-উপাদান বিশিষ্ট ডাস্ট মাল্টি-আয়ন নন-এক্সটেনসিভ প্লাজমা বিবেচনা করা হয়। এই লক্ষ অর্জনের জন্য, Burgers সমীকরণের পরিবর্তিত রুপ গঠন করে যার মধ্যে কোয়ার্টিক ননলিন্যারিটি রয়েছে এবং রিডাক্টিভ পারটাবের্শন পদ্ধতির মাধ্যমে এর বিশ্লেষণাত্মক সমাধান পাওয়া গেছে। এটি পাওয়া যায় যে, সংকোচনমূলক ইলেক্ট্রোস্ট্যাটিক শকগুলি শধুমাত্র সুপার ক্রিটিক্যাল মানগুলির চারপাশেই নয় বরং নির্দিষ্ট প্যারামিটারগুলির সুপার ক্রিটিক্যাল মানগুলিতেও সমর্থিত। এই থিসিসের ফলাফলগুলি সাধারণভাবে অনেক অ্যাস্ট্রোফিজিকাল এবং মহাকাশ পরিবেশে শক ওয়েভ উত্তেজনা গভীরভাবে বোঝার জন্য অবদান রাখবে বলে আশা করা হচ্ছে। অধিকন্তু, এটি প্রত্যাশিত যে এই গবেষণা কাজের ফলাফলগুলি প্লাজমাগুলিতে শক ওয়েভ প্রচারের প্রকৃতি বুঝতে এবং আরো পরীক্ষাগার যাচাইকরণে কার্যকর হবে।

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Nomenclature

Index of Abbreviations

Nonlinear evolution equations	NLEEs
Velocity distribution	VD
Maxwell velocity distribution	MVD
Burgers equation	BE
Modified Burgers equation	mBE
Mixed modified Burgers equation	mmBE
Korteweg-de Vries Burger	KdVB
Gardner's equation	GE
Critical values	CVs
Super Critical Values	SCVs
Reductive perturbation technique	RPT
Shock wave excitations	SWEs
Ion acoustic	IA
Ion acoustic waves	IAWs
Ion acoustic shock waves	IASWs
Dust acoustic	DA
Dust ion acoustic	DIA
Dust ion acoustic waves	DIAWs
Dust ion acoustic shock waves	DIASWs
Reductive perturbation technique	RPT
Astrophysical and space plasma systems	ASPSs
Dusty plasmas	DPs
Positive ions	PIs
Negative ions	NIs

Maxwellian light ions	MLIs
Stationary dusts	SDs
Stationary dust Ions	SDIs
Heavy ions	HIs

Index of Symbols

Electron temperature	T_e
Neutral temperature	T_h
Ion temperature	T_i
Electron density	n_e, N_e
Neutral density	n_a
Ion density	n_i, N_i, ρ_{N_i}
Debye length	λ_D
Linear dimensions	L
Electron charge	e
Fluid density	n, n_j
Charge density	ρ
Permittivity of vacuum	ε_0
Electric field	$\boldsymbol{E}$
Magnetic field	$\boldsymbol{B}$
Electron velocity	v_e
Ion velocity	v_i
Number of plasma particles within Debye's sphere	N_D
Plasma parameter	g_p
Frequency of plasma oscillation	ω
Mean time between collisions with neutral atoms	τ
Time period of oscillation of plasma particles	T

Velocity of the waves	v_s
Boltzmann constant	k_B
Ion mass	M_i
Electron Debye radius	λ_{De}
Wave frequency	ω
Wave number, Normalized constant	k, K_j
Plasma frequency	ω_{pi}
Dust plasma frequency	ω_{pd}
Unperturbed ion density	n_{i0}, N_{i0}, $\rho_{N\,i0}$
Unperturbed electron density	n_{e0}, N_{e0}
Phase velocity	V_p, V_{ph}
Thermal speed of ion	V_{Ti}
Velocity vector	v_x
Thermal speed of electron	V_{Te}, v_t
Ion acoustic speed	c_s
Population of electrons	α
Strength of nonextensivity	q
Electrostatic potential function	Φ, Ψ
Fluid mass	m
Fluid velocity	u, v, u_j
Fluid pressure	p
Positive ion mass	M_{+i}
Negative ion mass	M_{-i}
Positive-to-negative ion mass ratio	$M_{\pm}$
Positive ion temperature	T_{+i}
Negative ion temperature	T_{-i}
Unperturbed positive ion density	N_{+i0}
Unperturbed negative ion density	N_{-i0}

Positive ion velocity	U_{+i}
Negative ion velocity	U_{-i}
Positive ion acoustic speed	C_{+is}
Positive ion kinematic viscosity	μ_{+i}
Negative ion kinematic viscosity	μ_{-i}
Space variable	x
Time variable	t
Measuring the weakness of dissipation	ε
Speed of the reference frame	V_r
Amplitude of shock wave excitations	Φ_A, Φ_{mA}, Ψ_A, Ψ_{mA}
Width of shock wave excitations	Φ_W, Φ_{mW}, Ψ_W, Ψ_{mW}
Critical values of strength of nonextensivity	q_c
Unperturbed heavy ion density	N_{hi0}, $\rho_{N_{hi0}}$
Unperturbed immobile dust density	N_{d0}, $\rho_{N_{d0}}$
Number of electrons residing on the dust grain surface	Z_d
Heavy ion density	N_{hi}, $\rho_{N_{hi}}$
Heavy ion velocity	U_{hi}, W_{hi}
Heavy ion acoustic speed	C_{hi}
Heavy ion Debye length	λ_{Dhi}
Heavy ion kinematic viscosity	μ_{hi}, ν_{hi}
Electron-to-ion temperature ratio	δ_{ei}, τ_{ei}
Negative-to-positive, Heavy-to-ion number density ratio	N_{r1}, $\rho_{N_{r1}}$
Electron-to-positive, Electron-to-ion number density ratio	N_{r2}, $\rho_{N_{r2}}$

Chapter 1

Introduction

1.1 Background and motivation

When the blood is free to the number of its corpuscles, then it appears like crystal clear fluid, which was named by the great researcher Johannes Purkinje as 'plasma' which means 'moldable substance' or 'jelly'. Foremost, Nobel Prize winning American chemist I. Langmuir mentioned the word plasma in 1928 at the time of his research on gas characteristics in tubes that carry charge in the case of the ion swinging to and fro noticed in the tube in viscous nature. Plasma is the 4th physical condition of matter, is defined as a quasi-neutral ionized gas consisting of charged particles as well as neutral molecules, and is usually supposed to be a different phase of matter. As per [1], a suitable expression is like this: a plasma is considered a quasi-neutral ionized gas (partially ionized gas) along with charged and neutral molecules, which demonstrates collective attributes. It is able to transmit electricity because of its internal electric charges, which strongly interact with both magnetic and electric fields. It can be elaborately categorized as partially ionized plasma based on the low degree of ionization, fully ionized plasma based on the high degree of ionization. Based on temperature, plasma is classified as thermal plasma ($T_e \sim T_h$, where T_e denote electron temperature and T_h denote neutral temperatures) and non-thermal plasma ($T_e \gg T_h$). Plasma can also be classified as unmagnetized plasmas, in which neither the self-consistent magnetic field nor the ambient magnetic field exist, and magnetized plasmas, in which the magnetic field is strong enough to effect the motion of charged particles and is unaffected by Debye shielding. In addition, plasmas can enter the complex regime when they contain dust of nanometer to micrometer size, which is called the dusty plasma [2]. The mentioned plasmas can be widely divided based on their presence and position as space and astrophysical plasmas (interstellar medium, the sun and other stars, Saturn rings, sun-from core to corona, solar wind, interstellar space stars, nuclear fusion, interplanetary medium, intergalactic medium, accretion discs and so on), terrestrial plasmas (lightning, ball lightning, flames as plasmas, auroras, magnetosphere, ionosphere, polar wind and so on) and artificially produced plasma (fluorescent lights, neon signs, confined fusion plasma, plasmas displays, plasma ball, ion thruster, medical science, textile industries, agriculture, materials processing, water and wastewater treatments, etc.). More than 99% of the matter in the apparent Universe is probably in the plasma state. The problem studied herein will focus on astrophysical and space plasmas. Significant variations are evident in plasma sources, particle flow velocities, average thermal energy, and a number of other physical properties in the broad and multi-scale physical domain. To fully comprehend the associated processes, there are still significant physical challenges to be overcome.

Due to a couple of issues, including the first fact that plasmas are electrostatic fluids and their molecules come into contact with each other through a long-range both magnetic and electrical fields rather than just collisions, plasma science has a reputation for being extremely challenging to fully comprehend. This is almost certainly correct when compared to fluid dynamics or electro-magnetic in dielectric media. The fields are altered by the plasma as a whole, and plasma particles might move to protect one another from forced electric fields, making this more challenging than handling charged ions at that moment, like in an electron beam. Second, most plasmas are too thin and hot to be regarded to continuous fluids like air or liquid (both of which have an approximate density of $3 \times 10^{22}\ cm^{-3}$ or $3 \times 10^{19}\ cm^{-3}$[1, 2]). Plasmas with high particle densities ($10^{9-13} cm^{-3}$) don't necessarily behave like continuous fluids. Since plasma contains free charge particles (e.g., electrons, protons, and ions), it responds to either electric or electromagnetic fields, can carry a current of electricity, and has a distinct space potential. The densities (e.g., ion density (n_i), neutral density (n_a) and electron density (n_e)) are applicable to describe such plasma. Instead of charge amounts, the densities of particles are applicable to determine the degree of ionization. The key criteria of plasma are its quasi-neutrality, which refers to the fact that the densities of positive and negative species are nearly equal, despite the fact that charged particles will always find a way to move in order to avoid coming into contact with strong potentials. It mostly refers to the Debye length (λ_D) at which plasma's mobile charge carrier's screen out electric fields. In other words, the Debye length is the distance over which the thermal particle energy usually disrupts charge neutrality and the electrostatic potential energy coming due to charge separation, which seeks to restore charge neutrality, are in equilibrium.

The existence of plasma must satisfy the following four basic criteria:

(i) Quasi-neutrality: For the existence of plasma, the quasi neutrality condition is $\lambda_D \ll L$, where L denote linear dimension caused by the ionized gas [1].

(ii) Collective behaviors: Existence of plasma requires the criteria "collective behaviors", which arises because of long-range interactions (i.e., Coulomb potential and magnetic fields), which means that local disturbances in equilibrium can have a strong influence on remote regions of the plasma. In other words, microscopic fields usually dominate over short-lived microscopic fluctuations, and a net charge imbalance in the overall density as $\rho = e(n_i - n_e)$ will immediately give rise to an electrostatic field, according to

$$\nabla . \boldsymbol{E} = \frac{\rho}{\varepsilon_0},$$

where $\boldsymbol{E}$ is the electric field, and ε_0 is the permittivity of vacuum. Likewise, the same set of charges moving with motion v_e (electron velocity) and v_i (ion velocity) will give rise to a current density $j = e(n_i v_i - n_e v_e)$. Therefore, elements of force exerted on a particle will affect not only in the intermediate area but also much further afield. The condition for collective behavior demands $N_D \gg 1$, where N_D denotes the number of plasma particles within Debye's sphere.

(iii) Plasma parameter (g_p): The ratio of the potential energy of a plasma particle on the Debye's sphere to translational kinetic energy of the plasma particle on the surface of Debye's sphere is known as the plasma parameter g_p. For the existence of strongly coupled plasma, the value of g_p should be much less than one i.e., $g_p \ll 1$.

(iv) Electromagnetic force: There is another criterion for the existence of plasma is electromagnetic force acting on plasma particles should dominate over the forces due to collisions.

Let, $\omega =$ frequency of plasma oscillation and $\tau =$mean time between collisions with neutral atoms. Then, for existence of plasma

$$\omega\tau > 1 \text{ i.e., } \frac{2\pi}{T} \times \tau > 1 \text{ i.e., } \tau > \frac{T}{2\pi}$$

where $T =$ time period of oscillation of plasma particles.

1.2 Waves in plasma

Plasma waves, or plasma oscillations, are the terms used to describe the sound waves in a charged particle liquid. Usually fluids can uphold the characteristics of acoustic waves associated with pressure, temperature, and velocity variations, whereas plasma waves are considered an interrelated bunch of molecules and fields that are characterized as frequent fashion that repeated periodically. Plasma is also considered a fluid in complex form, so it ensures different wave modes, and in that case, the recovery energies are thought to be composed of dynamic pressure and electrical and magnetic forces. Therefore, the propagation of not merely electrostatic waves but also electromagnetic waves exists in plasma. So, the existence of an enormous amount is plausible because the wave phase velocity depends on the wave frequency as well as its angle of propagation with regards to the background magnetic field. As a consequence, in the absence or presence of an oscillating magnetic field, waves in plasmas can be categorized as electrostatic or electromagnetic. An electrostatic wave necessarily be completely longitudinal, while an electromagnetic wave is probably partially longitudinal in that it contains transverse material. On the other hand, if plasma consists of the attributes of oscillation, then waves could be pursued by differentiability. Based on the hypothesis, the electron temperature is greater than the ion temperature, and in the case of a significantly lower mass of the electron, one can observe that the movement of the electrons is much more rapid than that of the ions. The mode closely related to the motility of the electron depends on the mass of the electron, but the ions may be supposed to be immobile due to their greater inertia. The mode of an ion depends on its mass, whereas electrons are considered to be massless, and based on the Boltzmann relation, they reassign themselves instantly. Different modes can be categorized as corresponding to the event that they generate in an unmagnetized plasma or as parallel, vertical, or diagonal to an immobile magnetic field. Plasma is also supported by nonlinear waves because it is supposed to be a medium of complex form. The aforementioned nonlinear effects are responsible for generating wavelike perturbations, which are not the likely form of the above. Plasma waves are observed in almost all objects of the solar system, such as planets and their satellites, comets, the interplanetary medium, the earth's ionosphere, interstellar media, protostellar disks, molecular clouds, asteroid areas, and nebula [3-6]. Plasma has many degrees of freedom, so it supports many wave modes, e. g., ion acoustic waves (IAWs), dust ion acoustic waves (DIAWs), electron acoustic waves, positron acoustic waves, dust acoustic waves, etc. The description of some of the acoustic wave modes, especially IAWs and DIAWs, is given below.

1.2.1 Ion acoustic wave

In the physics of plasma, an Ion acoustic wave (IAW) is considered a kind of longitudinal wave in which the ions and electrons are oscillated, similar to acoustic waves or regular sound waves moving through neutral gas. In that case, the restoring force is determined by the stress of lighter species (such as electrons, positrons, etc.), and the inertia is provided by the mass density of the ion species. Such waves are generated due to the compression and rarefaction of the medium. However, normal collisions happen when waves propagate through positively charged ions, according to the interaction of IAWs and their electromagnetic fields. Again, IAWs can travel across the medium, which is thought to be collision-free since the ions interact with electrostatic or electromagnetic fields at long distances. Plasma contains electrons, and because of their high mobility relative to ions, electrons have an impact on wave dispersion. They do this by quickly observing the mobility of ions, which preserves the neutrality condition. The slight electric field that is produced by the plasma is responsible for the mobility of electrons due to the variation in the local ion density. If one considers a single ion species as well as a large wavelength boundary, the velocity of the waves in the case of dispersionless $(\omega = v_s k)$ is $v_s = \sqrt{\gamma_e k_B T_e / M_i}$, where the notation k_B is the Boltzmann constant, M_i is the ion mass and T_e is the electron temperature. Generally, γ_e is assumed as unity because the case that satisfied the condition of isothermality, which relies on the ion acoustic wave time scale, is that electrons are massive enough. In collisionless plasma, the ions are frequently cooler than the electrons. It was found that the group velocity of IAWs is equal to the phase velocity, and it exists only when there are thermal motions of charged particles. In IAWs, the ions may oscillate, overcoming their large inertia owing to the restoring force, which is determined by the stress of lighter species. Nonlinear IAWs have considered in Ref. [7], where he studied the basic features were studied using a mechanical analogy. In space plasmas, the highly energetic particles streaming upstream of planetary bow shock fronts are observed as IAWs [8]. There are various types of IAWs in plasmas, such as solitons, shock waves, double layers, and so on, that are important for the knowledge of the physics concerned.

1.2.2 Dust-ion acoustic wave

The presence of dust grains modifies the regular ion acoustic waves, which are known as dust ion acoustic waves (DIAWs), which are a very low-frequency, longitudinal compressional wave originating from a balance of dust particle inertia and plasma pressure and involving the motions of the dust particles. Rao et al. [9] have announced the existence of dust acoustic waves in multi-component collisionless dusty plasma made up of negatively charged electrons, ions and dust grains. The electron and ion thermal speeds are substantially faster than the phase velocity of such waves. Shukla and Silin [6] first identified the DIA waves in 1992. They have shown that the electron thermal speeds as well as the thermal speeds of ions, and dust are significantly lighter (heavier) than the phase velocity of DIA waves. They have also established the following DIA wave dispersion relation:

$$1 + \frac{k_{De}^2}{k^2} - \frac{\omega_{pi}^2 + \omega_{pd}^2}{\omega^2} = 0, \tag{1.1}$$

where, $k_{De} = 1/\lambda_{De}$ and λ_{De} denote electron Debye radius. Furthermore, ω and k are the frequency and the wave vector of the DIA wave, respectively. Since, the ion plasma frequency (ω_{pi}) is significantly larger than the dust plasma frequency (ω_{pd}) due to the massive bulk of the dust grains. Thus, they have also expressed the DIA wave frequencies as follows:

$$\omega^2 = \frac{k^2 C_s^2}{1 + k^2 \lambda_{De}^2}, \tag{1.2}$$

where $C_s = \omega_{pi}\lambda_{De} = (n_{i0}/n_{e0})^{1/2}\, c_s$ is dust ion acoustic speed and $c_s = k_B T_e/M_i$ is the ion acoustic speed. In the long wavelength limit (namely $k^2\lambda_{De}^2 \ll 1$), Eq. (1.2) reduces to

$$\omega = k\left(\frac{n_{i0}}{n_{e0}}\right)^{\frac{1}{2}} c_s.$$

Where, n_{i0} and n_{e0} represent the unperturbed ion density and unperturbed electron density, respectively. If the condition $n_{i0} > n_{e0}$ holds for negatively charged dust grains, the phase speed $(V_p = \omega/k)$ of the DIA waves will greater than c_s. The decrease in the number of electrons in the surrounding plasma, which causes the electron Debye radius to grow, is blamed for the change in phase speed. Further, the greater phase speed of the DIA waves is caused by what seems to be a more powerful space charge electric field. If the condition $kV_{Ti} \ll \omega \ll kV_{Te}$ (V_{Ti} and V_{Te} represent thermal speed for ion and electron, respectively) holds in dusty plasmas, the latter are subject to negligible electron and ion Landau damping [2]. Nakamura [8] and Barkan et al. [10] have experimentally observed DIA waves in laboratory.

1.2.3 Shock wave

Shock waves are a special class of nonlinear waves, also known as strong pressure waves, produced by a quick transfer of pressure, temperature, and density in a thin area moving through a material medium, particularly air, caused by an explosion or by a body traveling faster than the speed of sound. The formation of shock waves is obtained in nature when it satisfies the equations of continuity of mass, momentum, and energy. Due to this, one can distinguish the propagation of shock waves from that of ordinary acoustic waves. The shock wave’s energy decreases more quickly than that of a sound wave since the energy of the shock wave is used to heat the medium through which it travels. When the amplitude of a strong shock wave is high, it reduces by obeying the law of inverse square as long as the wave turns weak, and then it follows the law of acoustic waves. Lightning, thunder, earthquakes, meteor strikes, solar wind, volcanic eruptions, etc. are naturally generated shocks, while on the other hand, nuclear or chemical explosions, the sonic boom of a supersonic aircraft, any supersonic flying projectile, a bullet pushing the air in the barrel of a rifle, etc. are artificially generated shocks [11].

Shock formation can be understood by considering the nonlinear terms in the fluid equations. Due to the high-velocity component's tendency to overtake the low-velocity wave profile caused by these nonlinear terms, a discontinuity finally forms and causes wave steepening. This process of steeping continues only until nonlinearity is balanced by dissipative mechanisms, for instance, kinematic viscosity and thermal conductivity. In plasma, shock waves can be excited when a large-amplitude mode propagates in the presence of strong dissipation, such as due to collisions with neutrals, viscosity, or Landau damping. In both collisional and collisionless plasmas, shock waves can occur. It is assumed that dilute hot gases are in fully ionized plasma states, which are the appropriate states where collisionless shocks appear [12]. The solar wind, plasma flow from the sun, etc. are considered collisionless relativistic shocks [13] that are immensely supersonic at the time of facing sphere magnetic fields. Anyway, a bulk of shock types in astrophysics are produced in highly dilute matter that is generally collisionless on earth, for instance, the earth bow shock. The evidence of astrophysical shock phenomena reproduced in the laboratory is given in Figure 1.1. Since shock wave excitations are supported not only in astrophysical and space environments but also in laboratories, one can study the various kinds of acoustic shock wave excitations in the plasmas. But this work will focus on the propagation of IA and DIA shock wave excitation in collisionless unmagnetized plasmas under various plasma assumptions.

Figure 1.1: An interstellar collisionless bow shock in the Orion Nebula. Image credit: NASA and the Hubble Heritage Team (STScI/AURA). Link: https://news.mit.edu/2019/collisionless-shock-reproduced-astrophysics-0807.

1.3 Distribution functions in plasma

The single-particle descriptions serve to demonstrate the fundamental kinematical phenomena in a plasma, but due to the large number of particles included, they are inadequate for a comprehensive explanation of the plasma dynamics. Each species of plasma is represented in fluid modelling by a distribution function $f(x, v, t)$ over the six-dimensional phase space, which varies with time. There are different distribution functions, such as Maxwell distribution (thermal), Crains distribution (non-thermal), Kappa distribution (super-thermal), q-distribution (nonextensive), (α, q)-distribution (generalized), etc. Furthermore, this work is confined to the (α, q)-velocity distribution (VD) function; hence, only the (α, q)-VD function is described in the next section.

1.3.1 (α, q)-velocity distribution function

The (α, q)-velocity distribution functions are very useful to describe the energy of electrons in all cases of thermality. Since the energies of the electrons may be isothermal, nonthermal, or have a smaller (subthermal) or superior (superthermal) amount of isothermality. As a result, the (α, q)-velocity distribution (VD) of electrons is assumed [14]. The (α, q)-VD function is defined by the composition of Tsallis and Cairns VD functions as

$$y(v_x) = k\left(1 + \alpha\frac{v_x^4}{v_t^4}\right) \times \left\{1 - (q-1)\frac{v_x^2}{2v_t^2}\right\}, \tag{1.3}$$

where $v_t = (k_B T_e/m_e)^{1/2}$ is the electron thermal velocity, $v_x = (2e\Phi/m_e)^{1/2}$ is the velocity vector, q is the nonextensivity strength, α represents the population of faster electrons, k_B is defined as the Boltzmann constant, and k is the normalized constant [14]. Hence, the electron density (N_e) function can be written by integrating (which includes an additional potential term of interacting electrons) the above equation over velocity space [14] as

$$N_e = N_{e0}\left[1 + (q-1)\left(\frac{e\Phi}{k_B T_e}\right)\right]^{\frac{(q+1)}{2(q-1)}} \times$$
$$\left[1 - \frac{16q\alpha}{3 - 14q + 15q^2 + 12\alpha}\left(\frac{e\Phi}{k_B T_e}\right) + \frac{16q(2q-1)\alpha}{3 - 14q + 15q^2 + 12\alpha}\left(\frac{e\Phi}{k_B T_e}\right)^2\right], \tag{1.4}$$

where Φ, T_e, and e are the electrostatic potential function, electron temperature, and magnitude of the electron charge, respectively. One can determine the Maxwellian [9], nonextensive [15] and Cairns et al. [16] distributed electron density functions from the above expression as

$$N_e = N_{e0}\exp\left(\frac{e\Phi}{k_B T_e}\right), \tag{1.5}$$

$$N_e = N_{e0}\left[1 + (q-1)\left(\frac{e\Phi}{k_B T_e}\right)\right]^{\frac{(q+1)}{2(q-1)}}, \tag{1.6}$$

and

$$N_e = N_{e0}\left[1 - \left(\frac{4\alpha}{1 + 3\alpha}\right)\left(\frac{e\Phi}{k_B T_e}\right) + \left(\frac{4\alpha}{1 + 3\alpha}\right)\left(\frac{e\Phi}{k_B T_e}\right)^2\right]\exp\left(\frac{e\Phi}{k_B T_e}\right), \tag{1.7}$$

for the case of $\alpha = 0$, $q = 1$; $\alpha = 0$ and $q = 1$, respectively. The proper ranges of α and q are obtained based on the physical cut-off obligatory by $q \geq 5/7$, and $\alpha_{Max} = (2q - 1)/4$ as (i) $q = 1$, $0 < \alpha < 0.35$ (nonthermality case), (ii) $q = 1$, $\alpha = 0$ (isothermality case), (iii) $0.33 < q < 1$, $\alpha = 0$ (superthermality case), and (iv) $q > 1$, $\alpha = 0$ (subthermality case), respectively.

On the other hand, one major characteristic associated with collisoinless space plasmas is the development of non-Maxwellian velocity distribution that in many circumstances can be described by κ function characterized by the κ parameters. Hence, the electron density (N_e) function can be written by integrating Kappa velocity distribution function [17] over velocity space [14] as

$$N_e = N_{e0}\left[1 - \left(\frac{e\Phi}{k_B T_e}\right)/(\kappa - \frac{3}{2})\right]^{-\kappa+1/2}, \tag{1.8}$$

where $\kappa > 1.5$. It is noted that the presence of superthermal electrons in astrophysical plasmas that are modelled by not only nonextensive distribution along with $-1 < q < 1$ but also Kappa distribution along with $\kappa > 1.5$.

1.4 Formation of fluid model equation

To study the aforementioned physical issues (especially shock wave phenomena), one can use the equations which are briefly discussed in the following under various types of plasma assumptions to study the basic feature of acoustic wave phenomena in the unmagnetized collisonless multi-component plasma.

1.4.1 Conservation of mass or fluid equation of continuity

According to the principle of conservation of matter, a volume V is limited to a change in the total amount of particles N if there is a net particle flux across the surface S enclosing that volume. The law of conservation of mass allows one to acquire

$$\frac{\partial n}{\partial t} + \nabla.(n\boldsymbol{u}) = 0, \tag{1.9}$$

where n and $\boldsymbol{u}$ denote the number density and thermal velocity of the particles, respectively. This well-known equation of continuity, which first arose in plasmas, expresses the concept of mass conservation in differential form. It should be noted that the second term in Eq. (1.9) indicates the divergence of net particle flux out of volume, whereas the first term in the equation (1.9) represents the rate of change of particle concentrations within a volume.

1.4.2 Conservation of energy-momentum or equation of motion neglecting collisions and thermal motion

(i) Equation of motion for a single particle with velocity $\vec{v}$

$$m\frac{d\vec{v}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right), \tag{1.10}$$

All particles in a fluid element move together with average velocity $\vec{u}$, because one can neglect collisions and thermal effects. We can also take $\vec{u} = \vec{v}$, and equation of motion for fluid element of particle density n is:

$$mn\frac{d\vec{u}}{dt} = nq\left(\vec{E} + \vec{u} \times \vec{B}\right), \tag{1.11}$$

where time derivative is to be taken at the position of the particles (fluid element) which not very convenient. We wish to have an equation for fluid elements fixed in space.

(ii) Transform to variables in a fixed frame that move with fluid element.

To make the transformation to variables in a fixed frame, consider $\vec{G}(x,t)$ to be any property of a fluid in one-dimensional x space. The change of $\vec{G}$ with time in a frame moving with the fluid is the sum of two terms:

$$\frac{d\vec{G}(x,t)}{dt} = \frac{\partial\vec{G}}{\partial t} + \frac{\partial\vec{G}}{\partial x}\frac{dx}{dt} = \frac{\partial\vec{G}}{\partial t} + u_x\frac{\partial\vec{G}}{\partial x}, \tag{1.12}$$

The first term on the right represents the change of $\vec{G}$ at a fixed point in space, and the second term represents the change of $\vec{G}$ as the observer moves with the fluid into a region in which $\vec{G}$ is different. In three dimensions Eq. (1.12) one can be written as:

$$\frac{d\vec{G}}{dt} = \frac{\partial\vec{G}}{\partial t} + (\vec{u}.\nabla)\vec{G}$$

This is called the convective derivative.

In the case of a plasma, we take $\vec{G}$ to be the fluid velocity $\vec{u}$,

$$mn\left[\frac{\partial\vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u}\right] = nq(\vec{E} + \vec{u} \times \vec{B})$$

In absence of ambient magnetic field or self-consistent magnetic field due to plasma current then $B \rightarrow 0$ one can obtained as:

$$\left[\frac{\partial\vec{u}}{\partial t} + (\vec{u}.\nabla)(m\vec{u})\right] = q\vec{E}$$

In the similar manner, the equation of motion including thermal effects (pressure term, which is denoted by p) one can be obtained as:

$$mn\left[\frac{\partial\vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u}\right] = nq(\vec{E} + \vec{u} \times \vec{B}) - \nabla p$$

1.4.3 Poisson's equation

The Maxwell equations is written as

$$\nabla.\boldsymbol{E} = \frac{\rho}{\varepsilon_0}, \tag{1.13}$$

where ρ and ε_0 are the overall charge density and permeability, respectively. The electrical potential ϕ is directly connected to the electric field $\boldsymbol{E}$ and is defined as

$$\boldsymbol{E} = -\nabla\phi, \tag{1.14}$$

Using equation (1.13) and (1.14), we have

$$\nabla^2 \phi = \frac{\rho}{\varepsilon_0}$$

This is the well-known Poisson's equation. It should be emphasized that the charge quasi-neutrality condition can be used to derive a governing equation from which one can investigate the wave structure and propagation properties of plasmas.

1.5 Literature review

The wave propagation in a multi-component plasma has become an interesting focus point for plasma physics researchers over the last few decades. Already, a large number of authors [18-36] have studied the properties of nonlinear waves in plasmas by considering different plasma models. Because nonlinearities contribute to the localization of waves, they lead to different types of interesting coherent nonlinear wave structures, namely solitary waves, shock waves, double layers, vortices, etc., which are important from both the theoretical and experimental points of view. For instance, wave structures have been reported in laser plasma interactions in the form of nonlinear electrostatic [28] or electromagnetic waves [29], collisionless shocks [30], ions and electrons in phase space holes [31], etc. It may arise out of the interplay between some of the mechanisms in physical systems, such as diffraction, dispersion, weakly transverse dispersion, and dissipation, in the presence of nonlinearity. Solitary waves (propagates without any temporal evolution in shape or size when viewed in the reference frame moving with the group velocity of the wave) are mainly formed due to the balance between the effects of nonlinearity and dispersion, where the dissipation is negligible. On the other hand, a soliton (nonlinear solitary wave with the additional property that the wave retains its permanent structure, even after interacting with another soliton) or solitary wave is a self-reinforcing hump or dip-shaped nonlinear wave that maintains its shape and size while it propagates at a constant speed. When the dissipative effects are comparable to or even dominant over the dispersive effects, nonlinear propagation of acoustic waves may appear in the form of shock structures instead of solitary structures. The shock wave can occur both in collisional and collisionless plasmas. Collisionless shocks appear [30] basically in dilute hot gases, which are in the state of fully ionized plasmas. Owing to the significance of IA- and DIA-shock waves for understanding the physical issues in many environments, such wave phenomena have studied by many researchers in different types of plasma environments [31-42]. Luo et al. [36, 37], have confirmed the production of ion acoustic and dust ion acoustic shock waves in the Q-machine with negative ions. Adak et al. [38, 39] have reported the nonlinear IA shock in a pair-ion ($C_{60}^{+}+, C_{60}^{-}$) plasma by deriving the Korteweg–de Vries equation Burgers (KdVB) equation in the presence of weakly dissipative media. Jannat et al. [40] have investigated the ion-acoustic shock waves (IASWs) in pair-ion plasma in the presence of nonextensive electrons. Hussain et al. [43] have reported the propagation of IASWs in a plasma with inertial pair-ions having kinematic viscosities of both positive and negative ion species and inertialess non-extensive electrons. Hafez et al. [44] have investigated the oblique propagation of ion acoustic shock waves in weakly and highly relativistic plasmas with nonthermal electrons and positrons. Hafez et al. [45] have reported ion acoustic shock in highly relativistic plasmas with nonextensive electrons and positrons. Shukla and Silin [6] initially theorized the presence of low-frequency DIA waves, the researchers became interested in DIAWs because of their significance in space as well as their laboratory of study [10, 46-48]. Bharuthram et al. [49] have studied how the large amplitude of DIAWs propagates. Shukla [50] has explored DIA shocks made of dust ions and sound. In addition, Borah et al. [51] have investigated DIA shock waves in a three-component dusty plasma system and found that as the dynamic viscosity of the plasma constituent increases, the steepness of the shock structures decreases. In their analysis of DIA shock waves and solitary waves in the presence of super-thermal electrons, Haider et al. [52] have shown that the beneficial influence associated with DIASWs rises as the super-thermality of the charged particles develops. Furthermore, Along with the Maxwellian electron, an investigation of DIA shocks have conducted by Duha and Mamun [53]. Several authors [54-58] have investigated the impact of confined ion species on DA shock waves in a dusty plasma context.

Most of the researchers in their earlier literature ignored studying the acoustic wave not only around the critical values (CVs) but also at the composition of CVs for any specific plasma parameters due to the cumbersome mathematical formulation in the plasmas. On the other hand, Mamun [59] has clearly shown that the investigation of a DA or DIA shock wave by using the stretching of not only the kinematic viscosity coefficient but also any parameters is not usually valid. The valid stretching coordinates supported only the Burgers equation but not the Korteweg-de Vries Burger (KdVB) equation [59], which divulges the formation of shock structures in the plasmas. It is therefore essential to study shock wave propagation in the plasmas by deriving the useful nonlinear evolution equations (NLEEs) via the valid stretching coordinates along with their appropriate solutions. Thus, this thesis work explores how to study the electrostatic nonlinear propagation of IA and DIA shock wave excitations by deriving useful NLEEs along with their appropriate solutions based on valid stretching coordinates. The effects of some parameters on shock wave excitation will also be investigated.

1.6 Research methodology

It is well confirmed that the reductive perturbation analysis is widely applicable to investigate small but finite-amplitude nonlinear waves. By means of this technique, one can derive various types of nonlinear evolution equations (NLEEs) through a single dependent variable that look very simple in structure from a physical system. However, the original model equations used to describe the physical system are not simple and generally contain several dependent variables. To illustrate it, suppose the governing equations contain the fluid density $n_j(x,t)$ and the fluid velocity $u_j(x,t)$ and perhaps several other variables and equations of state as well, depending on whether thermodynamic considerations are taken into account or not. One requires procedure in a systematic way for reducing such sets of equations to simpler forms. The reductive perturbation technique (RPT) [60] is one such method because such procedures are usually perturbative in nature.

1.6.1 Reductive perturbation technique

In the reductive perturbation method, one introduces new stretching coordinates for space and time variables that are appropriate for the description of long wave-length phenomena. This rescaling gives the system isolation from the relevant equations, which describe how the system reacts on the new space and time scales. It is to be noted that RPT has the limitation that it rests on experience in knowing how to pick the relevant scale. To overcome such limitations, one expands all the dependent variables in terms of a small perturbation parameter ε based on the general principles of the reductive perturbation theory [60] on multiple scale expansion. As for example:

$$\left.\begin{aligned} n_j &= n_j^{(0)} + \varepsilon n_j^{(1)} + \varepsilon^2 n_j^{(2)} + \cdots \\ u_j &= \varepsilon u_j^{(1)} + \varepsilon^2 u_j^{(2)} + \cdots \end{aligned}\right\}. \tag{1.15}$$

The boundary conditions can usually be controlled by the presence or absence of a first term in the above equations. In most of the cases, $n_j \to n_j^{(0)}$ as $x \to \pm\infty$ because the density is normally perturbed about its equilibrium value [61] and $u_j \to 0$ if there is background flow supports, it is actually dependent on the physical circumstances. Besides, the harmonic wave solution can be defined as $n_j, u_j, \sim exp(i\Omega)$ with $\Omega = K_j x - \omega(K_j)t$ for finding the dispersion or dissipation relations. Note that, $\omega(K_j)$ satisfies the dispersion or dissipation relation by the wave number K_j. Now, one can define the new wave number as $K_j = \varepsilon^j k_j$, $j = 1,2,3 \ldots\ldots$ for long waves without the loss of generality. As a result, Ω becomes $\Omega = \varepsilon^j k_j x - \omega(\varepsilon^j k_j)t$. For purely dissipative media, the Taylor expansion [61] of $\omega(\varepsilon^j k_j)$ yields $\omega(\varepsilon^j k_j) = \omega'(0)\varepsilon^j k_j + \omega''(0)\varepsilon^{2j} k_j^2 + \cdots\cdots$. In such case, Ω is obtained as $\Omega = \varepsilon^j k_j(x - \omega'(0)\, t) - \omega''(0)\varepsilon^{2j} {k_i}^2 t$, where the other terms are neglected because ε is small quantity. Finally, one may consider the new stretching for x and t as $X = \varepsilon^j(x - V_p\, t)$ and $T = \varepsilon^{2j} t$ since $\omega'(0)$ and $\omega''(0)$ are constants [62]. Thus, one can derive various evolution equations to study the shock wave excitations in the plasmas by taking the stretching $X = \varepsilon^j(x - V_p\, t)$ and $T = \varepsilon^n t$ along with $n = 2 * j$; $j = 1, 2, 3, \cdots$. If the evolution equation obtained by taking $j = 1$ is unable to examine the basic features of shock wave phenomena, then one can derive another evolution equation by taking $j = 2$ to overcome such difficulty.

1.7 Outline of thesis

The outline of this thesis is as follows:

Chapter 1 covers the introductory discussions, background and motivations, the fundamentals of plasmas, the existence of shock wave excitations, a literature review, and reduction perturbation techniques.

Chapter 2 deals with the features of nonlinear propagation of ion shock wave excitations (IASWEs) by deriving Burger's equations involving different nonlinear terms via the reductive perturbation method in an unmagnetized collisionless pair-ion plasma with -distributed electrons. The effect of plasma parameters on the nonlinear propagation of IASWs is investigated by determining the solutions of Burger's equations involving various kinds of nonlinearity.

Chapter 3 concerns the nonlinear features of dust ion acoustic shock wave excitations (DIASWEs) by deriving Burger's equations involving different nonlinear terms via the reductive perturbation method in a collisionless, four-component, unmagnetized, dusty plasma system having generalized (α, q)-distributed electrons, positively charged Maxwellian light ions, negatively charged inertial heavy ions, and negatively charged stationary dust ions. The effect of plasma parameters on the nonlinear propagation of DIASWEs is investigated by determining the solutions of Burger's equations involving various kinds of nonlinearity.

Chapter 4 gives a clear idea how to study DIASWEs for the super-critical values of any particular parameter in an unmagnetized collisionless four-component dusty multi-ion nonextensive plasma by deriving a Burger's-type equation with quartic nonlinearity.

Chapter 5 presents the concluding remarks that are found in our previous chapters. Additionally, the potential future works for further investigation in the plasmas are discussed.

Chapter-2

Ion acoustic shock wave excitations in multicomponent collisionless unmagnetized plasmas

2.1 Introduction

Rigorous experimental and theoretical investigations on unmagnetized collisionless plasmas having pair-ions and electrons are reported by many researchers because of their significant applications in understanding several types of collective processors in diverse environments, especially plasmas produced by Q-machines [63-66], neutral beam sources [67], semiconductor and material processing [68], and so on. To illustrate it, Ichiki et al. [69] and Weingarten et al. [70] have experimentally confirmed the production of negative ions (NIs) at low temperatures. Sato [64] has also confirmed that the NIs with low temperature ($\approx 0.2eV$) are generated in potassium plasma in the Q-machine. Goeler et al. [71] have detected pair ions (Cs^+, Cl^-) in the hot tungsten plate of a Q-machine. Furthermore, the NIs exist in space and astrophysical objects [72, 73], e.g., in the D-region altitudes, the F-region of the Earth's ionosphere, etc. Wong et al. [74] have experimentally reported ion acoustic (IA) waves in the presence of SF_6^- plasma species. In addition, Song et al. [75] have already described the IA wave propagation in a plasma produced by the Q-machine, which has a pair of ions (K^+, SF_6^-) and electrons. They have found that the phase velocity of the IA "fast" mode increases with the increase in the negative to positive ion density ratio. It is to be noted that the negative ion plasma system is mainly formed naturally, and is composed of electrons, positive ions (PIs), and NIs in space and astrophysical environments [76-78]. For instance, NIs are produced in the lower ionosphere (D region) by the attachment of electrons to atoms or molecules in the presence of relatively high pressure [79]. The decrease in electron density allows the night-time reflection of radio waves from the E-layer (region of the ionosphere). Andersen et al. [80] have also experimentally observed that Landau damping prevented the formation of a shock and only a "spreading" of the pulse with equal electron and NI temperatures. However, they have found that the Landau damping is reduced and the shock structure is formed by increasing electron temperature and cooling the ions via ion-neutral collisions. With the increase in wave phase velocity, reduced by the wave particle resonance due to the involvement of NIs when the electron temperature is greater than the NIs temperature [75]. Due to the existence of pair ions not only in laboratories but also in many space and astronomical environments, many researchers [37, 39, 43, 81, 82] have investigated the propagation of IA shock waves in plasmas composed of various types of PIs (e.g.K^+, Cs^+, C_{60}^+, etc.) and NIs (e.g., SF_6^-, Cs^-, C_{60}^- etc.). Actually, ion acoustic shock waves (IASWs) are produced in plasmas because the relative velocity between the rarefaction wave and the plasma overtakes its IA speed, where the frequency of the rarefaction wave and the surrounding medium is comparatively the same. Adak et al. [39] have reported IASWs in the (C_{60}^+, C_{60}^-) plasma by obtaining the KdV Burgers equation. Hussain et al. [43] have theoretically described IASWs in negative ion plasma with nonextensive electrons. Recently, Alam and Talukder [81] investigated IASWs in collisionless plasma, which consist of pair ions and isothermal electrons.

Further, the presence of energetic electrons in a variety of environments and measurements of their distribution functions revealed them to be highly nonthermal, sub-thermal or superthermal [14, 44, 45, 82-85]. Due to the energetic electrons, the electrons have long range interactions with other plasma species because the plasma species actually occur in different phase spaces [86]. Already, many researchers have examined the IA structures in the plasma system by assuming the plasma species are in thermal equilibrium, which follows the Maxwell–Boltzmann distribution. However, such an ideal thermal equilibrium assumption is no longer valid when some external agents, such as force fields present in natural space environments, wave–particle interaction, turbulence, etc., disturb the thermal equilibrium of the plasma systems. The fast ions and electrons mode in plasma environments also suggests that the particles have a deviated velocity distribution far from the Maxwellian distribution. Moreover, systems with the presence of long-range interactions as well as long-time memory are intractable within the Boltzmann–Gibbs statistics [87]. At these stages, the non-Maxwellian velocity distribution functions are an arena for obtaining the electron density function when the electron energy is higher or lower than the isothermal energies or the electron energy becomes nonthermal. When the energy of electrons becomes subthermal or superthermal, one can use the nonextensive velocity distribution function to determine the density function of the electron as

$$N_e = N_{e0}[1 + (q-1)(e\Phi/k_B T_e)]^{\frac{(q+1)}{2(q-1)}},$$

where Φ, N_{e0}, q, T_e, e and k_B are the electrostatic potential, unperturbed electron density, strength of nonextensivity, electron temperature, electron charge, and Boltzmann constant, respectively [84]. The effects of electron nonextensivity on IA shock waves (IASWs) have already been studied by many researchers [45, 84, 85, 88, 89]. For instance, Ferdousi et al. [89] have reported the effects of electron nonextensivity on the properties of IASWs in an unmagnetized three-component plasma. They have shown that shock wave excitations (SWEs) are significantly modified, and both compressive and rarefective shock waves are supported by the influence of the strength of nonextensivity. But, the nonextensive distribution function fails to address the physical issues for the nonthermal electron populations. As a result, one can consider modeling an electron distribution with a population of fast particles by taking the Cairns velocity distribution function [90-92] leading to

$$N_e = N_{e0}\left[1 - \left(\frac{4\alpha}{1+3\alpha}\right)\left(\frac{e\Phi}{k_B T_e}\right) + \left(\frac{4\alpha}{1+3\alpha}\right)\left(\frac{e\Phi}{k_B T_e}\right)^2\right]\exp\left(\frac{e\Phi}{k_B T_e}\right),$$

where α is a parameter determining the fast particles present in the model considered. Later, Tribeche et al. [14] have proposed a unique distribution function, the so-called the (α, q)-velocity distribution function, by generalizing the work of Cairns et al. [16], which provides a better fit of the space observations due to the flexibility provided by the nonextensivity parameter. They have obtained electron density by integrating (α, q)-velocity distribution function over all velocity spaces as

$$N_e = N_{e0}\left[1 + (q-1)\left(\frac{e\Phi}{k_B T_e}\right)\right]^{\frac{(q+1)}{2(q-1)}} \times$$
$$\left[1 - \frac{16q\alpha}{3 - 14q + 15q^2 + 12\alpha}\left(\frac{e\Phi}{k_B T_e}\right) + \frac{16q(2q-1)\alpha}{3 - 14q + 15q^2 + 12\alpha}\left(\frac{e\Phi}{k_B T_e}\right)^2\right]. \quad (2.1)$$

One can easily reduce the above electron density to the nonextensive and nonthermal electron densities by setting $\alpha = 0$ and $q \rightarrow 1$, respectively. Thus, a unique distribution function, like the (α, q)-velocity distribution function [14] can be assumed for examining all the cases of electron energies. El-wakil et al. [92] have already described IA modulation instability characteristics in negative ion plasma with nonthermal electrons. They have also shown that the instability conditions are affected by nonthermal electron parameters in the D and F regions of the Earth's ionosphere. It is therefore suggested to extend the hydrodynamic fluid model by considering (α, q)-distributed electrons [14] and pair-ions because the (α, q)-velocity distribution function is not only used for superthermal, subthermal, and nonthermal, but also Maxwellian distributed electrons. Very recently, Hafez et al. [93] have proposed the NI plasma for understanding the nature of overtaking collisions of multi-shocks by deriving a Burgers-like equation. They reported that the compressive and rarefactive electrostatic shocks are supported in the aforementioned plasma system. But, they ignored the features of electrostatic shocks not only around the critical values (CVs) but also at the CVs. Also, the shock wave phenomena around CVs are reported by incorrectly defined solutions of modified Burgers (mB) and mixed modified Burgers (mmB) equations in most of the previous studies [94-96], which is not useful for further verification in laboratory plasmas. Mamun [59] has clearly shown that the stretching of kinematic viscosity coefficient parameters is not usually valid. The valid stretching coordinates supported only the Burgers equation but not the KdV Burgers equation, which divulges the formation of shock structures in the plasmas. It is therefore essential to study shock wave propagation in the plasmas by deriving the useful nonlinear evolution equations (NLEEs) via the valid stretching coordinates along with their appropriate solutions. Thus, this work explores how to study the electrostatic nonlinear propagation of IASWs by deriving useful NLEEs along with their appropriate solutions based on the valid stretching coordinates in a pair ion plasma system. The effects of some parameters on the SWEs are also investigated.

2.2 Theoretical model equations with plasma assumptions

An unmagnetized collisionless plasma composed of inertial pair-ion (e.g., PI (K^{+}) with mass (M_{+i}) and temperature (T_{+i}), and NI (SF_6^-) with mass (M_{-i}) and temperature (T_{-i}) and inertial-less (α, q)-distributed electrons, where α and q are treated as the population of nonthermal and the strength of non-extensivity electrons, respectively. For the above plasma assumption, one obtains the charge neutrality condition as $1 = N_{r1} + N_{r2}$, where $N_{r1} = N_{-i0}/N_{+i0}$, $N_{r2} = N_{e0}/N_{+i0}$. Since $N_{+i0}(N_{-i0})$ and N_{e0} are the PIs (NIs) and electrons unperturbed densities, respectively. To study the nonlinear phenomena in such plasmas, the following normalized hydrodynamic continuity and momentum equations are obtained by implementing the mass and momentum conservation laws along with the above plasma assumptions for PIs and NIs [93]:

$$\frac{\partial N_{+i}}{\partial t} + \frac{\partial (N_{+i} U_{+i})}{\partial z} = 0, \tag{2.2}$$

$$\frac{\partial N_{-i}}{\partial t} + \frac{\partial (N_{-i} U_{-i})}{\partial z} = 0, \tag{2.3}$$

$$\frac{\partial U_{+i}}{\partial t}+U_{+i}\frac{\partial U_{+i}}{\partial z}+\frac{\partial \Phi}{\partial z}+\mu_{+i}\frac{\partial^2 U_{+i}}{\partial z^2}=0, \tag{2.4}$$

$$\frac{\partial U_{-i}}{\partial t}+U_{-i}\frac{\partial U_{-i}}{\partial z}+-M_{\pm}\frac{\partial \Phi}{\partial z}+\frac{\delta_{ei}}{N_{-i}}\frac{\partial N_{-i}}{\partial z}+\mu_{-i}\frac{\partial^2 U_{-i}}{\partial z^2}=0, \tag{2.5}$$

$$\frac{\partial^2 \Phi}{\partial z^2}=N_{pi}-N_{ni}-N_{r2}\times\Big\{[1+(q-1)\Phi]^{\frac{q+1}{2(q-1)}}$$
$$\times\left[1-\frac{16q\alpha}{3-14q+15q^2+12\alpha}\Phi+\frac{16q(2q-1)\alpha}{3-14q+15q^2+12\alpha}\Phi^2\right]\Big\}, \tag{2.6}$$

where

$$M_{\pm}=\frac{M_{+}}{M_{-}},\qquad \delta_{ei}=\frac{T_e}{(1-N_{r1})T_{-i}}. \tag{2.7}$$

Here, $N_{+i}(N_{-i})$ and N_e are respectively the normalized PIs (NIs) and electrons densities normalized by N_{+i0}, $U_{+i}(U_{-i})$ is the normalized PIs (NIs) velocity normalized by the PI speed $C_{+is}=\sqrt{k_BT_e/M_{\pm}}\sqrt{(N_{r1}+N_{r2})/N_{r2}(1-N_{r1})}$, Φ is the normalized electrostatic potential, $\Phi\rightarrow e\Phi/k_BT_e$, t is the time variable normalized by $\omega_{pi}^{-1}=\lambda_{De}/C_{+is}$ is the space variable normalized by $\lambda_{De}=\sqrt{k_BT_e/4\pi N_{e0}e^2}$, and $\mu_{+i}(\mu_{-i})$ is the normalized PIs (NIs) kinematic viscosity coefficient normalized by $\omega_{+i}^{-1}/M_{\pm}\,N_{+i0}C_{+is}^2(\omega_{+i}^{-1}/M_{-i}\,N_{-i0}C_{+is}^2)$. Additionally, one can use (i) $q\rightarrow 1$ and $\alpha=0$ for the Maxwell–Boltzmann distributed electrons, (ii) $q\rightarrow 1$ and $\alpha\neq 0$ for the Cairns distributed electrons and (iii) $\alpha=0$ for superthermal $(0<q<1)$ and subthermal $(q>1)$ electrons, respectively, where q is the strength of nonextensivity and α is measuring the population of nonthermal electrons.

2.3 Formation of Burgers equation

It is well established that the nonlinear IA wave mode cannot be easily described by solving Eqs. (2.2)-(2.6) directly. At this stage, one can use the tedious mathematical technique for deriving the nonlinear evolution equations (NLEEs) to study the basic features of physical phenomena in diverse environment. To derive a NLEE form the aforementioned model, the reduction perturbation technique [59, 84] is allowed to consider the new coordinates instead of the scaling of variables as

$$X = \varepsilon\left(z - V_p t\right), T = \varepsilon^2 t, \tag{2.8}$$

From Eq. (2.8), the operators are defined as

$$\frac{\partial}{\partial t} = \varepsilon^2 \frac{\partial}{\partial T} - \varepsilon V_p \frac{\partial}{\partial X}, \qquad \frac{\partial}{\partial z} = \varepsilon \frac{\partial}{\partial X}, \tag{2.9}$$

Eqs. (2.2)-(2.6) are then converted with the aid of Eq. (2.9) to the following:

$$\varepsilon^2 \frac{\partial N_{+i}}{\partial T} - \varepsilon V_p \frac{\partial N_{-i}}{\partial X} + \varepsilon \frac{\partial}{\partial X}(N_{+i}U_{+i}) = 0, \tag{2.10}$$

$$\varepsilon^2 \frac{\partial N_{-i}}{\partial T} - \varepsilon V_p \frac{\partial N_{-i}}{\partial X} + \varepsilon \frac{\partial}{\partial X}(N_{-i}U_{-i}) = 0, \tag{2.11}$$

$$\varepsilon^2 \frac{\partial U_{+i}}{\partial T} - \varepsilon V_p \frac{\partial U_{+i}}{\partial X} + \varepsilon U_{+i}\left(\frac{\partial U_{+i}}{\partial X}\right) + \varepsilon \frac{\partial \Phi}{\partial X} + \varepsilon^2 \mu_{+i} \frac{\partial^2 U_{+i}}{\partial X^2} = 0, \tag{2.12}$$

$$\varepsilon^2 \frac{\partial U_{-i}}{\partial T} - \varepsilon V_p \frac{\partial U_{-i}}{\partial X} + \varepsilon U_{-i}\left(\frac{\partial U_{-i}}{\partial X}\right) - \varepsilon M_{\pm} \frac{\partial \Phi}{\partial X} + \varepsilon \frac{\delta_{ei}}{N_{-i}} \frac{\partial N_{-i}}{\partial X} + \varepsilon^2 \mu_{-i} \frac{\partial^2 U_{-i}}{\partial X^2} = 0, \tag{2.13}$$

and

$$\varepsilon^4 \frac{\partial^2 \Phi}{\partial X^2} = -N_{r2}\left\{[1 + (q-1)\Phi]^{\frac{q+1}{2(q-1)}} \times [1 + B_1\Phi + B_2\Phi^2]\right\} + N_{pi} - N_{ni}, \tag{2.14}$$

where

$$B_1 = -\frac{16q\alpha}{3 - 14q + 15q^2 + 12\alpha} \text{ and } B_2 = \frac{16q(2q-1)\alpha}{3 - 14q + 15q^2 + 12\alpha}.$$

Again, the expansions for physical quantities are considered [59, 84] as

$$\begin{bmatrix} N_{+i} \\ N_{-i} \\ U_{+i} \\ U_{-i} \\ \Phi \end{bmatrix} = \begin{bmatrix} 1 \\ N_{r1} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \sum_{i}^{\infty} \varepsilon^i \begin{bmatrix} N_{+i}^{(i)} \\ N_{-i}^{(i)} \\ U_{+i}^{(i)} \\ U_{-i}^{(i)} \\ \Phi^{(i)} \end{bmatrix}, \tag{2.15}$$

where V_p and ε are the linear phase speed of the IA mode and a small quantity measuring the weakness of dissipation, respectively. By employing Eq. (2.15) into Eqs. (2.10)-(2.14), one can covert Eqs. (2.10)-(2.14) by including the different orders of ε, that is $O(\varepsilon^r)$, $r = 2,3,4,\cdots\cdots$, as well as the first, second, third, and so on perturbed quantities.

For $O(\varepsilon^2)$:

$$-V_p \frac{\partial N_{+i}^{(1)}}{\partial X} + \frac{\partial U_{+i}^{(1)}}{\partial X} = 0, \tag{2.16}$$

$$-V_p \frac{\partial N_{-i}^{(1)}}{\partial X} + N_{r1} \frac{\partial U_{-i}^{(1)}}{\partial X} = 0, \tag{2.17}$$

$$-V_p \frac{\partial U_{+i}^{(1)}}{\partial X} + \frac{\partial \Phi^{(1)}}{\partial X} = 0, \tag{2.18}$$

$$-V_p \frac{\partial U_{-i}^{(1)}}{\partial X} - M_{\pm} \frac{\partial \Phi^{(1)}}{\partial X} + \frac{\delta_{ei}}{N_{r1}} \frac{\partial N_{-i}^{(1)}}{\partial X} = 0, \tag{2.19}$$

and

$$-N_{r2}\Omega_1 \Phi^{(1)} + N_{+i}^{(1)} - N_{-i}^{(1)} = 0, \tag{2.20}$$

where

$$\Omega_1 = \frac{q+1}{2} + B_1.$$

For $O(\varepsilon^3)$:

$$\frac{\partial N_{+i}^{(1)}}{\partial T} - V_p \frac{\partial N_{+i}^{(2)}}{\partial X} + \frac{\partial U_{+i}^{(2)}}{\partial X} + \frac{\partial}{\partial X}\left(N_{+i}^{(1)} U_{+i}^{(1)}\right) = 0, \tag{2.21}$$

$$\frac{\partial N_{-i}^{(1)}}{\partial T} - V_p \frac{\partial N_{-i}^{(2)}}{\partial X} + N_{r1} \frac{\partial U_{-i}^{(2)}}{\partial X} + \frac{\partial}{\partial X}\left(N_{-i}^{(1)} U_{-i}^{(1)}\right) = 0, \tag{2.22}$$

$$\frac{\partial U_{+i}^{(1)}}{\partial T} - V_p \frac{\partial U_{+i}^{(2)}}{\partial X} + U_{+i}^{(1)} \frac{\partial U_{+i}^{(1)}}{\partial X} + \frac{\partial \Phi^{(2)}}{\partial X} + \mu_{+i} \frac{\partial^2 U_{+i}^{(1)}}{\partial X^2} = 0, \tag{2.23}$$

$$\frac{\partial U_{-i}^{(1)}}{\partial T} - V_p \frac{\partial U_{-i}^{(2)}}{\partial X} + U_{-i}^{(1)} \frac{\partial U_{-i}^{(1)}}{\partial X} - M_{\pm} \frac{\partial \Phi^{(2)}}{\partial X} +$$

$$\frac{\delta_{ei}}{N_{r1}} \frac{\partial N_{-i}^{(2)}}{\partial X} - \frac{\delta_{ei}}{N_{r1}^2} N_{-i}^{(1)} \frac{\partial N_{-i}^{(1)}}{\partial X} + \mu_{-i} \frac{\partial^2 U_{-i}^{(1)}}{\partial X^2} = 0, \qquad (2.24)$$

and

$$-N_{r2}\Omega_1 \Phi^{(2)} - N_{r2}\Omega_2 \left[\Phi^{(1)}\right]^2 + N_{+i}^{(2)} - N_{-i}^{(2)} = 0, \qquad (2.25)$$

where

$$\Omega_2 = \frac{q+1}{2} B_1 + \frac{(q+1)(3-q)}{8} + B_2.$$

Now, the solutions for $O(\varepsilon^2)$ from Eqs. (2.16)-(2.19) are obtained as

$$\left.\begin{aligned} N_{+i}^{(1)} &= \frac{1}{V_p^2} \Phi^{(1)}, U_{+i}^{(1)} = \frac{1}{V_p} \Phi^{(1)} \\ N_{-i}^{(1)} &= -\frac{N_{r1} M_{\pm}}{V_p^2 - \delta_{ei}} \Phi^{(1)}, U_{-i}^{(1)} = -\frac{V_p M_{\pm}}{V_p^2 - \delta_{ei}} \Phi^{(1)} \end{aligned}\right\}, \qquad (2.26)$$

By inserting the different values of Eq. (2.26) in Eq. (2.20), the linear phase velocity is obtained as

$$V_p = \pm \sqrt{\frac{(N_{r2}\Omega_1 \delta_{ei} + N_{r1} M_{\pm} + 1) \pm \sqrt{R}}{2 N_{r2} \Omega_1}}. \qquad (2.27)$$

The positive value of phase velocity indicates fast mode, and the negative value of phase velocity indicates slow mode. But in our work, we consider it fast mode. Where $R = (N_{r2}\Omega_1\delta_{ei} + N_{r1}M_{\pm} + 1)^2 - 4N_{r2}\Omega_1\delta_{ei}$. Eq. (2.27) clearly indicates that V_p is strongly dependent only on N_{r1}, $M_{\pm}$, δ_{ei}, N_{r2}, α and q. Moreover, it is validated only if $R = (N_{r2}\Omega_1\delta_{ei} + N_{r1}M_{\pm} + 1)^2 - 4N_{r2}\Omega_1\delta_{ei} \geq 0$. Now, one can easily derive the following equations from Eqs. (2.21)-(2.25) by using the values of Eq. (2.26):

$$\frac{1}{V_p^2} \frac{\partial \Phi^{(1)}}{\partial T} - V_p \frac{\partial N_{+i}^{(2)}}{\partial X} + \frac{\partial U_{+i}^{(2)}}{\partial X} + \frac{2}{V_p^3} \Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial X} = 0, \qquad (2.28)$$

$$\frac{1}{V_p}\frac{\partial\Phi^{(1)}}{\partial T}-V_p\frac{\partial U_{+i}^{(2)}}{\partial X}+\frac{\partial\Phi^{(2)}}{\partial X}+\frac{1}{V_p^2}\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}+\frac{\mu_{+i}}{V_p}\frac{\partial^2\Phi^{(1)}}{\partial X^2}=0, \tag{2.29}$$

$$-\frac{N_{r1}M_\pm}{V_p^2-\delta_{ei}}\frac{\partial\Phi^{(1)}}{\partial T}-V_p\frac{\partial N_{-i}^{(2)}}{\partial X}+N_{r1}\frac{\partial U_{-i}^{(2)}}{\partial X}+\frac{2N_{r1}V_pM_\pm^2}{(V_p^2-\delta_{ei})^2}\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}=0, \tag{2.30}$$

$$-\frac{V_pM_\pm}{V_p^2-\delta_{ei}}\frac{\partial\Phi^{(1)}}{\partial T}-V_p\frac{\partial U_{-i}^{(2)}}{\partial X}+\frac{\delta_{ei}}{N_{r1}}\frac{\partial N_{-i}^{(2)}}{\partial X}-M_\pm\frac{\partial\Phi^{(2)}}{\partial X}+\frac{V_p^2M_\pm^2}{(V_p^2-\delta_{ei})^2}\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}$$
$$-\frac{\delta_{ei}M_\pm^2}{(V_p^2-\delta_{ei})^2}\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}-\frac{\mu_{-i}V_pM_\pm}{V_p^2-\delta_{ei}}\frac{\partial^2\Phi^{(1)}}{\partial X^2}=0, \tag{2.31}$$

and

$$-N_{r2}\Omega_1\Phi^{(2)}-N_{r2}\Omega_2\left[\Phi^{(1)}\right]^2+N_{+i}^{(2)}-N_{-i}^{(2)}=0. \tag{2.32}$$

Multiplying Eq. (2.28) by V_p and then adding Eq. (2.29), one can obtain as

$$\frac{2}{V_p}\frac{\partial\Phi^{(1)}}{\partial T}-V_p^2\frac{\partial N_{+i}^{(2)}}{\partial X}+\frac{3}{V_p^2}\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}+\frac{\partial\Phi^{(2)}}{\partial X}+\frac{\mu_{+i}}{V_p}\frac{\partial^2\Phi^{(1)}}{\partial X^2}=0, \tag{2.33}$$

Again, multiplying Eq. (2.30) by V_p and multiplying Eq. (2.31) by N_{r1}, then adding, one can obtain as

$$-\frac{2N_{r1}M_\pm}{V_p^2-\delta_{ei}}\frac{\partial\Phi^{(1)}}{\partial T}-\left(V_p^2-\delta_{ei}\right)\frac{\partial N_{-i}^{(2)}}{\partial X}+\left[\frac{3N_{r1}V_p^2M_\pm^2}{(V_p^2-\delta_{ei})^2}-\frac{\delta_{ei}N_{r1}M_\pm^2}{(V_p^2-\delta_{ei})^2}\right]\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}$$
$$-N_{r1}M_\pm\frac{\partial\Phi^{(2)}}{\partial X}-\frac{\mu_{-i}V_pM_\pm}{V_p^2-\delta_{ei}}\frac{\partial^2\Phi^{(1)}}{\partial X^2}=0, \tag{2.34}$$

Differentiating Eq. (2.32) with respect to X and multiplying Eq. (2.33) by $\frac{1}{V_p^2}$, then adding, one can obtain

$$\frac{2}{V_p^3}\frac{\partial\Phi^{(1)}}{\partial T}+\left[\frac{3}{V_p^4}+2N_{r2}\Omega_2\right]\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}+\left[\frac{2}{V_p^2}-N_{r2}\Omega_1\right]\frac{\partial\Phi^{(2)}}{\partial X}+\frac{\mu_{+i}}{V_p^3}\frac{\partial^2\Phi^{(1)}}{\partial X^2}\Phi^{(2)}$$
$$-\frac{\partial N_{-i}^{(2)}}{\partial X}=0, \tag{2.35}$$

Finally, multiplying Eq. (2.34) by $\frac{1}{(V_p^2-\delta_{ei})}$ and then subtracting Eq. (2.35) from it, the following NLEEs is obtained as follows:

$$\frac{\partial\Phi^{(1)}}{\partial T}+B\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}=C\frac{\partial^2\Phi^{(1)}}{\partial X^2}, \tag{2.36}$$

where

$$B=\left[\frac{3}{V_p^4}-\frac{3N_{r1}V_p^2M_\pm^2}{(V_p^2-\delta_{ei})^3}+\frac{\delta_{ei}N_{r1}M_\pm^2}{(V_p^2-\delta_{ei})^3}-2N_{r2}\Omega_2\right]\div\left[\frac{2}{V_p^3}+\frac{2N_{r1}V_pM_\pm}{(V_p^2-\delta_{ei})^2}\right], \tag{2.37}$$

$$C=-\left[\frac{\mu_{+i}}{V_p^3}+\frac{\mu_{-i}N_{r1}V_pM_\pm}{(V_p^2-\delta_{ei})^2}\right]\div\left[\frac{2}{V_p^3}+\frac{2N_{r1}V_pM_\pm}{(V_p^2-\delta_{ei})^2}\right], \tag{2.38}$$

It is to be noted that Eq. (2.36) divulges only the shock wave structures in plasmas because it is a Burgers equation.

2.3.1 Solution of Burgers equation

To determine the shock wave solution of Burgers equation, one can convert Eq. (2.36) by using the travelling wave transform as $\Phi^{(1)}(X,T)=f(\xi)$ with $\xi=X-V_rT$ (V_r is the speed of the reference frame) with $f\to 0$, $f'\to 0$, $f''\to 0$ as $\xi\to\pm\infty$ to the following form:

$$-V_r\frac{df}{\partial\xi}+Bf\frac{df}{\partial\xi}=C\frac{d^2f}{d\xi^2}$$

$$\text{or,}\frac{Cdf}{\frac{1}{2}Bf^2-V_rf}=d\xi$$

$$\text{or,}\frac{2C}{B}\frac{df}{f\left(f-\frac{2V_r}{B}\right)}=d\xi$$

$$\text{or,}\frac{C}{V_r}\left[\frac{1}{f-\frac{2V_r}{B}}-\frac{1}{f}\right]df=d\zeta$$

$$\text{or,}\frac{C}{V_r}\int\left[\frac{1}{f-\frac{2V_r}{B}}-\frac{1}{f}\right]df=\int d\xi$$

$$\text{or,} \; ln\left(f - \frac{2V_r}{B}\right) - ln\, f = \frac{V_r}{C}\xi$$

$$\text{or,} \; f - \frac{2V_r}{B} = f\, exp\left(\frac{V_r}{C}\xi\right)$$

$$\text{or,} \; f\left[exp\left(\frac{-V_r}{C}\xi\right) - 1\right] = \frac{2V_r}{B} exp\left(\frac{-V_r}{C}\xi\right)$$

$$\text{or,} \; f = -\frac{2V_r}{B}\frac{exp\left(\frac{-V_r}{C}\xi\right)}{1 - exp\left(\frac{-V_r}{C}\xi\right)}$$

$$\text{or,} \; f = -\frac{2V_r}{B}\frac{exp\left(\frac{-V_r}{2C}\xi\right)}{exp\left(\frac{V_r}{2C}\xi\right) - exp\left(\frac{-V_r}{2C}\xi\right)}$$

$$\text{or,} \; f = \frac{V_r}{B}\left[1 - \frac{exp\left(\frac{AV_r}{2C}\xi\right) + exp\left(\frac{-AV_r}{2C}\xi\right)}{exp\left(\frac{AV_r}{2C}\xi\right) - exp\left(\frac{-AV_r}{2C}\xi\right)}\right]$$

$$\therefore f = \frac{V_r}{B}\left[1 - tan\, h\left(\frac{V_r}{2C}\xi\right)\right]$$

Therefore the solution of Eq. (2.36) is obtained as

$$\Phi^{(1)} = \Phi_A\left[1 - tan\, h\left(\frac{\xi}{\Phi_W}\right)\right], \tag{2.39}$$

where $\Phi_A = (V_r/B)$ and $\Phi_W = (2C/V_r)$ are the amplitude and width of SWEs.

2.4 Formation of modified Burgers type equation

Eq. (2.39) is clearly shown that Burgers equation is unable to describe the shock wave phenomena when $\Phi_A \to \infty$ as $B \to 0$. By considering $B = 0$, one can easily determine the critical values of any specific parameter. To overcome such difficulty, one may derive Burgers equation involving higher order nonlinearity. As a result, in order to study the shock wave phenomena around the critical values, one can consider the stretching coordinates by taking the higher-order correction of the reductive perturbative method [59, 84] as

$$X = \varepsilon^2(z - V_p t), \qquad T = \varepsilon^4 t, \tag{2.40}$$

From Eq. (2.40), the operators are defined as

$$\frac{\partial}{\partial t}=\varepsilon^4\frac{\partial}{\partial T}-\varepsilon^2 V_p\frac{\partial}{\partial X},\qquad \frac{\partial}{\partial z}=\varepsilon^2\frac{\partial}{\partial X}, \tag{2.41}$$

Eqs. (2.2)-(2.6) are then converted with the aid of Eq. (2.41) to the following:

$$\varepsilon^4\frac{\partial N_{+i}}{\partial T}-\varepsilon^2 V_p\frac{\partial N_{+i}}{\partial X}+\varepsilon^2\frac{\partial}{\partial X}(N_{+i}U_{+i})=0, \tag{2.42}$$

$$\varepsilon^4\frac{\partial N_{-i}}{\partial T}-\varepsilon^2 V_p\frac{\partial N_{-i}}{\partial X}+\varepsilon^2\frac{\partial}{\partial X}(N_{-i}U_{-i})=0, \tag{2.43}$$

$$\varepsilon^4\frac{\partial U_{+i}}{\partial T}-\varepsilon^2 V_p\frac{\partial U_{+i}}{\partial X}+\varepsilon^2 U_{+i}\left(\frac{\partial U_{+i}}{\partial X}\right)+\varepsilon^2\frac{\partial \Phi}{\partial X}+\varepsilon^4\mu_{+i}\frac{\partial^2 U_{+i}}{\partial X^2}=0, \tag{2.44}$$

$$\varepsilon^4\frac{\partial U_{-i}}{\partial T}-\varepsilon^2 V_p\frac{\partial U_{-i}}{\partial X}+\varepsilon^2 U_{-i}\left(\frac{\partial U_{-i}}{\partial X}\right)-\varepsilon^2 M_{\pm}\frac{\partial \Phi}{\partial X}+\varepsilon^2\frac{\delta_{ei}}{N_{-i}}\frac{\partial N_{-i}}{\partial X}+\varepsilon^4\mu_{-i}\frac{\partial^2 U_{-i}}{\partial X^2}=0, \tag{2.45}$$

and

$$\varepsilon^4\frac{\partial^2\Phi}{\partial X^2}=-N_{r2}\left\{[1+(q-1)\Phi]^{\frac{q+1}{2(q-1)}}\times[1+B_1\Phi+B_2\Phi^2]\right\}+N_{pi}-N_{ni}, \tag{2.46}$$

Inserting Eq. (2.15) into Eqs. (2.42)-(2.46), one can coverts Eqs. (2.42)-(2.46) by including the different orders of ε, that is $O(\varepsilon^r)$, $r=3,4,\cdots\cdots$. For $O(\varepsilon^3)$, one obtains the same equations as in Eqs. (2.16)-(2.20) and their solutions are given in Eq. (2.26). The linear phase velocity is obtained in the same form as in Eq. (2.27).

For $O(\varepsilon^4)$:

$$-V_p\frac{\partial N_{+i}^{(2)}}{\partial X}+\frac{\partial U_{+i}^{(2)}}{\partial X}+\frac{\partial}{\partial X}\left(N_{+i}^{(1)}U_{+i}^{(1)}\right)=0, \tag{2.47}$$

$$-V_p\frac{\partial N_{-i}^{(2)}}{\partial X}+N_{r1}\frac{\partial U_{-i}^{(2)}}{\partial X}+\frac{\partial}{\partial X}\left(N_{-i}^{(1)}U_{-i}^{(1)}\right)=0, \tag{2.48}$$

$$-V_p\frac{\partial U_{+i}^{(2)}}{\partial X}+U_{+i}^{(1)}\frac{\partial U_{+i}^{(1)}}{\partial X}+\frac{\partial \Phi^{(2)}}{\partial X}=0, \tag{2.49}$$

$$-V_p\frac{\partial U_{-i}^{(2)}}{\partial X}+U_{-i}^{(1)}\frac{\partial U_{-i}^{(1)}}{\partial X}-M_{\pm}\frac{\partial \Phi^{(2)}}{\partial X}+\frac{\delta_{ei}}{N_{r1}}\frac{\partial N_{-i}^{(2)}}{\partial X}-\frac{\delta_{ei}}{N_{r1}^2}N_{-i}^{(1)}\frac{\partial N_{-i}^{(1)}}{\partial X}=0, \tag{2.50}$$

and

$$-N_{r2}\Omega_1\Phi^{(2)}-N_{r2}\Omega_2\left[\Phi^{(1)}\right]^2+N_{+i}^{(2)}-N_{-i}^{(2)}=0. \tag{2.51}$$

For $O(\varepsilon^5)$:

$$\frac{\partial N_{+i}^{(1)}}{\partial T} - V_p \frac{\partial N_{+i}^{(3)}}{\partial X} + \frac{\partial U_{+i}^{(3)}}{\partial X} + \frac{\partial}{\partial X}\left(N_{+i}^{(1)} U_{+i}^{(2)}\right) + \frac{\partial}{\partial X}\left(N_{+i}^{(2)} U_{+i}^{(1)}\right) = 0, \quad (2.52)$$

$$\frac{\partial N_{-i}^{(1)}}{\partial T} - V_p \frac{\partial N_{-i}^{(3)}}{\partial X} + N_{r1} \frac{\partial U_{-i}^{(3)}}{\partial X} + \frac{\partial}{\partial X}\left(N_{-i}^{(1)} U_{-i}^{(2)}\right) + \frac{\partial}{\partial X}\left(N_{-i}^{(2)} U_{-i}^{(1)}\right) = 0, (2.53)$$

$$\frac{\partial U_{+i}^{(1)}}{\partial T} - V_p \frac{\partial U_{+i}^{(3)}}{\partial X} + U_{+i}^{(1)} \frac{\partial U_{+i}^{(2)}}{\partial X} + U_{+i}^{(2)} \frac{\partial U_{+i}^{(1)}}{\partial X} + \frac{\partial \Phi^{(3)}}{\partial X} + \mu_{+i} \frac{\partial^2 U_{+i}^{(1)}}{\partial X^2} = 0, (2.54)$$

$$\frac{\partial U_{-i}^{(1)}}{\partial T} - V_p \frac{\partial U_{-i}^{(3)}}{\partial X} + U_{-i}^{(1)} \frac{\partial U_{-i}^{(2)}}{\partial X} + U_{-i}^{(2)} \frac{\partial U_{-i}^{(1)}}{\partial X} - M_{\pm} \frac{\partial \Phi^{(3)}}{\partial X} + \frac{\delta_{ei}}{N_{r1}} \frac{\partial N_{-i}^{(3)}}{\partial X}$$

$$- \frac{\delta_{ei}}{N_{r1}^2} N_{-i}^{(1)} \frac{\partial N_{-i}^{(2)}}{\partial X} - \frac{\delta_{ei}}{N_{r1}^2} N_{-i}^{(2)} \frac{\partial N_{-i}^{(1)}}{\partial X} + \frac{\delta_{ei}}{N_{r1}^3} \left(N_{-i}^{(1)}\right)^2 \frac{\partial N_{-i}^{(1)}}{\partial X} + \mu_{-i} \frac{\partial^2 U_{-i}^{(1)}}{\partial X^2} = 0, (2.55)$$

and

$$-N_{r2} \Omega_1 \Phi^{(3)} - 2N_{r2} \Omega_2 \Phi^{(1)} \Phi^{(2)} - N_{r2} \Omega_3 \left[\Phi^{(1)}\right]^3 + N_{+i}^{(3)} - N_{-i}^{(3)} = 0, \quad (2.56)$$

where,

$$\Omega_3 = \frac{q+1}{2} B_2 + \frac{(q+1)(3-q)(5-3q)}{48} + \frac{(q+1)(3-q)}{8} B_1.$$

Now, one can easily derive the following equations from Eqs. (2.47)-(2.51) by using the values of Eq. (2.26):

$$-V_p \frac{\partial N_{+i}^{(2)}}{\partial X} + \frac{\partial U_{+i}^{(2)}}{\partial X} + \frac{2}{V_p^3} \Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial X} = 0, \quad (2.57)$$

$$-V_p \frac{\partial N_{-i}^{(2)}}{\partial X} + N_{r1} \frac{\partial U_{-i}^{(2)}}{\partial X} + \frac{2N_{r1} V_p M_{\pm}^2}{(V_p^2 - \delta_{ei})^2} \Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial X} = 0, \quad (2.58)$$

$$-V_p \frac{\partial U_{+i}^{(2)}}{\partial X} + \frac{\partial \Phi^{(2)}}{\partial X} + \frac{1}{V_p^2} \Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial X} = 0, \quad (2.59)$$

$$-V_p \frac{\partial U_{-i}^{(2)}}{\partial X} + \frac{\delta_{ei}}{N_{r1}} \frac{\partial N_{-i}^{(2)}}{\partial X} - M_{\pm} \frac{\partial \Phi^{(2)}}{\partial X} + \frac{V_p^2 M_{\pm}^2}{(V_p^2 - \delta_{ei})^2} \Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial X}$$

$$- \frac{\delta_{ei} M_{\pm}^2}{(V_p^2 - \delta_{ei})^2} \Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial X} = 0, \quad (2.60)$$

and

$$-N_{r2} \Omega_1 \Phi^{(2)} - N_{r2} \Omega_2 \left[\Phi^{(1)}\right]^2 + N_{+i}^{(2)} - N_{-i}^{(2)} = 0, \quad (2.61)$$

The solutions of the Eqs. (2.57)-(2.60) are obtained as

$$N_{+i}^{(2)} = \frac{1}{V_p^2}\left[\frac{3}{2V_p^2}\left\{\Phi^{(1)}\right\}^2 + \Phi^{(2)}\right], \tag{2.62}$$

$$U_{+i}^{(2)} = \frac{1}{V_p}\left[\frac{1}{2V_p^2}\left\{\Phi^{(1)}\right\}^2 + \Phi^{(2)}\right], \tag{2.63}$$

$$N_{-i}^{(2)} = \frac{N_{r1}M_\pm}{V_p^2 - \delta_{ei}}\left[\frac{M_\pm\left(3V_p^2 - \delta_{ei}\right)}{2(V_p^2 - \delta_{ei})^2}\left\{\Phi^{(1)}\right\}^2 - \Phi^{(2)}\right], \tag{2.64}$$

$$U_{-i}^{(2)} = \frac{V_p M_\pm}{V_p^2 - \delta_{ei}}\left[\frac{M_\pm(V_p^2 + \delta_{ei})}{2(V_p^2 - \delta_{ei})^2}\left\{\Phi^{(1)}\right\}^2 - \Phi^{(2)}\right], \tag{2.65}$$

and inserting the values of Eq. (2.62) and Eq. (2.64) in Eq. (2.61), one can obtain

$$\left[\frac{3}{V_p^4} - \frac{3N_{r1}M_\pm^2 V_p^2}{(V_p^2 - \delta_{ei})^3} + \frac{N_{r1}M_\pm^2\delta_{ei}}{(V_p^2 - \delta_{ei})^3} - 2N_{r2}\Omega_2\right]\left\{\Phi^{(1)}\right\}^2 = 0. \tag{2.66}$$

Now, Eq. (2.66) gives

$$-C_f\left\{\Phi^{(1)}\right\}^2 = 0, \tag{2.67}$$

where

$$C_f = \frac{3}{V_p^4} - \frac{3N_{r1}M_\pm^2 V_p^2}{(V_p^2 - \delta_{ei})^3} + \frac{N_{r1}M_\pm^2\delta_{ei}}{(V_p^2 - \delta_{ei})^3} - 2N_{r2}\Omega_2 = 0. \tag{2.68}$$

Finally, using the values from Eq. (2.26) and from Eqs. (2.62)-(2.65), Eqs. (2.52)-(2.56) provides the following equations:

$$\frac{1}{V_p^2}\frac{\partial\Phi^{(1)}}{\partial T} - V_p\frac{\partial N_{+i}^{(3)}}{\partial X} + \frac{\partial U_{+i}^{(3)}}{\partial X} + \frac{6}{V_p^5}\left\{\Phi^{(1)}\right\}^2\frac{\partial\Phi^{(1)}}{\partial X} + \frac{2}{V_p^3}\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right] = 0, \tag{2.69}$$

$$-\frac{N_{r1}M_\pm}{V_p^2 - \delta_{ei}}\frac{\partial\Phi^{(1)}}{\partial T} - V_p\frac{\partial N_{-i}^{(3)}}{\partial X} + N_{r1}\frac{\partial U_{-i}^{(3)}}{\partial X} - \frac{6N_{r1}V_p^3 M_\pm^3}{(V_p^2 - \delta_{ei})^4}\left\{\Phi^{(1)}\right\}^2\frac{\partial\Phi^{(1)}}{\partial X}$$
$$+ \frac{2N_{r1}V_p M_\pm^2}{(V_p^2 - \delta_{ei})^2}\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right] = 0, \tag{2.70}$$

$$\frac{1}{V_p}\frac{\partial\Phi^{(1)}}{\partial T} - V_p\frac{\partial U_{+i}^{(3)}}{\partial X} + \frac{\partial\Phi^{(3)}}{\partial X} + \frac{3}{2V_p^4}\left\{\Phi^{(1)}\right\}^2\frac{\partial\Phi^{(1)}}{\partial X} + \frac{1}{V_p^2}\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right]$$
$$+ \frac{\mu_{+i}}{V_p}\frac{\partial^2\Phi^{(1)}}{\partial X^2} = 0, \tag{2.71}$$

$$-\frac{V_pM_\pm}{V_p^2-\delta_{ei}}\frac{\partial\Phi^{(1)}}{\partial T}-V_p\frac{\partial U_{-i}^{(3)}}{\partial X}+\frac{\delta_{ei}}{N_{r1}}\frac{\partial N_{-i}^{(3)}}{\partial X}-M_\pm\frac{\partial\Phi^{(3)}}{\partial X}+\frac{M_\pm^2}{V_p^2-\delta_{ei}}\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right]$$
$$-\left[-\frac{3V_p^2M_\pm^3\left(V_p^2+\delta_{ei}\right)}{2(V_p^2-\delta_{ei})^4}+\frac{3\delta_{ei}M_\pm^3\left(3V_p^2-\delta_{ei}\right)}{2(V_p^2-\delta_{ei})^4}-\frac{\delta_{ei}M_\pm^3}{(V_p^2-\delta_{ei})^3}\right]\times$$
$$\left\{\Phi^{(1)}\right\}^2\frac{\partial\Phi^{(1)}}{\partial X}-\frac{\mu_{-i}V_pM_\pm}{V_p^2-\delta_{ei}}\frac{\partial^2\Phi^{(1)}}{\partial X^2}=0, \tag{2.72}$$

and

$$-N_{r2}\Omega_1\Phi^{(3)}-2N_{r2}\Omega_2\Phi^{(1)}\Phi^{(2)}-N_{r2}\Omega_3\left[\Phi^{(1)}\right]^3+N_{+i}^{(3)}-N_{-i}^{(3)}=0, \tag{2.73}$$

Multiplying Eq. (2.69) by V_p and then adding Eq. (2.71), one can obtain as

$$\frac{2}{V_p}\frac{\partial\Phi^{(1)}}{\partial T}-V_p^2\frac{\partial N_{+i}^{(3)}}{\partial X}+\frac{15}{2V_p^4}\left\{\Phi^{(1)}\right\}^2\frac{\partial\Phi^{(1)}}{\partial X}+\frac{3}{V_p^2}\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right]+\frac{\partial\Phi^{(3)}}{\partial X}+\frac{\mu_{+i}}{V_p}\frac{\partial^2\Phi^{(1)}}{\partial X^2}$$
$$=0, \tag{2.74}$$

Again, multiplying Eq. (2.70) by V_p and multiplying Eq. (2.72) by N_{r1}, then adding, one can obtain as

$$-\frac{2N_{r1}M_\pm V_p}{V_p^2-\delta_{ei}}\frac{\partial\Phi^{(1)}}{\partial T}-\left(V_p^2-\delta_{ei}\right)\frac{\partial N_{-i}^{(3)}}{\partial X}+\left[\frac{2N_{r1}V_p^2M_\pm^2}{(V_p^2-\delta_{ei})^2}+\frac{N_{r1}M_\pm^2}{V_p^2-\delta_{ei}}\right]\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right]$$
$$-\left[\frac{6N_{r1}V_p^4M_\pm^3}{(V_p^2-\delta_{ei})^4}+\frac{3N_{r1}V_p^2M_\pm^3\left(V_p^2+\delta_{ei}\right)}{2(V_p^2-\delta_{ei})^4}-\frac{3N_{r1}\delta_{ei}M_\pm^3\left(3V_p^2-\delta_{ei}\right)}{2(V_p^2-\delta_{ei})^4}\right.$$
$$\left.+\frac{\delta_{ei}N_{r1}M_\pm^3}{(V_p^2-\delta_{ei})^3}\right]\left\{\Phi^{(1)}\right\}^2\frac{\partial\Phi^{(1)}}{\partial X}-N_{r1}M_\pm\frac{\partial\Phi^{(3)}}{\partial X}-\frac{N_{r1}\mu_{-i}V_pM_\pm}{V_p^2-\delta_{ei}}\frac{\partial^2\Phi^{(1)}}{\partial X^2}$$
$$=0, \tag{2.75}$$

Differentiating Eq. (2.73) with respect to X and multiplying Eq. (2.74) by $\frac{1}{V_p^2}$, then adding, one can obtain

$$\frac{2}{V_p^3}\frac{\partial\Phi^{(1)}}{\partial T}+\left[\frac{15}{2V_p^6}-3N_{r2}\Omega_3\right]\left\{\Phi^{(1)}\right\}^2\frac{\partial\Phi^{(1)}}{\partial X}+\left[\frac{3}{V_p^5}-3N_{r2}\Omega_2\right]\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right]-\frac{\partial N_{-i}^{(3)}}{\partial X}$$
$$+\left(\frac{1}{V_p^2}-N_{r2}\Omega_1\right)\frac{\partial\Phi^{(3)}}{\partial X}+\frac{\mu_{+i}}{V_p^3}\frac{\partial^2\Phi^{(1)}}{\partial X^2}=0, \tag{2.76}$$

Finally, multiplying Eq. (2.75) by $\frac{1}{(V_p^2-\delta_{ei})}$ and then subtracting Eq. (2.76) from it, the following NLEEs is obtained as follows:

$$\frac{\partial\Phi^{(1)}}{\partial T}+B'\left\{\Phi^{(1)}\right\}^2\frac{\partial\Phi^{(1)}}{\partial X}=C'\frac{\partial^2\Phi^{(1)}}{\partial X^2}, \tag{2.77}$$

which is the modified Burgers (mB)-type equation. The coefficients of Eq. (2.77) are determined as

$$B' = \left[\frac{15\left(V_p^2-\delta_{ei}\right)}{2V_p^4} - 3N_{r2}\Omega_3 V_p^2\left(V_p^2-\delta_{ei}\right) + \frac{3N_{r1}V_p^2 M_\pm^3\left(5V_p^4-2V_p^2\delta_{ei}+\delta_{ei}^2\right)}{2(V_p^2-\delta_{ei})^4} + \frac{\delta_{ei}N_{r1}M_\pm^3 V_p^2}{(V_p^2-\delta_{ei})^3}\right] \div \left[\frac{2\left(V_p^2-\delta_{ei}\right)}{V_p} + \frac{2N_{r1}V_p^3 M_\pm}{\left(V_p^2-\delta_{ei}\right)}\right], \tag{2.78}$$

$$C' = -\left[\frac{\mu_{+i}\left(V_p^2-\delta_{ei}\right)}{V_p} + \frac{N_{r1}\mu_{-i}V_p^3 M_\pm}{\left(V_p^2-\delta_{ei}\right)}\right] \div \left[\frac{2\left(V_p^2-\delta_{ei}\right)}{V_p} + \frac{2N_{r1}V_p^3 M_\pm}{\left(V_p^2-\delta_{ei}\right)}\right], \tag{2.79}$$

2.4.1 Solution of modified Burgers type equation

In the previous literature [94-96], the solution of mB-type equation (see Appendix A) is incorrectly defined, which is not useful for further verification in laboratory plasmas. To determine the correct stationary shock wave solution of Eq. (2.77), one can convert Eq. (2.77) by considering $\Phi^{(1)}(X,T) = \psi(\xi)$ with $\xi = X - V_r T$ (V_r is the speed of the reference frame) with $\psi \to 0$, $\psi' \to 0$, $\psi'' \to 0$ as $\xi \to \pm\infty$ to the following form:

$$-V_r \frac{d\psi}{\partial\xi} + B'\psi^2 \frac{d\psi}{\partial\xi} = C' \frac{d^2\psi}{d\xi^2}$$

$$\text{or,} -V_r\psi + \frac{1}{3}B'\psi^3 = C'\frac{d\psi}{d\xi}$$

$$\text{or,} \frac{C'd\psi}{\frac{1}{3}B'\psi^3 - V_r\psi} = d\xi$$

$$\text{or,} \frac{3C'}{B'}\frac{d\psi}{\psi\left(\psi^2 - \frac{3V_r}{B}\right)} = d\xi$$

$$\text{or,} \frac{C'}{2V_r}\left[\frac{2\psi}{\psi^2 - \frac{3V_r}{B'}} - \frac{2}{\psi}\right]d\psi = d\xi$$

$$\text{or,} \, ln\left(\psi^2 - \frac{3V_r}{B'}\right) - ln\,\psi^2 = \frac{2V_r}{C'}\xi$$

$$\text{or}, \psi^2 - \frac{3V_r}{B'} = \psi^2 \, exp\left(\frac{2V_r}{C'}\xi\right)$$

$$\text{or}, \psi^2 \left[exp\left(\frac{-2V_r}{C'}\xi\right) - 1\right] = \frac{3V_r}{B'} exp\left(\frac{-2V_r}{C'}\xi\right)$$

$$\text{or}, \psi^2 = -\frac{3V_r}{B'} \frac{exp\left(\frac{-2V_r}{C'}\xi\right)}{1 - exp\left(\frac{-2V_r}{C'}\xi\right)}$$

$$\text{or}, \psi^2 = -\frac{3V_r}{B'} \frac{exp\left(\frac{-V_r}{C'}\xi\right)}{exp\left(\frac{V_r}{C'}\xi\right) - exp\left(\frac{-V_r}{C'}\xi\right)}$$

$$\text{or}, \psi^2 = \frac{3V_r}{2B'}\left[1 - \frac{exp\left(\frac{V_r}{C'}\xi\right) + exp\left(\frac{-V_r}{C'}\xi\right)}{exp\left(\frac{V_r}{C'}\xi\right) - exp\left(\frac{-V_r}{C'}\xi\right)}\right]$$

$$\therefore \psi = \sqrt{\frac{3V_r}{2B'}\left[1 - tan\,h\left(\frac{V_r}{C'}\xi\right)\right]}, \tag{2.80}$$

From Eq. (2.80), the stationary shock wave solution of Eq. (2.77) is obtained as

$$\Phi^{(1)} = \sqrt{\Phi_A\left\{1 - tan\,h\left(\frac{\xi}{\Phi_W}\right)\right\}}, \tag{2.81}$$

Where $\Phi_A = (3V_r/2B')$ and $\Phi_W = (C'/V_r)$ are respectively the amplitude and width of IASWs around the critical values. The verification of the obtained solution is given in Appendix A.

2.5 Formation of mixed modified Burgers type equation

One can easily find that Eq. (2.80) is not useful to study the shock wave phenomena not only at CVs but also around CVs. In such situation, one needs to derive another evolution equation. To do so, one can take C_f^0 for q (say) around its q_c as

$$C_f^0 = h\left(\frac{\partial C_f}{\partial q}\right)_{q=q_c} |q - q_c| = SG\varepsilon, \tag{2.82}$$

where $|q-q_c|\equiv\varepsilon$ (because $|q-q_c|$ is small quantity). $S=1(-1)$ for $q>q_c(q<q_c)$ and $G=\left(\frac{\partial C_f}{\partial q}\right)_{q=q_c}$. As a result, one can re-evaluate from Eq. (2.56) by adding $\rho^{(2)}=\varepsilon\frac{1}{2}SG\Phi^2$ and yields

$$-N_{r2}\Omega_1\frac{\partial\Phi^{(3)}}{\partial X}+SG\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}-3N_{r2}\Omega_3\{\Phi^{(1)}\}^2\frac{\partial\Phi^{(1)}}{\partial X}-2N_{r2}\Omega_2\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right]$$
$$+\frac{\partial N_{+i}^{(3)}}{\partial X}-\frac{\partial N_{-i}^{(3)}}{\partial X}=0. \tag{2.83}$$

Mmultiplying Eq. (2.74) by $\frac{1}{V_p^2}$ and adding with Eq. (2.83), one can obtained as

$$\frac{2}{V_p^3}\frac{\partial\Phi^{(1)}}{\partial T}+\left[\frac{15}{2V_p^6}-3N_{r2}\Omega_3\right]\{\Phi^{(1)}\}^2\frac{\partial\Phi^{(1)}}{\partial X}+\left[\frac{3}{V_p^4}-2N_{r2}\Omega_2\right]\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right]$$
$$+SG\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}-\frac{\partial N_{-i}^{(3)}}{\partial X}+\left(\frac{1}{V_p^2}-N_{r2}\Omega_1\right)\frac{\partial\Phi^{(3)}}{\partial X}+\frac{\mu_{+i}}{V_p^3}\frac{\partial^2\Phi^{(1)}}{\partial X^2}=0, \tag{2.84}$$

Now, multiplying Eq. (2.75) by $\frac{1}{(V_p^2-\delta_{ei})}$ and then subtracting Eq. (2.84) from it, one obtained as follows:

$$\frac{\partial\Phi^{(1)}}{\partial T}+SD\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}+B'\{\Phi^{(1)}\}^2\frac{\partial\Phi^{(1)}}{\partial X}=C'\frac{\partial^2\Phi^{(1)}}{\partial X^2}, \tag{2.85}$$

$$\text{where } D=\left[\left(V_p^2-\delta_{ei}\right)/2\left\{\frac{(V_p^2-\delta_{ei})}{V_p}+\frac{N_{r1}V_p^3M_\pm}{(V_p^2-\delta_{ei})}\right\}\right]\left(\frac{\partial C_f}{\partial q}\right)_{q=q_c}, \tag{2.86}$$

Equation (2.85) is the mmB-type equation because the additional nonlinear term is occurred with the mBE. One can easily convert mmB-type equation not only to mB-type equation but also to Burger equation. It is noted that Eq. (2.85) may be supported the IASWs around CVs but also at CVs, which is analyzed later by deriving the analytical solution of it.

2.5.1 Solution of mixed modified Burgers type equation

To determine the stationary shock wave solution of Eq. (2.85), one can convert Eq. (2.85) by considering $\Phi^{(1)}=\Phi-\frac{SD}{2B'}$, one can obtain

$$\frac{\partial\Phi}{\partial T}-\left(\frac{SD}{2\sqrt{B'}}\right)^2\frac{\partial\Phi}{\partial X}+B'\{\Phi\}^2\frac{\partial\Phi}{\partial X}=C'\frac{\partial^2\Phi}{\partial X^2}, \tag{2.87}$$

Again assume that $\Phi = f(\xi)$ with $\xi = X - V_r T$ (V_r is the speed of the reference frame) with $f \to 0, f' \to 0, f'' \to 0$ as $\xi \to \pm\infty$ then Eq. (2.87) convert to the following form:

$$-V_r \frac{df}{\partial \xi} - \left(\frac{SD}{2\sqrt{B'}}\right)^2 \frac{df}{\partial \xi} + B' f^2 \frac{df}{\partial \xi} = C' \frac{d^2 f}{d\xi^2}$$

$$\text{or,} -V_r f - \left(\frac{SD}{2\sqrt{B'}}\right)^2 f + \frac{B'}{3} f^3 = C' \frac{df}{d\xi}$$

$$\text{or,} \frac{3C'}{B'} \frac{df}{f\left\{f^2 - \frac{3}{B'}\left(V_r + \frac{S^2D^2}{4B'}\right)\right\}} = d\xi$$

$$\text{or,} \frac{C'}{2\left(V_r + \frac{S^2D^2}{4B'}\right)} \left[\frac{2f}{f^2 - \frac{3}{B'}\left(V_r + \frac{S^2D^2}{4B'}\right)} - \frac{2}{f}\right] df = d\xi$$

$$\text{or,} \ln\left[f^2 - \frac{3}{B'}\left(V_r + \frac{S^2D^2}{4B'}\right)\right] - \ln f^2 = \frac{2\left(V_r + \frac{S^2D^2}{4B'}\right)}{C'} \xi$$

$$\text{or,} f^2 - \frac{3}{B'}\left(V_r + \frac{S^2D^2}{4B'}\right) = f^2 \exp\left[\frac{2\left(V_r + \frac{S^2D^2}{4B'}\right)}{C'} \xi\right]$$

$$\text{or,} f^2 - \frac{3}{B'}\left(V_r + \frac{S^2D^2}{4B'}\right) = f^2 \exp\left[\frac{2\left(V_r + \frac{S^2D^2}{4B'}\right)}{C'} \xi\right]$$

$$\text{or,} f^2\left(\exp\left[\frac{-2\left(V_r + \frac{S^2D^2}{4B'}\right)}{C'} \xi\right] - 1\right) = \frac{3}{B'}\left(V_r + \frac{S^2D^2}{4B'}\right) \exp\left[\frac{-2\left(V_r + \frac{S^2D^2}{4B'}\right)}{C'} \xi\right]$$

$$\text{or,} f^2 = -\frac{3}{B'}\left(V_r + \frac{S^2D^2}{4B'}\right) \frac{\exp\left[\frac{-\left(V_r + \frac{S^2D^2}{4B'}\right)}{C'} \xi\right]}{\exp\left[\frac{\left(V_r + \frac{S^2D^2}{4B'}\right)}{C'} \xi\right] - \exp\left[\frac{-\left(V_r + \frac{S^2D^2}{4B'}\right)}{C'} \xi\right]}$$

$$\text{or}, f^2 = \frac{3}{2B'}\left(V_r + \frac{S^2D^2}{4B'}\right)\left(1 - \frac{exp\left[\frac{\left(V_r+\frac{S^2D^2}{4B'}\right)}{C'}\xi\right] + exp\left[\frac{-\left(V_r+\frac{S^2D^2}{4B'}\right)}{C'}\xi\right]}{exp\left[\frac{\left(V_r+\frac{S^2D^2}{4B'}\right)}{C'}\xi\right] - exp\left[\frac{-\left(V_r+\frac{S^2D^2}{4B'}\right)}{C'}\xi\right]}\right)$$

$$\therefore f = \sqrt{\frac{3}{2B'}\left(V_r + \frac{S^2D^2}{4B'}\right)\left\{1 - tan\,h\frac{\left(V_r + \frac{S^2D^2}{4B'}\right)}{C'}\xi\right\}}. \tag{2.88}$$

The stationary shock wave solution of mmB-type equation defined as

$$\Phi^{(1)} = \sqrt{\Phi_{mA}\left\{1 - tan\,h\left(\frac{\xi}{\Phi_{mW}}\right)\right\}} - \frac{SD}{2B'}, \tag{2.89}$$

where

$$\Phi_{mA} = \frac{3}{2B'}\left(V_r + \frac{S^2D^2}{4B'}\right) \tag{2.90}$$

and

$$\Phi_{mW} = \left(\frac{C}{V_r} + \frac{S^2D^2}{4B'}\right), \tag{2.91}$$

where, Φ_{mA} and Φ_{mW} are the amplitude and width of IASWs at the critical values, respectively.

2.6 Results and discussions

To investigate the influences of N_{r1} (density ratio of NIs to PIs), N_{r2} (density ratio of electrons to PIs), μ_{+i} (viscosity coefficient of PIs), μ_{-i} (viscosity coefficient of NIs), α (population of nonthermal electrons), and q (strength of non-extensive electrons) on the basic features of shock wave phenomena, an unmagnetized collosionless three-component plasma having positive as well as negative ions and (α, q)-distributed electrons is considered. The Burgers equation with the exact solution as mentioned in Equation (2.39) is obtained to reveal such physical phenomena. In the presented analysis, the physical parameters are taken as $M_{\pm} = 3.75$, $T_{-i} = 0.05$, $T_e = 0.2$, $0 < N_{r1} < 1$, $0 < N_{r2} < 1$, $\mu_{+i} = 0.1 \sim 0.5$ and $\mu_{-i} = 0.001 \sim 0.4$ based on the work [43, 77] and $-1 < q < 1$ (for superthermality), $q > 1$ (subthermality), $q = 1$ (isothermality), and $0 < \alpha < 1$ based on the work [92].

2.6.1 Existence of critical values of strength of nonextensivity

One can determine the critical values (CVs) by setting $B = 0$. But, it is very difficult to formulate the mathematical expression for CV. Figure 2.1 shows the variation of B with regard to q and α by considering the other parameters constant. It is obviously found from Figure 2.1 that the CVs occur for the isothermal and subthermal electrons. For instance, the CV q_c for q is determined as $q_c = 3.098638852$ by considering $\alpha = 0$, $M_{\pm} = 3.75$, $T_{-i} = 0.05eV$, $T_e = 0.19eV$, $N_{r1} = 0.5$ and $N_{r2} = 0.01$.

2.6.2 Effects of the plasma parameters on phase velocity

Figure 2.2(a) shows the variation of phase velocity with regards to N_{r2} and α. It is observed from Figure 2.2(a) that the linear phase velocity (V_p) loses energy with increases the density ratio of electrons to positive ions. Whereas, V_p loses energy with increasing q for the case of subthermality. Moreover, V_p loses more energy with increases the population of nonthermal nonextensive electrons. Therefore when driving force increases then linear phase velocity increases and when restoring force increases then linear phase velocity decreases. Furtermore, Figure 2.2(b) demonstrate the variation of phase velocity for different values of N_{r1} and q. It is observed from Figure 2.2(b) that the linear phase velocity (V_p) gains energy with increases the density ratio of negative to positive ions. Whereas, V_p gains energy with increasing q for the case of superthermality. Moreover, V_p gains more energy with increases the population of nonthermal nonextensive electrons.

2.6.3 Effects of the plasma parameters on SWEs

Effects of the plasma parameter on SWEs described by Burgers equation, mB-, and mmB-type equation are discussed in this section. Effects of different plasma parameter described by Burgers equation (2.36) demonstrate in Figure 2.3 to Figure 2.7. Again, effects of the plasma parameter on SWEs described by mB-type equation (2.77) are displayed in Figure 2.8 to Figure 2.10. Furthermore, effects of plasma parameter on SWEs described by mmB-type equation (2.89) are shown in Figure 2.11 and Figure 2.12.

Variation of SWEs due to the changes of time: Figure 2.3 demonstrates the IA SWEs for different values of time T by choosing the other parameters constant. It is found that the amplitude and thickness of IASWs remains unchanged, but the position of shock wave changes with increases in time, as it is expected.

Effects of μ_{+i} and μ_{-i} on SWEs: Figure 2.4(a) and 2.4(b) demonstrates the influence of μ_{+i} and μ_{-i} on the nonlinear IA shock wave excitations by choosing the other parameters constant, respectively. It is evidently revelled from these figures that both μ_{+i} and μ_{-i} play a vital role by generating electrostatic shock waves in the negative ions plasmas. Because, the amplitude and width of shock wave increases with increases in μ_{+i} and μ_{-i}. Figure 2.4 shows that the amplitude and width of shock wave increase with increases in $\mu_{+i}$and μ_{-i}. If we ignore both of μ_{+i} and μ_{-i}, the system does not saturated, and as a result, the amplitude does not become sufficiently large. It is provided that μ_{+i} and μ_{-i} is only responsible for forming the shock wave excitations in the considered plasmas

Role of nonextensivity of ion on the SWEs: Figure 2.5(a) and 2.5(b) exhibits the influence of q on the nonlinear IASWEs by choosing other parameters constant with $T = 5$. It is clearly shown form these figures that the amplitude and thickness of SWEs decrease with increasing in q. Figures 2.4 are also obviously provided that the compressive and rarefactive electrostatic of SWEs are supported in the considered plasmas for the isothermal and superthermal energetic electrons only. From the above observations, it can be predicted that shock wave potential gains much more energies with the presence of superthermal rather than isothermal and subthermal electrons.

Role of nonthermality of ion on the SWEs: Figure 2.6 shows the influence of α on the nonlinear IA SWEs by choosing other parameters constant with time $T = 5$. It is clearly shown form Figure 2.6 that the amplitude and width of SWEs decrease with increasing in α. It is also found that SWEs losses energies with the increase of nonthermal population of electrons. From the physical point of view, one can minimize the energies of SWEs by increasing nonthermal populations of electrons in such environments.

Effects of N_{r1} and N_{r2} of the SWEs: Figure 2.7(a) and 2.7(b) displays the influence of N_{r1} and N_{r2} on the nonlinear electrostatic IA shock wave profile by assuming other parameters are constant. It is found from Figure 2.7 that both density ratios remarkably play distinct roles in the formation of SWEs in the considered plasmas. This is because the monotonically shock wave also occurred, in which the amplitude and width of shocks slightly decreases monotonically with increases in N_{r2}. However, the amplitude and thickness of shocks increases with increases in N_{r1}. From the physical point of view, the IA shock wave phenomena gains energies very slowly with the increase of PIs density. This actually happens because the driving force increases with the increase of PIs density. On the other hand, the restoring force provided by the pressure of electrons are only increased with the increase of isothermal electrons density. As a results, the electrostatic SWEs gains much more energies with the increase of isothermal electrons density.

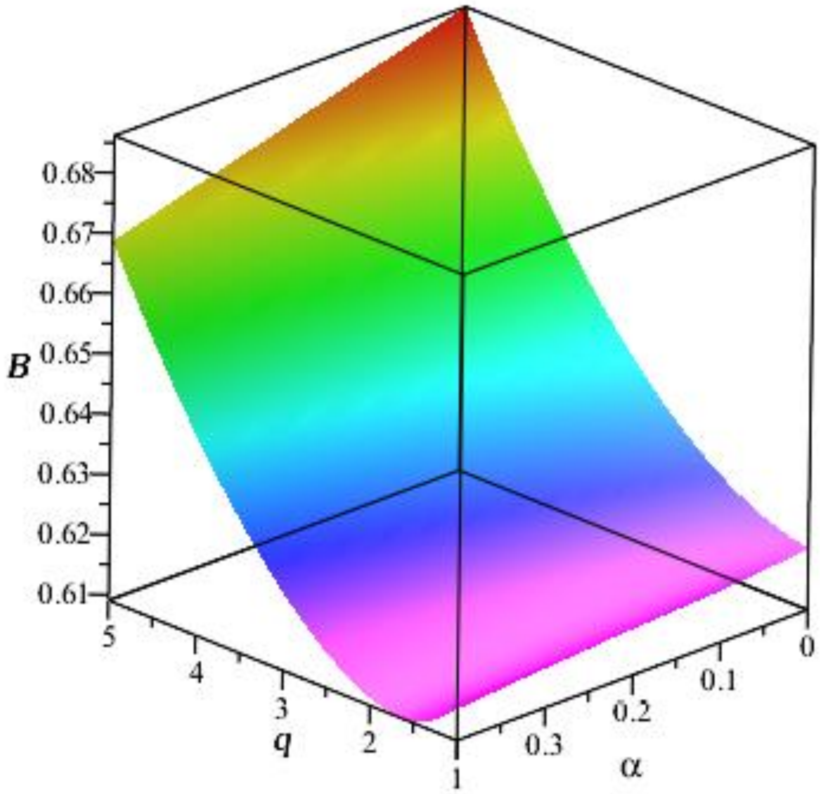
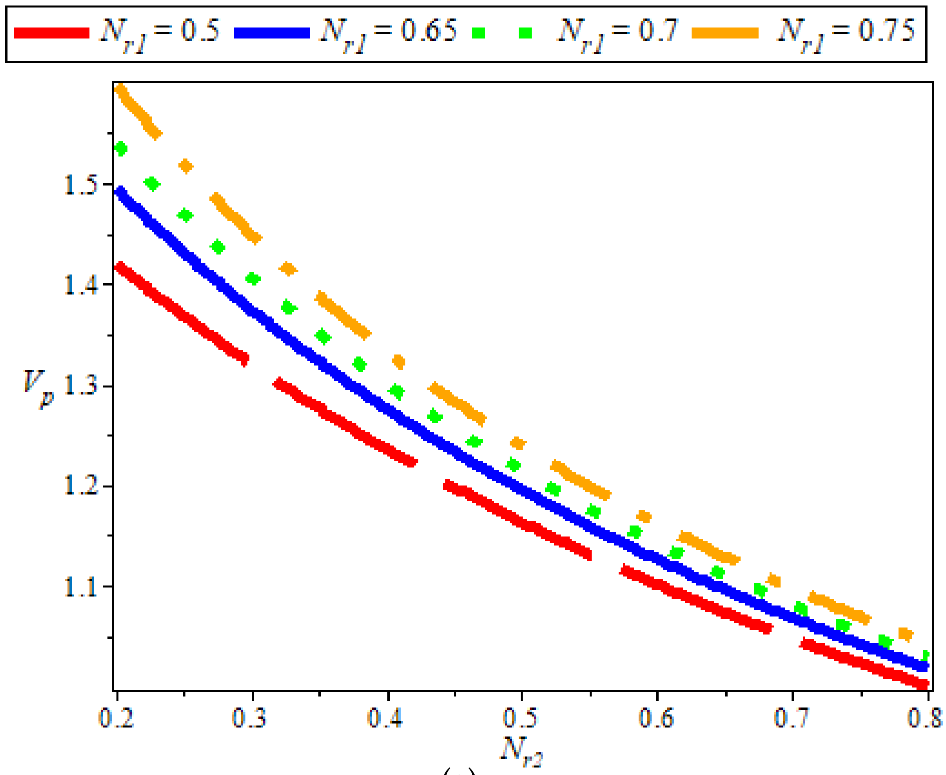
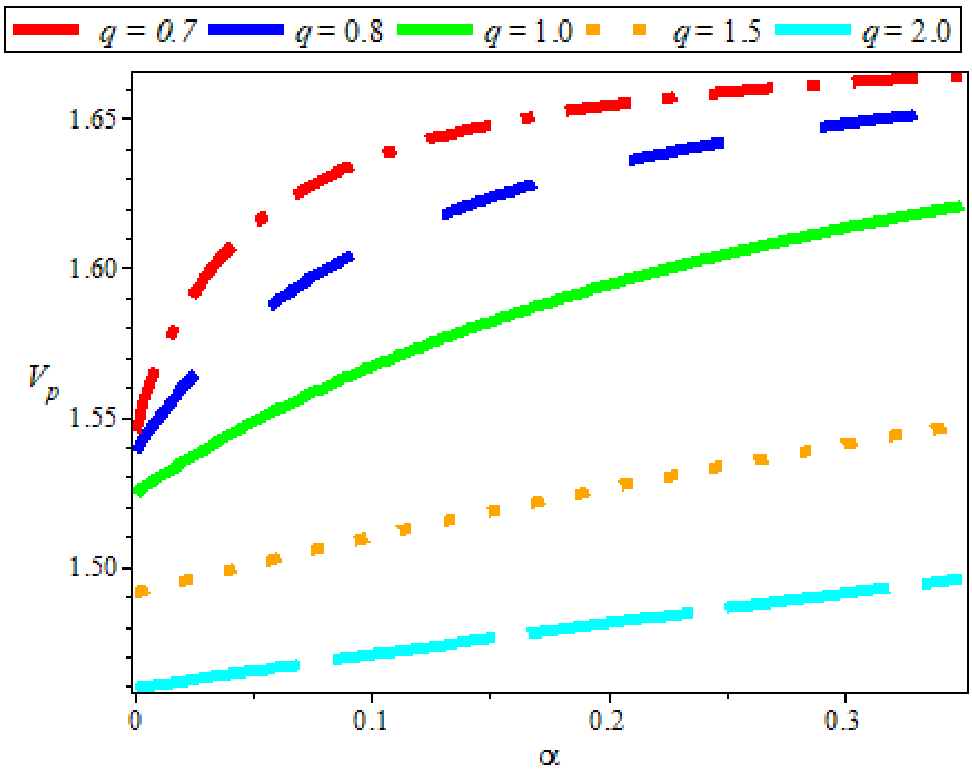


Figure 2.1: Variation of B with regard to q and α. The other parameters are considered as $M_{\pm} = 3.75$, $T_{-i} = 0.05eV$, $T_e = 0.2eV$, $N_{r1} = 0.5$ and $N_{r2} = 0.01$.



(a)



(b)

Figure 2.2: Variation of V_p with regards to (a) N_{r2} for different values of N_{r1} ($M_{\pm} = 3.75$, $\alpha = 0.35$, $T_{-i} = 0.05$, $T_e = 0.2$ and $q = 1.6$), and (b) α for different values of q ($M_{\pm} = 3.75$, $T_{-i} = 0.05$, $T_e = 0.2$, $N_{r1} = 0.5$ and $N_{r2} = 0.1$), respectively.

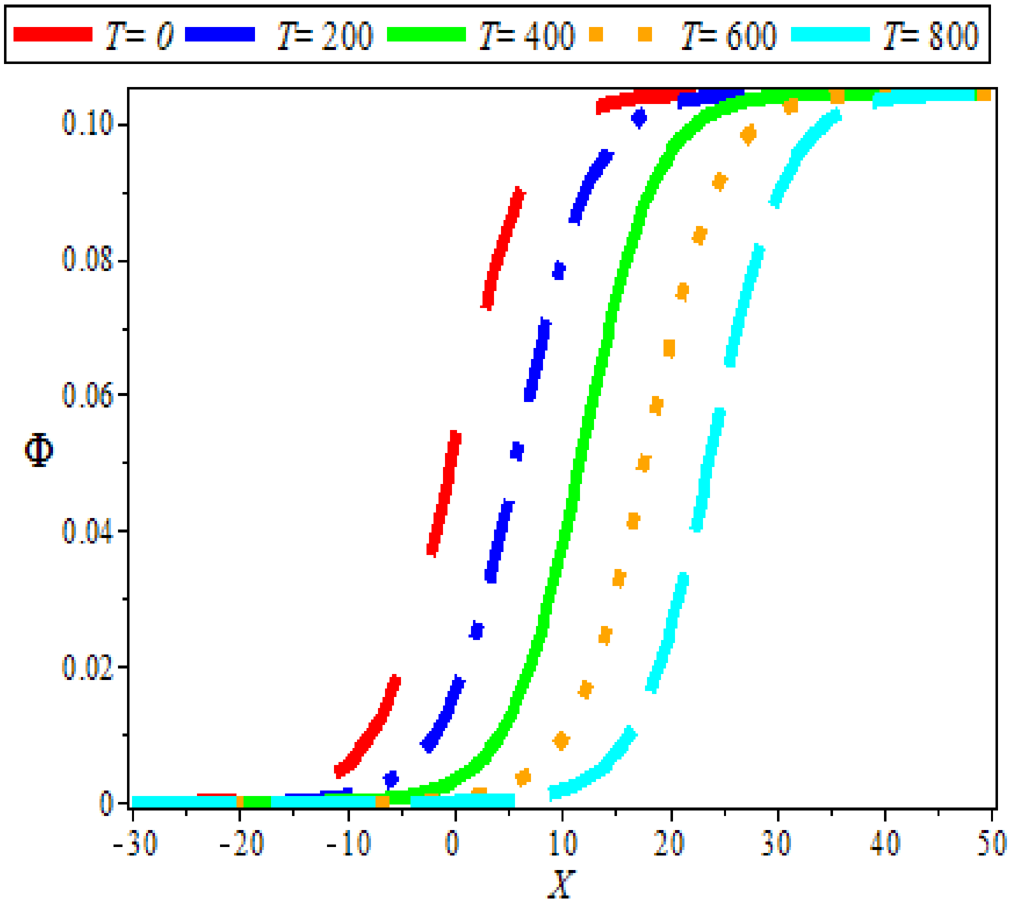
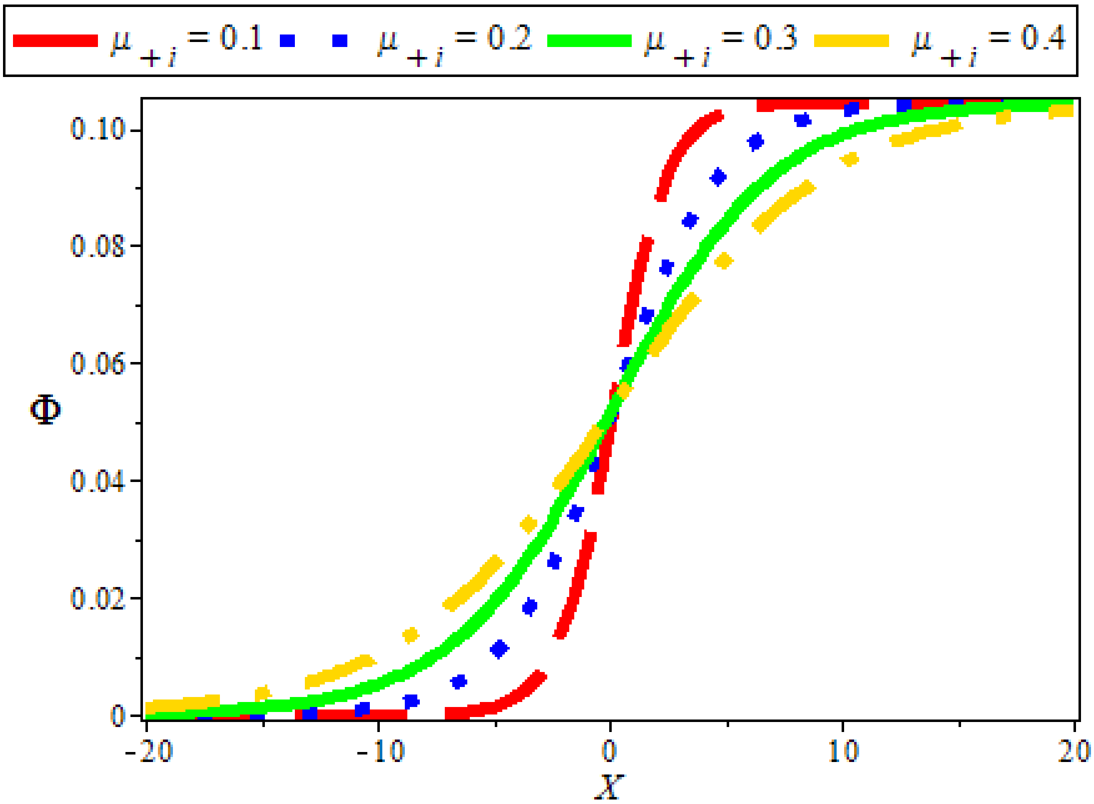
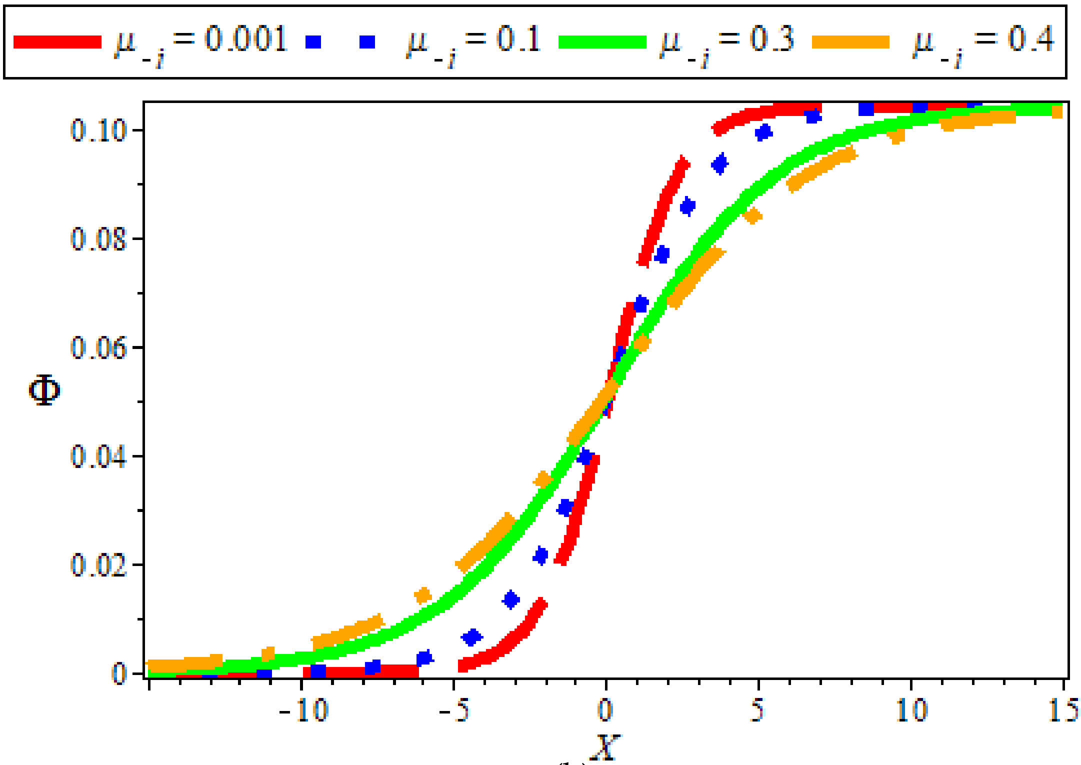


Figure 2.3: Electrostatic shock wave profile due to the variation of T with $\mu_{+i} = 0.3$ and $\mu_{-i} = 0.01$. The other parameters are chosen as $\alpha = 0.35$, $q = 1.6$, $M_{\pm} = 3.75$, $T_{-i} = 0.05$, $T_e = 0.2$, $N_{r1} = 0.5$, $N_{r2} = 0.1$ and $V_r = 0.03$.

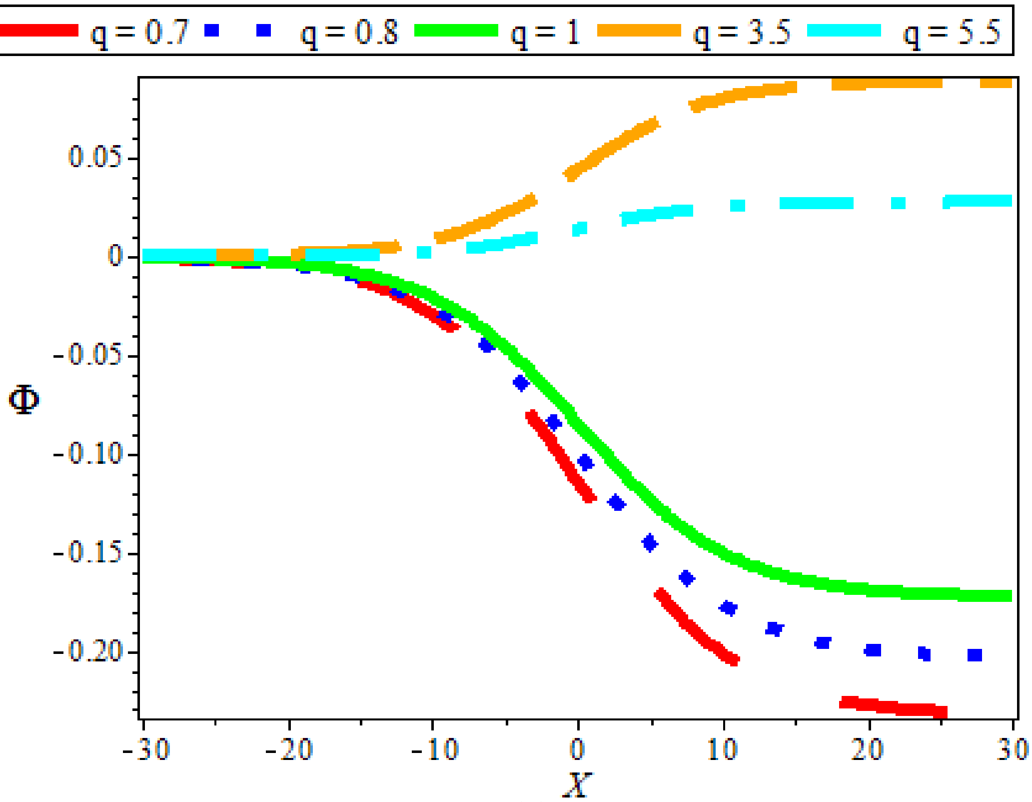


(a)

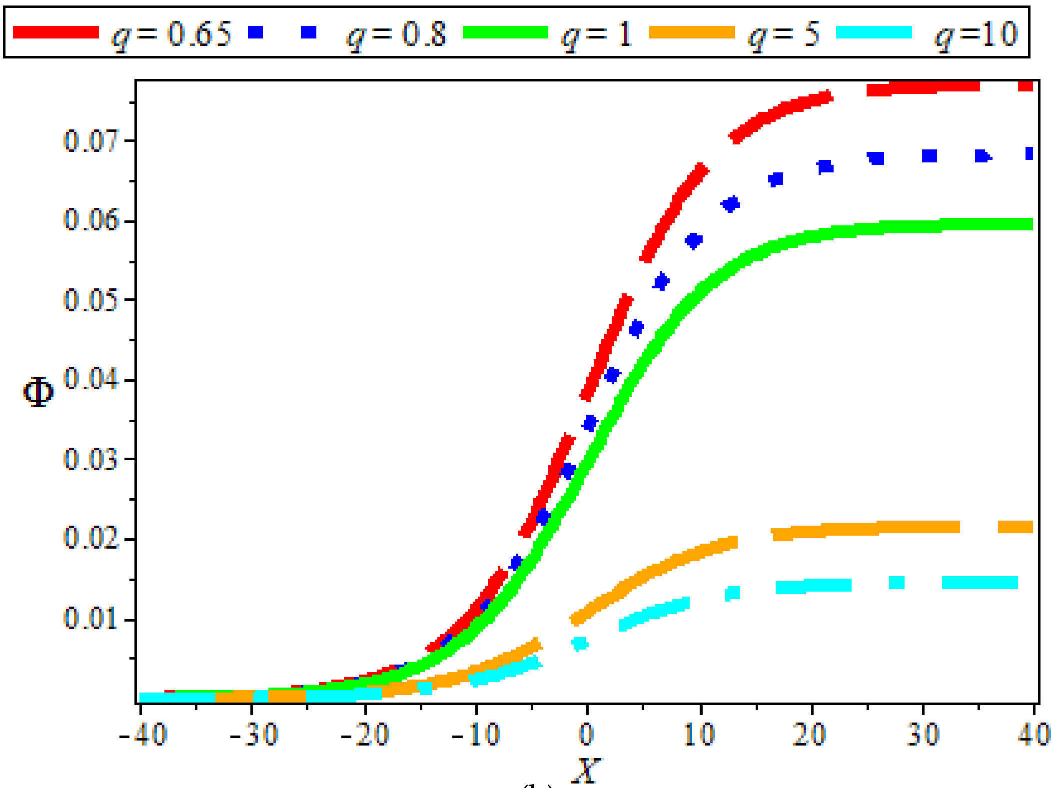


(b)

Figure 2.4: Electrostatic shock wave profile due to (a) the variation of μ_{+i} with $\mu_{-i} = 0.01$, and (b) the variation of μ_{-i} with $\mu_{+i} = 0.1$, respectively. The other parameters are chosen as $\alpha = 0.35$, $q = 1.6$, $M_{\pm} = 3.75$, $T_{-i} = 0.05$, $T_e = 0.2$, $N_{r1} = 0.5$, $N_{r2} = 0.1$, $T = 5$, and $V_r = 0.03$.



(a)



(b)

Figure 2.5: Electrostatic shock wave profile by choosing (a) $\alpha = 0$, $M_{\pm} = 3.75$, $T_{-i}0.02$, $T_e = 0.4$, $N_{r1} = 0.1$, $N_{r2} = 0.02$, $T = 5$, $\mu_{+i} = 0.25$, $\mu_{-i} = 0.4$, $V_r = 0.03$ and $q = 5.5$, and (b) $\alpha = 0$, $M_{\pm} = 3.75$, $T_{-i} = 0.05$, $T_e = 0.2$, $N_{r1} = 0.1$, $N_{r2} = 0.9$, $T = 5$, $\mu_{+i} = 0.35$, $\mu_{-i} = 0.05$, $V_r = 0.03$.

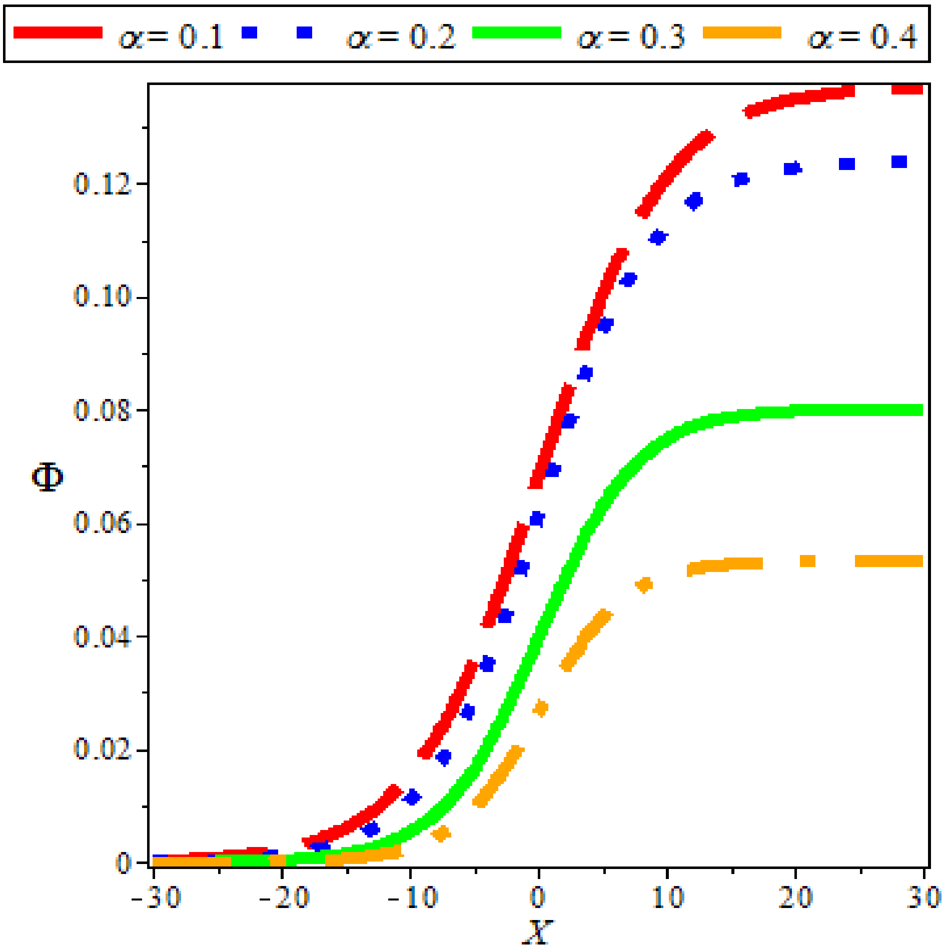
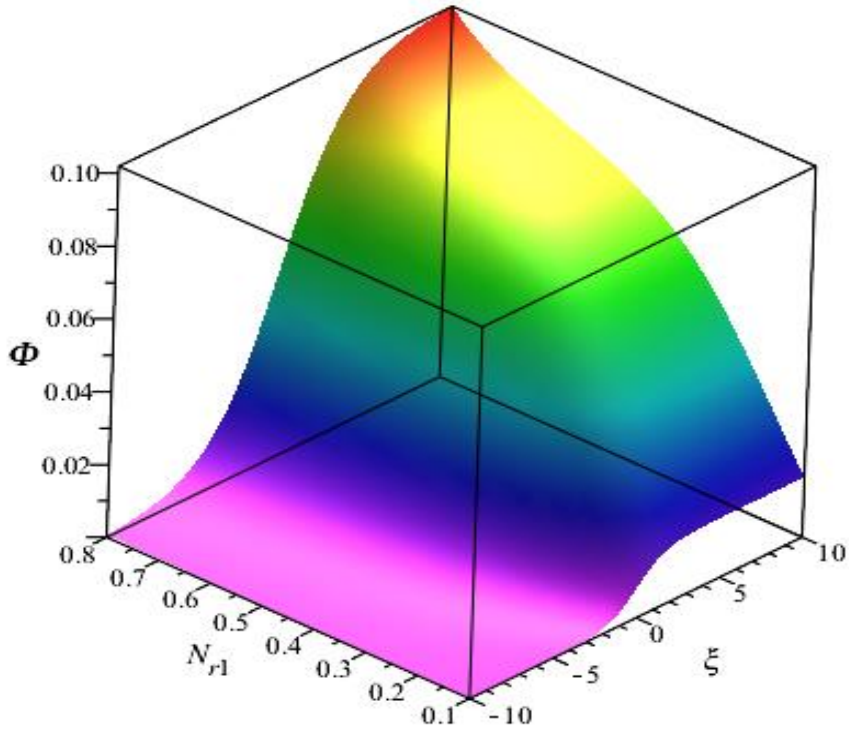
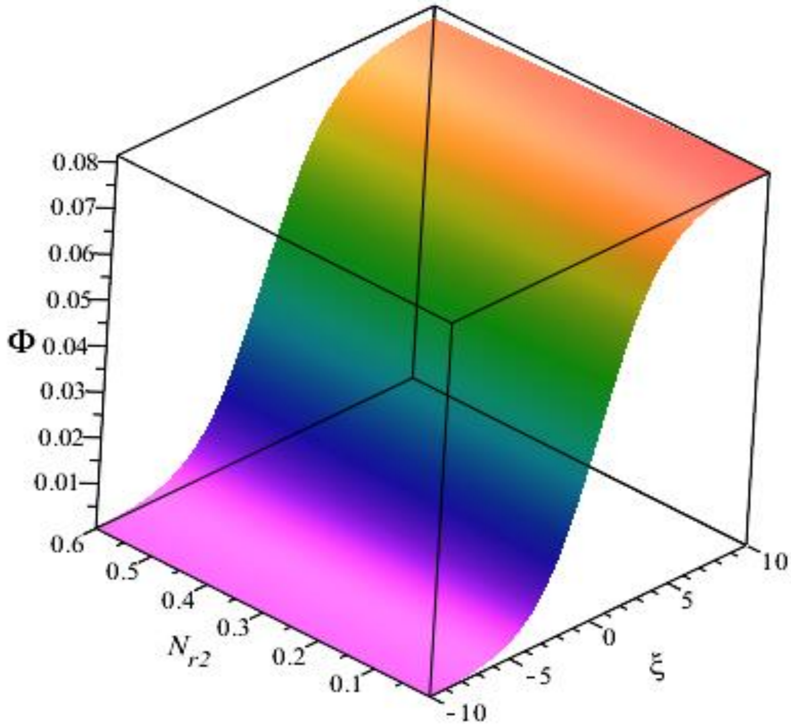


Figure 2.6: Effect of α on the electrostatic shock wave profile by choosing $M_{\pm} = 3.75$, $T_{-i} = 0.05$, $T_e = 0.2$, $N_{r1} = 0.1$, $N_{r2} = 0.5$, $\mu_{+i} = 0.35$, $\mu_{-i} = 0.05$, $V_r = 0.03$ and $q = 1$ with time $T = 5$.



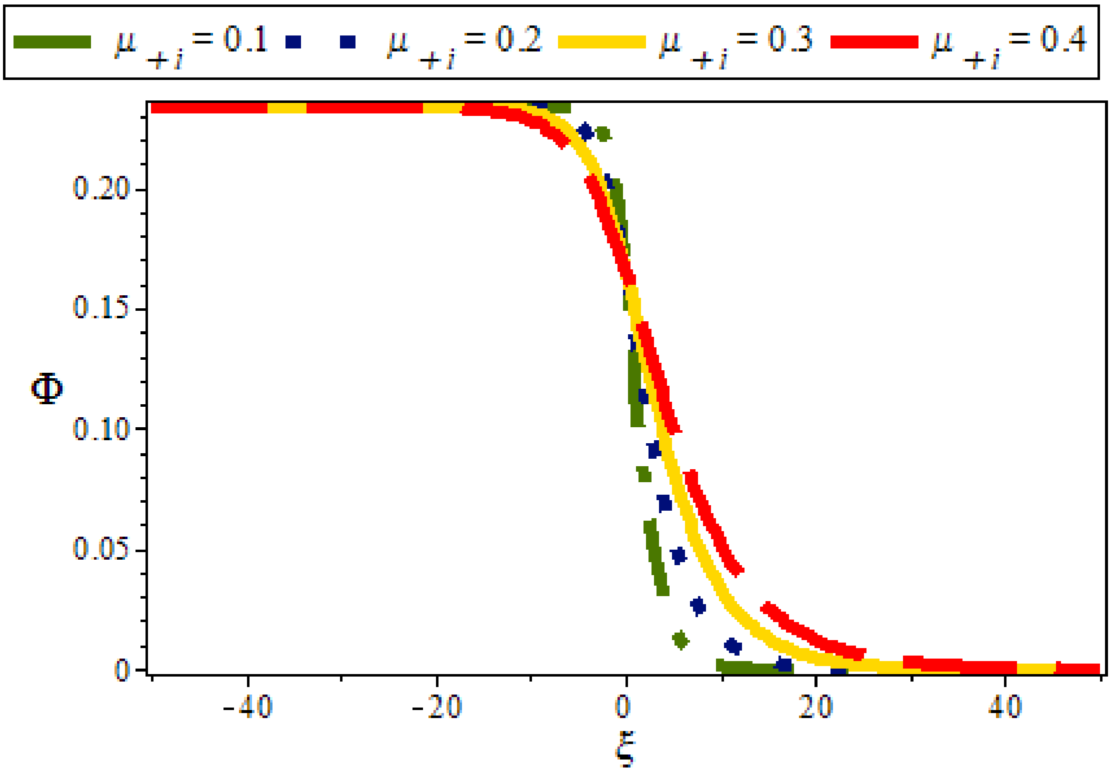
(a)



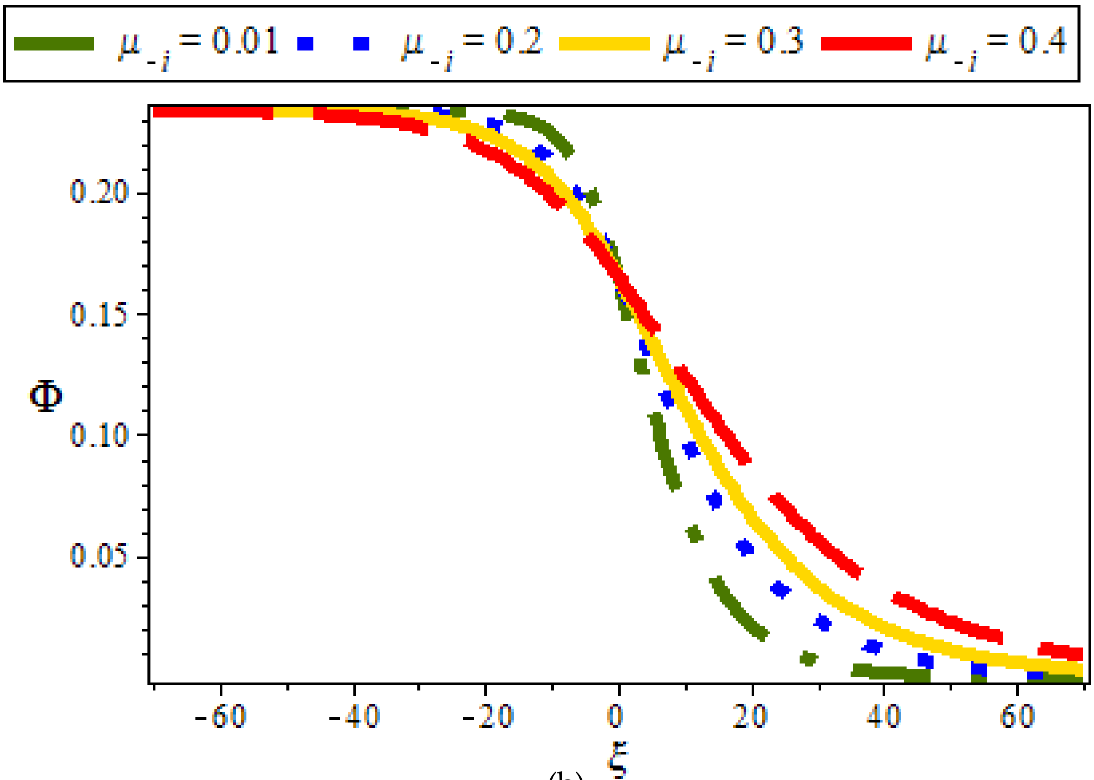
(b)

Figure 2.7: Electrostatic shock wave profile (a) due to the variations of N_{r1} with $N_{r2} = 0.5$, and (b) due to the variations of N_{r2} with $N_{r1} = 0.5$ .The other parameters are chosen as $\alpha = 0$, $M_{\pm} = 3.75$, $T_{-i} = 0.05$, $T_e = 0.2$, $\mu_{+i} = 0.5$, $\mu_{-i} = 0.01$, $V_r = 0.1$ and $q = 1$.

Figure 2.8(a) and 2.8(b) display the effect of μ_{+i} and μ_{-i} on the electrostatic IASWs around q_c. It is observed that μ_{+i} and μ_{-i} strongly plays an important role to the formation of monotonically shock waves around q_c. In addition, the width of shocks are remarkably increased and slightly increased with the increases of μ_{+i} and μ_{-i}, respectively. It is noted that the effect of the kinematic viscosity coefficient of NIs and PIs on shock wave excitation can be determined on the basis of collective friction between the layers of the plasma concentration system. In fact, viscosity is the force of collective friction between layers of fluid in the aforementioned plasmas. With the decrease of μ_{+i} and μ_{-i}, the collective friction force is decreased. As a result, the thickness of shocks is decreased. Figure 2.9 obviously shows that the amplitude and thickness of IASWs are significantly increased with the increase of V_r. Finally, the variation of normalized electric field ($E = -grad\Phi$) around q_c for different values of μ_{+i} and μ_{-i} is presented in Figure 2.10. It is observed that the normalized electric field becomes hump-shaped with the increase of μ_{+i} and μ_{-i}. Consequently, the electric field is propagating narrowly with the increase of μ_{+i}, but smoothly with the increase of μ_{-i}.



(a)



(b)

Figure 2.8: Electrostatic mB shocks around the critical value $q_c = 3.098638852$ that is $q = 3.5 > q_c$ (a) μ_{+i} with $\mu_{-i} = 0.01$, and (b) μ_{-i} with $\mu_{+i} = 0.5$, respectively with $\alpha = 0$, $M_{\pm} = 3.75$, $T_{-i} = 0.05$, $T_e = 0.19$, $N_{r1} = 0.5$, $N_{r2} = 0.01$ and $V_r = 0.01$.

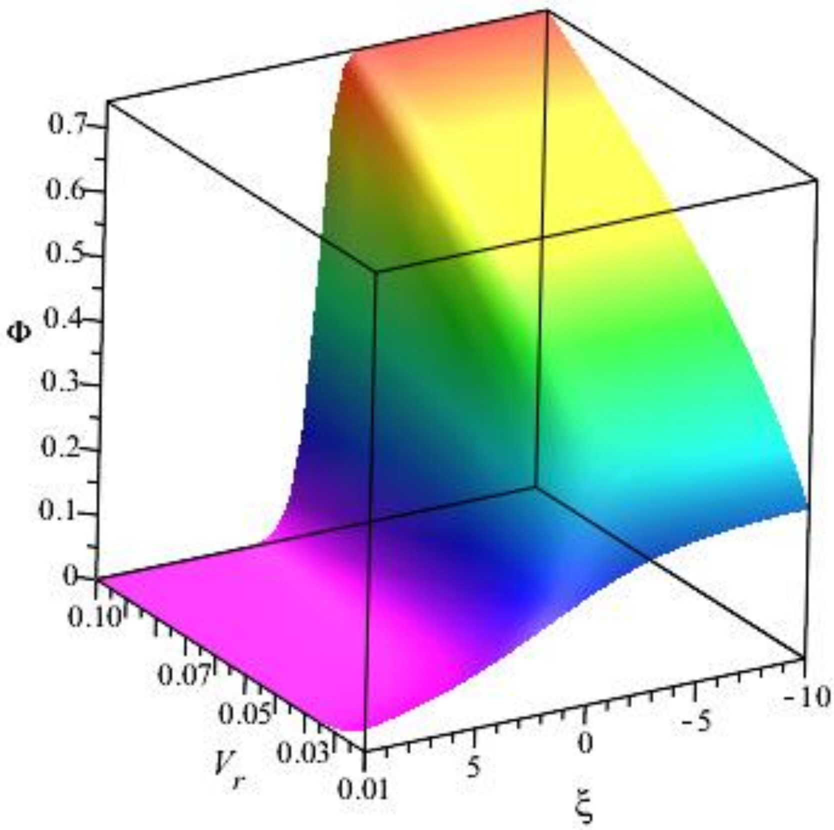
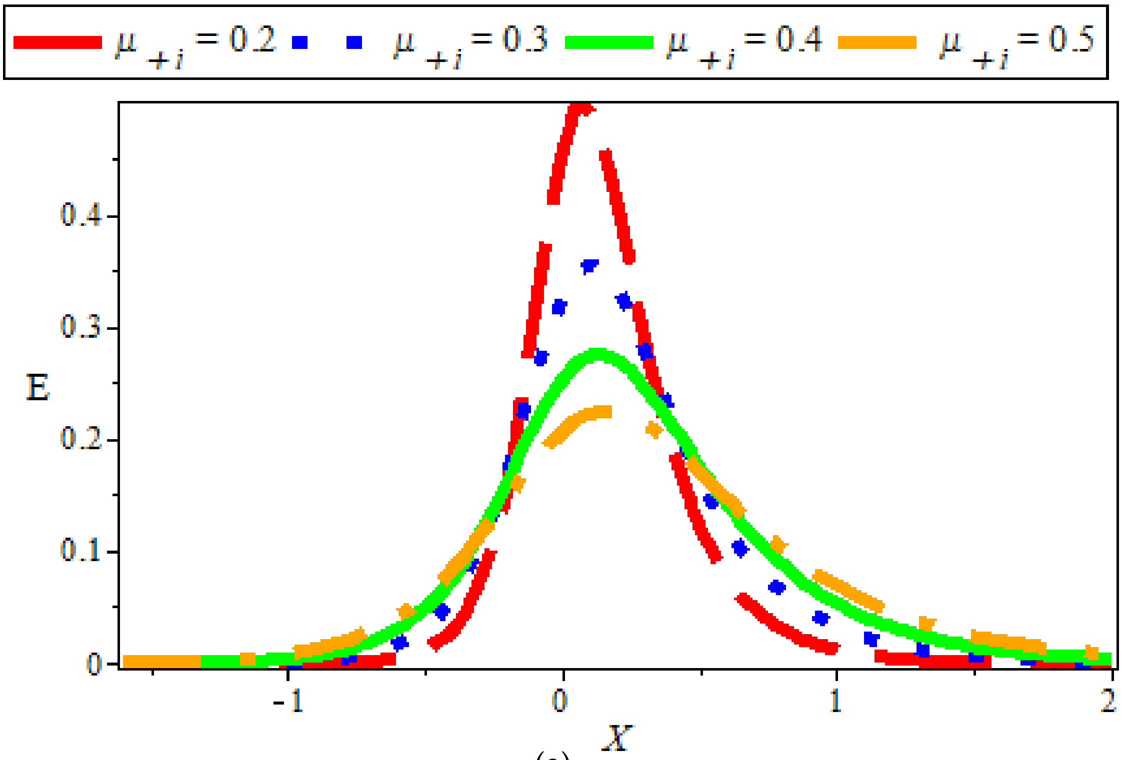
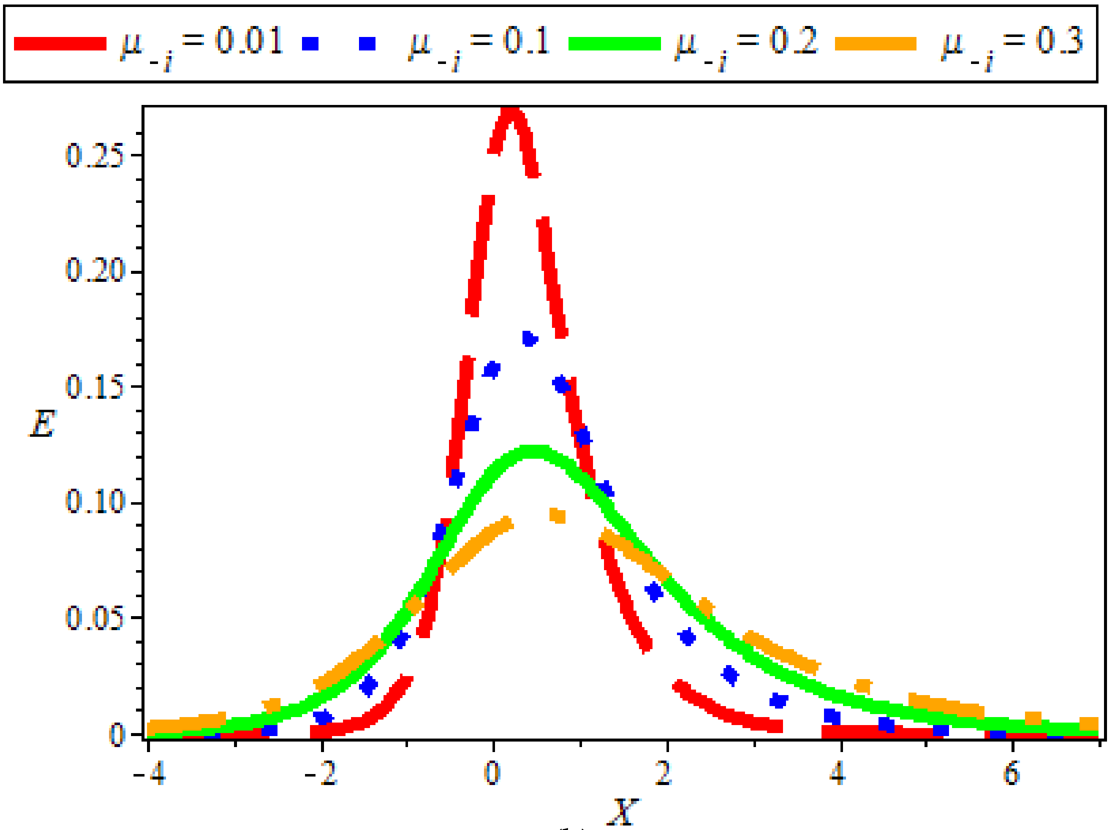


Figure 2.9: Effect of the reference speed V_r electrostatic mB shocks around the critical value $q_c = 3.098638852$ that is $q = 3.5 > q_c$ with $\alpha = 0$, $M_{\pm} = 3.75$, $T_{-i} = 0.05$, $T_e = 0.19$, $N_{r1} = 0.5$, $N_{r2} = 0.01$, $\mu_{-i} = 0.01$ and $\mu_{+i} = 0.3$.



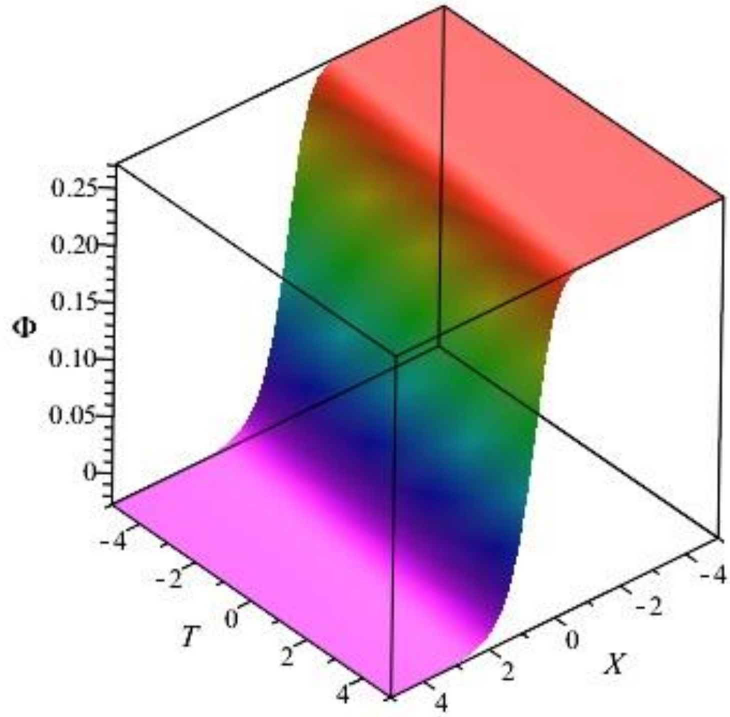
(a)



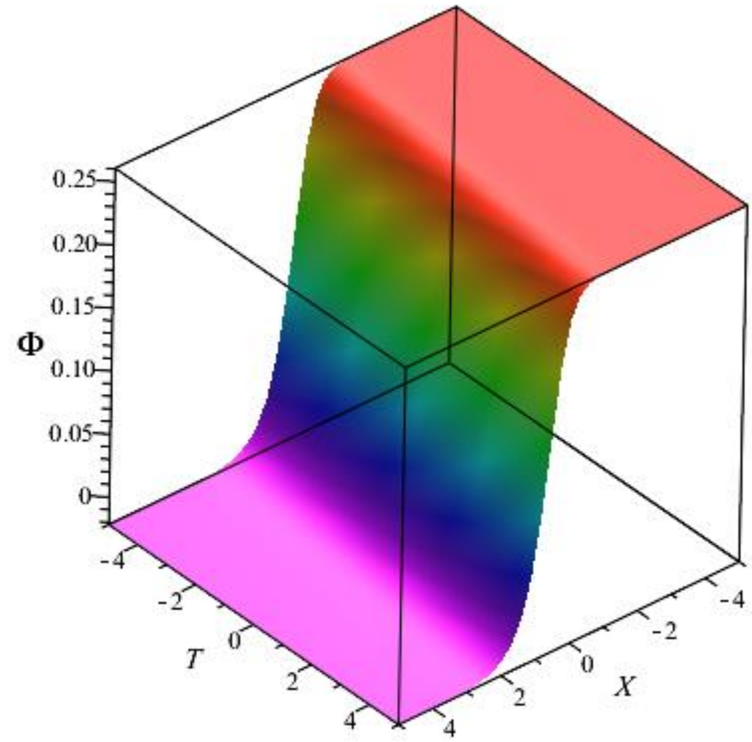
(b)

Figure 2.10: Variation of the normalized electric field around the critical value $q_c = 3.098638852$ that is $q = 3.5 > q_c$, for different values of (a) μ_{+i}, and (b) μ_{-i} with the same typical values of Figure 2.4.

Figure 2.11(a) and 2.11(b) demonstrates the shape of electrostatic ion acoustic mmB shocks for $q = 6 < q_c = 6.176859718$ and $q = 6.5 > q_c = 6.176859718$ with $M_{\pm} = 3.75$, $T_{-i} = 0.05$, $T_e = 0.1$, $N_{r1} = 0.5$, $N_{r2} = 0.1$, $V_r = 0.1$, $\mu_{+i} = 0.5$ and $\mu_{-i} = 0.001$. Whereas Figure 2.12(a) and 2.12(b) display the effect of μ_{+i} and μ_{-i} on the electrostatic IASWs at the neighbouring of q_c, that is at $q = 6.5$. It is found that the solution of the shocks are only generated for the viscosity coefficient of NIs in which both amplitude and width are increased with the increase of the kinematic viscosity coefficient of NIs at the neighbouring CVs. However, the amplitude and width of IASWs are decreased with the increase of the kinematic viscosity coefficient of PIs at the neighbouring CVs. It might be predicted from this work that one can study the real features of shock wave excitations around CVs by formulating the solution of Eqs. (2.77) and (2.89) of mB-type and mmB equations, respectively. Based on the consideration of the typical values of the parameters, it may be concluded that the obtained results are very useful to understand the features of broadband shock noise in the D- and F-regions of the Earth's ionosphere.

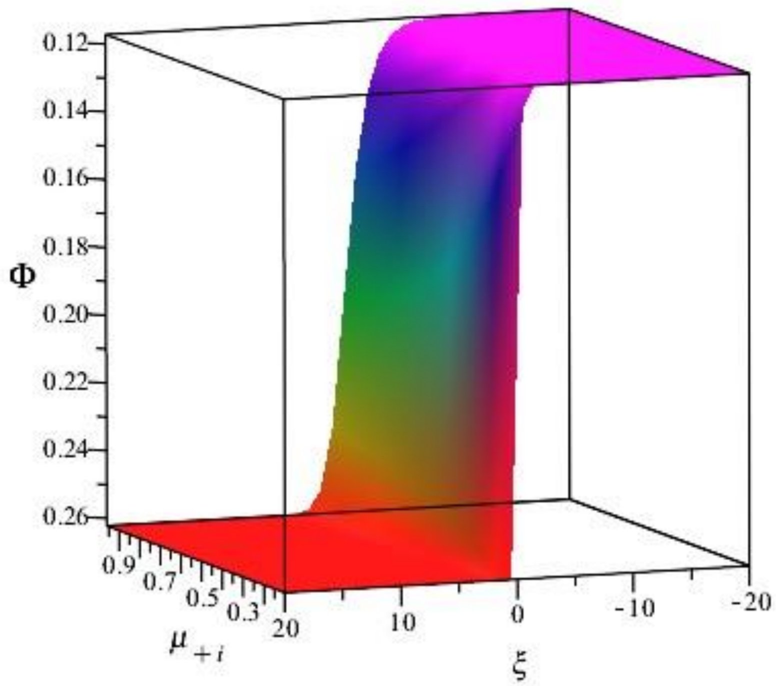


(a)

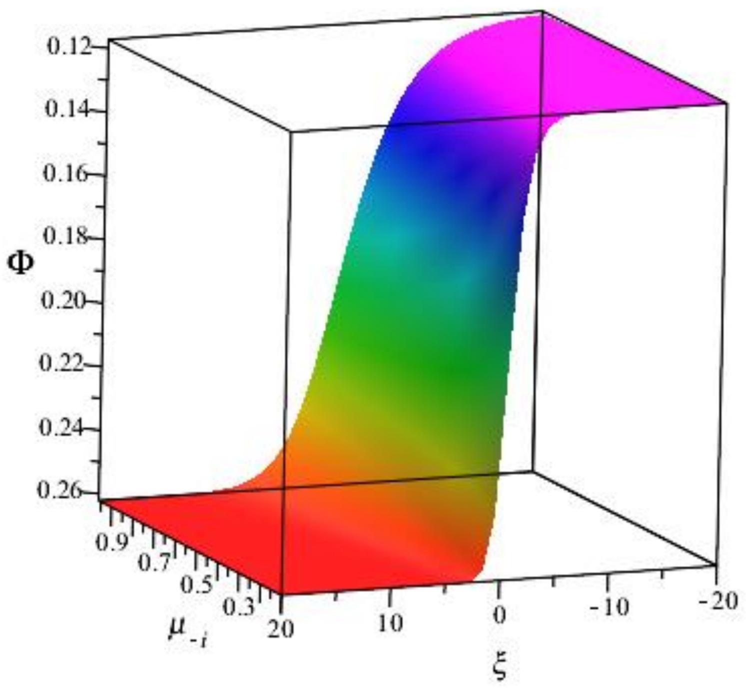


(b)

Figure 2.11: Electrostatic mmB shock profile (a) $q = 6 < q_c = 6.176859718$, and (b) $q = 6.5 > q_c = 6.176859718$, with $\alpha = 0$, $M_{\pm} = 3.75$, $T_{-i} = 0.05$, $T_e = 0.1$, $N_{r1} = 0.5$, $N_{r2} = 0.1$, $T = 5$, $\mu_{+i} = 0.5$, $\mu_{-i} = 0.01$, $V_r = 0.1$ and $q = 6$.



(a)



(b)

Figure 2.12: Variation of the normalized electric field around the critical value $q_c = 6.176859718$ that is $q = 6.5 > q_c$, for different values of (a) μ_{+i} with $\mu_{-i} = 0.01$ and (b) μ_{-i} with $\mu_{+i} = 0.5$, respectively with $\alpha = 0.5$, $M_{\pm} = 3.75$, $T_{-i} = 0.05$, $T_e = 0.19$, $N_{r1} = 0.5$, $N_{r2} = 0.01$ and $V_r = 0.01$.

2.7 Concluding remarks

A plasma system consisting of positive as well as negative ions and electrons is considered to study the nature of SWEs with the influences of plasma parameters, where electrons are assumed to follow not only iosthermality but also superthermality, subthermality and nonthermality with the presence of nonextensitity. The (α, q)-velocity distribution function is considered because it is very effective in all cases of thermality conditions. By employing the well-established reductive perturbation approach, the useful NLEEs, Burgers, mB and mmB equations are obtained which divulges the shocks in such plasmas. The solutions of these equations are determined by direct integration. In addition, the correct stationary shock wave solutions of mB and mmB equations are determined for the first time. It is found that the steepness and amplitude of shocks are sensitively increased (decreased) with the increase of $\mu_{+i}(N_{r2})$, but very slightly increased with the increase of μ_{-i}. In addition, both compressive and rarefactive shocks exist described by Burgers equation with the influences of parameters. On the other hand, the mB equation reveals the shocks only around the critical values, whereas the mmB equation reveals the shocks not only around the critical values but also at the critical values in the plasmas. The effect of μ_{+i}, μ_{-i} and V_r on the electrostatic mB and mmB shocks around and at the neighbouring of CVs are discussed. It is found that the mB-type equation is supporting monotonic shocks around CVs in which the amplitude is unchanged but the thickness is increased with the increase of μ_{+i} and μ_{-i}. Subsequently, the monotonically hump-shaped electric field is propagating narrowly and smoothly with the increase of μ_{+i} and μ_{-i}, respectively, due to the correct solution of the mB-type equation. However, the mmB-type equation supports monotonic shocks around CVs with the influence of μ_{-i} only, in which both the amplitudes and the thickness are increased with the increase of μ_{-i} but decreased with the increase of μ_{+i}. It may be concluded that the results presented in this work are used not only to understand the broadband shocks noise in the D- and F-regions of the Earth's ionosphere around the critical values, but also in laboratory experiments.

Chapter 3

Dust-ion-acoustic shock wave excitations in collisionless unmagnetized dusty plasma

3.1 Introduction

Most of the astrophysical and space plasma systems (ASPSs), e.g., Earth's ionosphere, planetary environments, interstellar media, protostellar disks, molecular clouds, asteroid regions, comet tails, nebulae, and so on [3-6], justify the existence of dust particles. As a result, one may not only study the ion-, electron-, and positron-acoustic wave phenomena but also the dust-ion acoustic (DIA) or DA wave phenomena with the existence of various charged particles in ASPSs. In addition, the development of dusty plasmas (DPs) is still mostly focused on the analysis of the propagation of DIA waves [6, 93, 97-99].The existence of low-frequency DIA waves was first proposed theoretically by Shukla and Silin in 1992 [6]. Shukla and Silin [6] have developed an important conclusion in his fundamental research that perhaps the inclusion of dense, enormous, and immovable charged dust species in electron-ion plasmas, which extensively affects the dynamics of the waves. Furthermore, Mamun et al. investigate the formation of low-frequency DIA waves, which take place over a duration of time that is also considerably smaller than the period of dusty plasma [100]. However, Shukla and Mamun [2] have reported that the influence of dust grains, which are considered to be immobile, impacts equilibrium quasineutrality. Further, researchers in plasma physics [37, 48, 101-105] concentrated attention on the study of acoustic waves in dusty multi-ion plasmas. Positive ions (PIs) and negative ions (NIs) are examples of multi-ions, and their presence has recently been well-confirmed by ASPSs [2, 48, 103, 104] and plasma laboratories [10, 105].For instance, in many plasma environments, such as the Earth's ionosphere and cometary comae, plasma is produced by the mixing of PIs and NIs in addition to electrons [106]. The sources for the successful generation of PI and NI plasmas are neutral beam sources [67], plasma processing reactors [107], and experiments in laboratories [69]. Additionally, it was shown that negatively charged ions performed better in plasma etching than positively charged ions. As a result, negative ion plasmas have become more and more significant in the field of plasma physics.

Moreover, the distribution of the velocity of associated plasma components affects the basic features of the plasma system. It is essential to describe the motion of lighter-charged particles. The Maxwellian velocity distribution (MVD) is the most familiar in a collisionless plasma. However, the VD of plasma particles in the laboratory and ASPs differs from the MVD. The particles follow either noxextensive [15, 84, 88], and kappa [108] or non-thermal [16, 44] distributions that deviate from MVD. Consequently, the interaction among the charged particles is mainly short-range based on simple statistical mechanics. But numerous plasma particles especially interaction nature via long ranges, i.e., long-range Coulomb interactions, where the extensible attribute is usually destroyed. To overcome such dynamic challenges, one can consider (α, q)-VD [14], which is applicable not only cases of superthermality, subthermality, and isothermality, but also for the case of nonthermality, depending on the appropriate values of index α and q. For instance, one would be considered q-nonextensive VD by setting $\alpha \rightarrow 0$ proposed by Tsallis [15] for analysing the circumstances where the MVD is unsuitable. A real-world example of the existence of the aforementioned complex plasma system is the ring of Saturn [4]. Because the fairly accurate number densities $N_{J0}(J = i$ for ions, $J = e$ for electrons, and $J = d$ for dust, etc.) and temperature T exist in the Saturn ring, that is $N_{i0} \sim 10^1 cm^{-3}$, $N_{d0} \sim 10^{-7} - 10^{-8} cm^{-3}$ and $T \sim 10^5 - 10^6 K$ for E-ring, $N_{i0} \sim 10^1 - 10^2 cm^{-3}$, $N_{d0} < 30\ cm^{-3}$ and $T \sim 10^5 - 10^6 K$ for F-ring and

N_{i0}~$0.1 - 10^2 cm^{-3}$, N_{d0}~$1\ cm^{-3}$ and T~2×10^4 for the Spokes [3]. The plasmas associated with the non-Maxwellian distributions have also recently attracted much attention due to their wide relevance to ASPSs such as quark-gluon [109] and hadronic matter plasmas [14], dark matter halos [110], Earth's bow shock [111], the magnetospheres of Jupiter and Saturn [112], etc.

Due to the existence of dust and multi-ions in ASPSs, some theoretical and experimental research [62, 67, 100, 113-118] has focused on the DIA waves in a multi-species collisionless plasma composed of electrons, PIs, and NIs with dust grains. Yasmin et al. [62] have analyzed the impact of nonextensive electrons on DIA shock waves (DIASWs) by taking into account a nonextensive plasma made of ions, nonextensive electrons, and stationary dust (negatively charged). The arbitrary amplitude of DIASWs in such plasma with PIs and NIs has been explored by Mamun et al. [100]. Ema et al. [118] have reported the propagation of DIASWs in a nonextensive complex plasma having nonextensive electrons, Maxwellian light ions (MLIs) having positive charges, heavy NIs and stationary dust (SD) having negative charges. They have reported the features of nonlinear propagation of DIASWs by deriving Burger's equation (BE) and a higher-order BE, that is, Gardner's equation (GE). However, the impact of -VD electrons on dusty multi-ion plasma has not been considered in any previously proposed theoretical research, to the best of our knowledge. Thus, in order to study the amplitude of DIASWs by deriving various higher-order BEs along with their solutions in a complex plasma, the plasma species have been considered as VD electrons, inertial heavy NIs, positively charged MLIs, and negatively charged SD. The effects of plasma parameters on the nonlinear propagation of DIASWs are also investigated with physical interpretations.

3.2 Theoretical model with plasma assumptions

Let us consider a collisionless four-component unmagnetized dusty plasma system, which is a mixture of the (α, q)-distributed electrons, inertial HIs (negatively charged), MLIs (positively charged), and SD known as immobile dust (negatively charged). As a result, the equilibrium charge neutrality condition is obtained as $N_{i0} - Z_{hi}N_{hi0} - N_{e0} - Z_d N_{d0} = 0$, where N_{s0} is the number of unperturbed densities of the speciess ($s = i, hi, e$, d for positive MLIs, negative HIs, electrons, and immobile dust, respectively), Z_{hi} is the number of electrons of HIs and Z_d is the number of electrons residing on the dust grain surface. The energies of the electrons may be thermal or nonthermal and if the electron energy have a smaller (subthermal) or equal (isothermal) or superior (superthermal) amount of neutral energy. As a result, the (α, q)-velocity distribution ofelectrons is assumed. Therefore, the (α, q)-VD function is defined by the composition of Tsallis and Cairns VD functions as [92]

$$y(v_x) = k\left(1 + \alpha \frac{v_x^4}{v_t^4}\right) \times \left\{1 - (q-1)\frac{v_x^2}{2v_t^2}\right\}, \tag{3.1}$$

where $v_t = (k_B T_e/m_e)^{1/2}$ is the electron thermal velocity, $v_x = (2e\Phi/m_e)^{1/2}$ is the velocity vector, q is the nonextensivity strength, α represents the population of faster electrons, k_B is defined as the Boltzmann constant, and k is the normalized constant [14]. Hence, the electron density (N_e) function can be written by integrating (which includes an additional potential term of interacting electrons) the above equation over velocity space [92] as

$$N_e = N_{e0}\left[1 + (q-1)\left(\frac{e\Phi}{k_B T_e}\right)\right]^{\frac{(q+1)}{2(q-1)}} \times$$
$$\left[1 - \frac{16q\alpha}{3 - 14q + 15q^2 + 12\alpha}\left(\frac{e\Phi}{k_B T_e}\right) + \frac{16q(2q-1)\alpha}{3 - 14q + 15q^2 + 12\alpha}\left(\frac{e\Phi}{k_B T_e}\right)^2\right], \tag{3.2}$$

where Φ, T_e, and e are the electrostatic wave potential, electron temperature, and magnitude of the electron charge, respectively. One can determine the nonextensive [15] and Cairns [16] distributed electron density functions from Eq. (3.2) as

$$N_e = N_{e0}\left[1 + (q-1)\left(\frac{e\Phi}{k_B T_e}\right)\right]^{\frac{(q+1)}{2(q-1)}}, \tag{3.3}$$

and $$N_e = N_{e0}\left[1 - \left(\frac{4\alpha}{1+3\alpha}\right)\left(\frac{e\Phi}{k_B T_e}\right) + \left(\frac{4\alpha}{1+3\alpha}\right)\left(\frac{e\Phi}{k_B T_e}\right)^2\right]\exp\left(\frac{e\Phi}{k_B T_e}\right), \quad (3.4)$$

for the case of $\alpha = 0$ and $q = 1$, respectively. The proper ranges of α and q are obtained based on the physical cut-off obligatory by $q \geq 5/7$, and $\alpha_{Max} = (2q-1)/4$ as (i) $q = 1$, $0 < \alpha < 0.35$ (nonthermality case), (ii) $q = 1$, $\alpha = 0$ (isothermality case), (iii) $0.33 < q < 1$, $\alpha = 0$ (superthermality case), and (iv) $q > 1$, $\alpha = 0$ (subthermality case), respectively. It is provided that the (α, q)-VD functions are very useful to describe the energy of electrons in all cases of thermality. However, the MLIs density (N_i) function can be written as

$$N_i = N_{i0} e^{\left(-\frac{e\Phi}{k_B T_i}\right)}. \quad (3.5)$$

To study the basic features of nonlinear DIASWs(where the mass of the negatively charged HIs provides inertia while the thermal pressure of the electrons acts as the restoring energy), the dimensionless continuity and momentum equations are obtained by implementing the mass and momentum conservation laws, respectively, in the following forms [118]:

$$\frac{\partial N_{hi}}{\partial t} + \frac{\partial (N_{hi} U_{hi})}{\partial x} = 0, \quad (3.6)$$

$$\frac{\partial U_{hi}}{\partial t} + U_{hi}\frac{\partial U_{hi}}{\partial x} = \frac{\partial \Phi}{\partial x} + \mu_{hi}\frac{\partial^2 U_{hi}}{\partial x^2}, \quad (3.7)$$

Since the plasma particles are interconnected to the electric field $(E = -\nabla\Phi)$, Eqs. (3.6) and (3.7) are supplemented with the Maxwell's equation, $\nabla \cdot E = -4\pi\rho$, where $\rho = (N_{i0} - N_{e0} - Z_{hi}N_{hi0} - Z_d N_{d0})$ is the overall charge density on the surface [118]. Here, N_{hi0} is the unperturbed HIs density, N_{d0} is the unperturbed SDs density, Z_{hi} is the number of electrons of His and Z_d is the number of electrons residing onto the dust grain surface. Based on the charge neutrality condition, the following dimensionless equation is obtained as

$$\frac{\partial^2 \Phi}{\partial x^2} = N_{r2}\left\{[1 + (q-1)\Phi]^{\frac{q+1}{2(q-1)}} \times [1 + B_1\Phi + B_2\Phi^2]\right\} + N_{r1}N_{hi} - e^{-\delta_{ei}\Phi} + N_{r3},$$

(3.8)

where

$$B_1 = -\frac{16q\alpha}{3-14q+15q^2+12\alpha} \text{ and } B_2 = \frac{16q(2q-1)\alpha}{3-14q+15q^2+12\alpha}. \tag{3.9}$$

It is noted that Eq. (3.8) is obtained in the similar form of the Ref. [123] with the presence of nonextensivity, i.e., $\alpha = 0$. Eqs. (3.6)-(3.8) are normalized by introducing $N_{hi} \to N_{hi}/N_{hi0}$, $U_{hi} \to U_{hi}/C_{hi}$ ($C_{hi} = (k_B T_e/m_{hi})^{1/2}$), $\Phi \to \Phi/(k_B T_e/e)$, $x \to x/\lambda_{Dhi}$ ($\lambda_{Dhi} = (k_B T_e/4\pi e^2 n_{hi0})^{1/2}$), $t \to t/\omega_{phi}^{-1}$ ($\omega_{phi}^{-1} = (m_{hi}/4\pi e^2 N_{hi0})^{1/2}$) and $\mu_{hi} \to \mu_{hi}/m_{hi} N_{hi0} \omega_{phi} \lambda_{Dhi}^2$, where U_{hi} is the HIs fluid speed, C_{hi} is the HIs acoustic speed, Φ is the electrostatic wave potential, μ_{hi} is the kinematic viscosity coefficient, k_B is the Boltzmann constant, T_e is the electron temperature, e is the magnitude of the electron charge, $t(x)$ is the time (space) variable, and $q(\alpha)$ is the strength of nonextensivity which measure the population of nonthermal electrons, respectively. Due to the normalization of the above equation, $N_{r1} = Z_{hi} N_{hi0}/N_{i0}$ (heavy-to-light ion number density ratio), $N_{r2} = N_{e0}/N_{i0}$ (electron-to-light ion number density ratio), $N_{r3} = N_d/N_{i0}$ (dust- to-light ion temperature ratio) and $\delta_{ei} = T_e/T_i$ (electron-to-light ion temperature ratio) are obtained. In order to avoid any encumbering effect, the phase speed of HIs is considered much smaller (larger) than the electrons (MLIs) thermal speed. It is noted that the plasma system is in good agreement that proposed in Ref. 118 for the case of $\alpha = 0$.

3.3 Formation of Burgers equation

To investigate the nonlinear wave propagation of DIASWs, one can derive the evolution equations from Eqs. (3.6)-(3.8) by using the appropriate stretching coordinates and the expansion of only perturb quantities but not any expansion of arbitrary quantities. To do it, one is allowed to consider the new coordinates instead of the scaling of variables as

$$X = \varepsilon(z - V_p t), \ T = \varepsilon^2 t, \tag{3.10}$$

where, V_p is linear phase speed and ε is a small quantity which measure the weakness of dissipation [93, 119].

From Eq. (3.10), the operator are defined as

$$\frac{\partial}{\partial t} = \varepsilon^2 \frac{\partial}{\partial T} - \varepsilon V_p \frac{\partial}{\partial X}, \quad \frac{\partial}{\partial z} = \varepsilon \frac{\partial}{\partial X}, \tag{3.11}$$

Eqs. (3.6)-(3.8) are converted with the aid of Eq. (3.11) to the following:

$$\varepsilon^2 \frac{\partial N_{hi}}{\partial T} - \varepsilon V_p \frac{\partial N_{hi}}{\partial X} + \varepsilon \frac{\partial}{\partial X}(N_{hi} U_{hi}) = 0, \tag{3.12}$$

$$\varepsilon^2 \frac{\partial U_{hi}}{\partial T} - \varepsilon V_p \frac{\partial U_{hi}}{\partial X} + \varepsilon U_{hi} \left(\frac{\partial U_{hi}}{\partial X}\right) - \varepsilon \frac{\partial \Phi}{\partial X} - \varepsilon^2 \mu_{hi} \frac{\partial^2 U_{hi}}{\partial X^2} = 0, \tag{3.13}$$

and

$$\varepsilon^4 \frac{\partial^2 \Phi}{\partial X^2} = N_{r2} \left\{ [1 + (q-1)\Phi]^{\frac{q+1}{2(q-1)}} \times \left[1 + B_1 \Phi + B_2 \Phi^2\right] \right\} + N_{r1} N_{hi} - e^{-\delta_{ei}\Phi} + N_{r3}. \tag{3.14}$$

By inserting the following expanded perturb quantities

$$\begin{bmatrix} N_{hi} \\ U_{hi} \\ \Phi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \sum_{i}^{\infty} \varepsilon^i \begin{bmatrix} N_{hi}^{(i)} \\ U_{hi}^{(i)} \\ \Phi^{(i)} \end{bmatrix}, \tag{3.15}$$

into Eqs. (3.12)-(3.14). As a result, one coverts Eqs. (3.12)-(3.14) by including the different orders of ε, that is $O(\varepsilon^r), r = 2,3,4,5, \cdots \cdots$.

For $O(\varepsilon^2)$:

$$-V_p \frac{\partial N_{hi}^{(1)}}{\partial X} + \frac{\partial U_{hi}^{(1)}}{\partial X} = 0, \tag{3.16}$$

$$-V_p \frac{\partial U_{hi}^{(1)}}{\partial X} - \frac{\partial \Phi^{(1)}}{\partial X} = 0, \tag{3.17}$$

and

$$N_{r2} \Omega_1 \Phi^{(1)} + \delta_{ei} \Phi^{(1)} + N_{r1} N_{hi}^{(1)} = 0, \tag{3.18}$$

Now, the solutions of Eqs. (3.16) and (3.17) yields [118]

$$\left.\begin{aligned} N_{hi}^{(1)} &= -\frac{1}{V_p^2}\Phi^{(1)} \\ U_{hi}^{(1)} &= -\frac{1}{V_p}\Phi^{(1)} \end{aligned}\right\}. \tag{3.19}$$

Inserting the values from Eq. (3.19) in Eq. (3.18), one can obtained V_p as

$$V_p = \pm\sqrt{\frac{N_{r1}}{N_{r2}\Omega_1 + \delta_{ei}}}\,, \tag{3.20}$$

where

$$\Omega_1 = \frac{q+1}{2} + B_1.$$

The positive value of phase velocity indicates fast mode, and the negative value of phase velocity indicates slow mode. But in this work, we consider it fast mode. Eq. (3.20) indicated that V_p is strongly dependent on N_{r1}, δ_{ei}, N_{r2}, α and q but not on μ_{hi} and validated only if $N_{r1}/(N_{r2}\Omega_1 + \delta_{ei}) \geq 0$. Eq. (3.20) is also in good agreement with the investigation in Ref. 118 for $\alpha = 0$.

For $O(\varepsilon^3)$:

$$\frac{\partial N_{hi}^{(1)}}{\partial T} - V_p\frac{\partial N_{hi}^{(2)}}{\partial X} + \frac{\partial U_{hi}^{(2)}}{\partial X} + \frac{\partial}{\partial X}\left(N_{hi}^{(1)}U_{hi}^{(1)}\right) = 0, \tag{3.21}$$

$$\frac{\partial U_{hi}^{(1)}}{\partial T} - V_p\frac{\partial U_{hi}^{(2)}}{\partial X} + U_{hi}^{(1)}\frac{\partial U_{hi}^{(1)}}{\partial X} - \frac{\partial \Phi^{(2)}}{\partial X} - \mu_{hi}\frac{\partial^2 U_{hi}^{(1)}}{\partial X^2} = 0, \tag{3.22}$$

and

$$N_{r2}\Omega_1\Phi^{(2)} + N_{r2}\Omega_2\left[\Phi^{(1)}\right]^2 + \delta_{ei}\Phi^{(2)} - \delta_{ei}^2\left[\Phi^{(1)}\right]^2 + N_{r1}N_{hi}^{(2)} = 0, \tag{3.23}$$

where

$$\Omega_2 = \frac{q+1}{2}B_1 + \frac{(q+1)(3-q)}{8} + B_2.$$

Now, one can easily derive the following equations from Eqs. (3.21)-(3.23) by using the values from Eq. (3.19):

$$-\frac{1}{V_p^2}\frac{\partial \Phi^{(1)}}{\partial T}-V_p\frac{\partial N_{hi}^{(2)}}{\partial X}+\frac{\partial U_{hi}^{(2)}}{\partial X}+\frac{2}{V_p^3}\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial X}=0, \tag{3.24}$$

$$-\frac{1}{V_p}\frac{\partial \Phi^{(1)}}{\partial T}-V_p\frac{\partial U_{hi}^{(2)}}{\partial X}+\frac{1}{V_p^2}\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial X}-\frac{\partial \Phi^{(2)}}{\partial X}+\frac{\mu_{hi}}{V_p}\frac{\partial^2 \Phi^{(1)}}{\partial X^2}=0, \tag{3.25}$$

and

$$N_{r2}\Omega_1\frac{\partial \Phi^{(2)}}{\partial X}+2N_{r2}\Omega_2\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial X}+\delta_{ei}\frac{\partial \Phi^{(2)}}{\partial X}-2\delta_{ei}^2\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial X}+N_{r1}\frac{\partial N_{hi}^{(2)}}{\partial X}=0. \tag{3.26}$$

Multiplying Eq. (3.24) by V_p and then adding with Eq. (3.25), one can obtained as

$$-\frac{2}{V_p}\frac{\partial \Phi^{(1)}}{\partial T}-V_p^2\frac{\partial N_{hi}^{(2)}}{\partial X}+\frac{3}{V_p^2}\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial X}-\frac{\partial \Phi^{(2)}}{\partial X}+\frac{\mu_{hi}}{V_p}\frac{\partial^2 \Phi^{(1)}}{\partial X^2}=0. \tag{3.27}$$

Multiplying Eq. (3.26) by V_p^2 and Eq. (3.27) by N_{r1}, then adding, the Burgers Equation is obtained as

$$\frac{\partial \Phi^{(1)}}{\partial T}+B\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial X}=C\frac{\partial^2 \Phi^{(1)}}{\partial X^2}, \tag{3.28}$$

where

$$B=\left[\frac{V_p^3\delta_{ei}^2}{2N_{r1}}-\frac{V_p^3N_{r2}\Omega_2}{N_{r1}}-\frac{3}{2V_p}\right], C=\frac{\mu_{hi}}{2}. \tag{3.29}$$

It is to be noted that Eq. (3.28) divulges only the shock wave structures in plasmas. It noted that the nonlinear coefficient B is in good agreement with the Ref. [118] for the presence of nonextensivity only, that is $\alpha=0$.

3.3.1 Exact solution of Burgers equation

The solution of Eq. (3.28) is obtained as

$$\Phi^{(1)}=\Phi_A\left[1-tan\,h\left(\frac{\xi}{\Phi_W}\right)\right], \tag{3.30}$$

where $\xi = X - V_r T$, V_r is the speed of the frame of reference, $\Phi_A = (V_r/B)$ is the amplitude and $\Phi_W = (2C/V_r)$ is width of shocks. The details calculation of finding Eq. (3.30) demonstrate in section 2.3.1. Based on the above solution, Eq. (3.30) is only applicable to study SWEs in the considered plasmas for $B > 0$ or $B < 0$. But its failed to address the SWEs when $B \rightarrow 0$, which provides one needs to derive another nonlinear evolution equations by taking more higher order correction into account.

3.4 Formation of modified Burgers equation

In order to study electrostatic DIASWs at the critical values (CVs), the stretching coordinates are considered as

$$X = \varepsilon^2 (z - V_p t), \qquad T = \varepsilon^4 t, \tag{3.31}$$

From Eq. (3.31), the operators are defined as

$$\frac{\partial}{\partial t} = \varepsilon^4 \frac{\partial}{\partial T} - \varepsilon^2 V_p \frac{\partial}{\partial X}, \quad \frac{\partial}{\partial z} = \varepsilon^2 \frac{\partial}{\partial X}. \tag{3.32}$$

Eqs. (3.6)-(3.8) are then converted with the aid of Eq. (3.32) to the following:

$$\varepsilon^4 \frac{\partial N_{hi}}{\partial T} - \varepsilon^2 V_p \frac{\partial N_{hi}}{\partial X} + \varepsilon^2 \frac{\partial}{\partial X}(N_{hi} U_{hi}) = 0, \tag{3.33}$$

$$\varepsilon^4 \frac{\partial U_{hi}}{\partial T} - \varepsilon^2 V_p \frac{\partial U_{hi}}{\partial X} + \varepsilon^2 U_{hi} \left(\frac{\partial U_{hi}}{\partial X}\right) - \varepsilon^2 \frac{\partial \Phi}{\partial X} - \varepsilon^4 \mu_{hi} \frac{\partial^2 U_{hi}}{\partial X^2} = 0, \tag{3.34}$$

and

$$\varepsilon^4 \frac{\partial^2 \Phi}{\partial X^2} = N_{r2} \left\{ [1 + (q-1)\Phi]^{\frac{q+1}{2(q-1)}} \times \left[1 + B_1 \Phi + B_2 \Phi^2\right] \right\} + N_{r1} N_{hi} - e^{-\delta_{ei}\Phi} + N_{r3}. \tag{3.35}$$

By applying the values from Eq. (3.15) into Eqs. (3.6)-(3.8) in a systematic way, one can coverts Eqs. (3.6)-(3.8) by including the different orders of ε, that is $O(\varepsilon^r)$, $r = 3,4,5, \cdots\cdots$. For $O(\varepsilon^3)$, one obtains the similar equations as in Eqs. (3.16)-(3.18) and their solutions are given in Eq. (3.19). The linear phase velocity is obtained in the same form as in Eq. (3.20).

For $O(\varepsilon^4)$:

$$-V_p\frac{\partial N_{hi}^{(2)}}{\partial X}+\frac{\partial U_{hi}^{(2)}}{\partial X}+\frac{\partial}{\partial X}\left(N_{hi}^{(1)}U_{hi}^{(1)}\right)=0, \tag{3.36}$$

$$-V_p\frac{\partial U_{hi}^{(2)}}{\partial X}+U_{hi}^{(1)}\frac{\partial U_{hi}^{(1)}}{\partial X}-\frac{\partial \Phi^{(2)}}{\partial X}=0, \tag{3.37}$$

and

$$N_{r2}\Omega_1\Phi^{(2)}+N_{r2}\Omega_2\left[\Phi^{(1)}\right]^2+\delta_{ei}\Phi^{(2)}-\delta_{ei}^2\left[\Phi^{(1)}\right]^2+N_{r1}N_{hi}^{(2)}=0. \tag{3.38}$$

Again, Eqs. (3.36)-(3.38) provides the following equations by inserting the value from Eqs. (3.19):

$$-V_p\frac{\partial N_{hi}^{(2)}}{\partial X}+\frac{\partial U_{hi}^{(2)}}{\partial X}+\frac{2}{V_p^3}\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial X}=0 \tag{3.39}$$

$$-V_p\frac{\partial U_{hi}^{(2)}}{\partial X}+\frac{1}{V_p^2}\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial X}-\frac{\partial \Phi^{(2)}}{\partial X}=0, \tag{3.40}$$

and

$$N_{r2}\Omega_1\Phi^{(2)}+N_{r2}\Omega_2\left[\Phi^{(1)}\right]^2+\delta_{ei}\Phi^{(2)}-\delta_{ei}^2\left[\Phi^{(1)}\right]^2+N_{r1}N_{hi}^{(2)}=0. \tag{3.41}$$

Now, the solutions of Eqs. (3.39) and (3.40) are derived as follows:

$$N_{hi}^{(2)}=\frac{1}{V_p^2}\left[\frac{3}{2V_p^2}\{\Phi^{(1)}\}^2-\Phi^{(2)}\right], \tag{3.42}$$

$$U_{hi}^{(2)}=\frac{1}{V_p}\left[\frac{1}{2V_p^2}\{\Phi^{(1)}\}^2-\Phi^{(2)}\right], \tag{3.43}$$

and inserting the values from Eqs. (3.42) and (3.43) in Eq. (3.41), one can obtained as

$$\left[N_{r2}\Omega_2-\frac{1}{2}\delta_{ei}^2+\frac{3N_{r1}}{2V_p^4}\right]\left\{\Phi^{(1)}\right\}^2+\left[N_{r2}\Omega_1+\delta_{ei}-\frac{N_{r1}}{V_p^2}\right]\Phi^{(2)}=0$$

yields

$$-C'_f\left\{\Phi^{(1)}\right\}^2=0, \tag{3.44}$$

where

$$C'_f=N_{r2}\Omega_2-\frac{1}{2}\delta_{ei}^2+\frac{3N_{r1}}{2V_p^4}.$$

For $O(\varepsilon^5)$:

$$\frac{\partial N_{hi}^{(1)}}{\partial T} - V_p \frac{\partial N_{hi}^{(3)}}{\partial X} + \frac{\partial U_{hi}^{(3)}}{\partial X} + \frac{\partial}{\partial X}\left(N_{hi}^{(1)} U_{hi}^{(2)}\right) + \frac{\partial}{\partial X}\left(N_{hi}^{(2)} U_{hi}^{(1)}\right) = 0, \tag{3.45}$$

$$\frac{\partial U_{hi}^{(1)}}{\partial T} - V_p \frac{\partial U_{hi}^{(3)}}{\partial X} + U_{hi}^{(1)} \frac{\partial U_{hi}^{(2)}}{\partial X} + U_{hi}^{(2)} \frac{\partial U_{hi}^{(1)}}{\partial X} - \frac{\partial \Phi^{(3)}}{\partial X} - \mu_{hi} \frac{\partial^2 U_{hi}^{(1)}}{\partial X^2} = 0, \tag{3.46}$$

and

$$N_{r2}\Omega_1 \Phi^{(3)} + 2N_{r2}\Omega_2 \Phi^{(1)}\Phi^{(2)} + N_{r2}\Omega_3 \left[\Phi^{(1)}\right]^3 + \delta_{ei}^3 \left[\Phi^{(1)}\right]^3 + \delta_{ei}\Phi^{(3)} - 2\delta_{ei}^2 \Phi^{(1)}\Phi^{(2)} + N_{r1} N_{hi}^{(3)} = 0, \tag{3.47}$$

where

$$\Omega_3 = \frac{q+1}{2} B_2 + \frac{(q+1)(3-q)(5-3q)}{48} + \frac{(q+1)(3-q)}{8} B_1.$$

Now, inserting the values from Eqs. (3.19), (3.42) and (3.43) in Eqs. (3.45)-(3.47), the following equations derived:

$$-\frac{1}{V_p^2}\frac{\partial \Phi^{(1)}}{\partial T} - V_p \frac{\partial N_{hi}^{(3)}}{\partial X} + \frac{\partial U_{hi}^{(3)}}{\partial X} - \frac{6}{V_p^5}\left\{\Phi^{(1)}\right\}^2 \frac{\partial \Phi^{(1)}}{\partial X} + \frac{2}{V_p^3}\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right] = 0, \tag{3.48}$$

$$-\frac{1}{V_p}\frac{\partial \Phi^{(1)}}{\partial T} - V_p \frac{\partial U_{hi}^{(3)}}{\partial X} - \frac{\partial \Phi^{(3)}}{\partial X} - \frac{3}{2V_p^4}\left\{\Phi^{(1)}\right\}^2 \frac{\partial \Phi^{(1)}}{\partial X} + \frac{1}{V_p^2}\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right] + \frac{\mu_{hi}}{V_p}\frac{\partial^2 \Phi^{(1)}}{\partial X^2} = 0, \tag{3.49}$$

and

$$N_{r2}\Omega_1 \frac{\partial \Phi^{(3)}}{\partial X} + 2N_{r2}\Omega_2 \frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right] + 3N_{r2}\Omega_3 \left[\Phi^{(1)}\right]^2 \frac{\partial \Phi^{(1)}}{\partial X} + \delta_{ei}\frac{\partial \Phi^{(3)}}{\partial X} + 3\delta_{ei}^3 \left[\Phi^{(1)}\right]^2 \frac{\partial \Phi^{(1)}}{\partial X} + 2\delta_{ei}^2 \frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right] + N_{r1}\frac{\partial N_{hi}^{(3)}}{\partial X} = 0. \tag{3.50}$$

Multiplying Eq. (3.48) by V_p and then adding with Eq. (3.49), one can obtained as

$$-\frac{2}{V_p}\frac{\partial \Phi^{(1)}}{\partial T} - V_p^2 \frac{\partial N_{hi}^{(3)}}{\partial X} - \frac{15}{2V_p^4}\left\{\Phi^{(1)}\right\}^2 \frac{\partial \Phi^{(1)}}{\partial X} + \frac{3}{V_p^2}\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right] - \frac{\partial \Phi^{(3)}}{\partial X} + \frac{\mu_{hi}}{V_p}\frac{\partial^2 \Phi^{(1)}}{\partial X^2} = 0, \tag{3.51}$$

Finally, multiplying Eq. (3.50) by V_p^2 and Eq. (3.51) by N_{r1}, then adding, the following modified BE is find as:

$$\frac{\partial \Phi^{(1)}}{\partial T} + B'\left\{\Phi^{(1)}\right\}^2 \frac{\partial \Phi^{(1)}}{\partial X} = C\frac{\partial^2 \Phi^{(1)}}{\partial X^2}. \tag{3.52}$$

The nonlinear coefficient of Eq. (3.52) is determined as

$$B' = \frac{V_p^3}{2N_{r1}}\left(\frac{15N_{r1}}{2V_p^6} - \frac{1}{2}\delta_{ei}^3 - 3\Omega_3 N_{r2}\right), \qquad C = \frac{\mu_{hi}}{2}. \tag{3.53}$$

3.4.1 Solution of modified Burgers equation

The useful stationary shock wave solution of Eq. (3.52) is obtained as

$$\Phi^{(1)} = \sqrt{\Phi'_A\left\{1 - tan\,h\left(\frac{\xi}{\Phi'_W}\right)\right\}}\,, \tag{3.54}$$

where $\Phi'_A = (3V_r/2B)$ and $\Phi'_W = (C/V_r)$ are the amplitude and width of DIASWs approximately around CVs. But its failed to address the DIASWs when $B' \to 0$, which provides one needs to derive another nonlinear evolution equation by taking more higher order correction into account. Or, one can overcome such difficulty by the composition of Burgers and modified Burgers equations. The details calculation of finding Eq. (3.54) are shown in section 2.4.1.

3.5 Formation of mixed modified Burgers equation

When the amplitude Φ'_A is approaching to infinity at CVs, then Eq. (3.54) is not valuable to report the propagation of DIASWs at the critical composition CVs. To prevail over such obscurity, one can take C'^0_f for q (say) around its q_c as

$$C'^0_f = \left(\frac{\partial C'_f}{\partial q}\right)_{q=q_c} |q - q_c| = SG\varepsilon\,, \tag{3.55}$$

where $|q - q_c| \equiv \varepsilon$ (because $|q - q_c|$ is small quantity). $S = 1(-1)$ for $q > q_c (q < q_c)$ and $G = \left(\frac{\partial C'_f}{\partial q}\right)_{q=q_c}$. As a result, one can re-evaluate Eqs. (3.45)-(3.47) by adding $\rho^{(2)} = \varepsilon^3 \frac{1}{2} SG\Phi^2$ and yields

$$-\frac{1}{V_p^2}\frac{\partial \Phi^{(1)}}{\partial T} - V_p \frac{\partial N_{hi}^{(3)}}{\partial X} + \frac{\partial U_{hi}^{(3)}}{\partial X} - \frac{6}{V_p^5}\left\{\Phi^{(1)}\right\}^2 \frac{\partial \Phi^{(1)}}{\partial X} + \frac{2}{V_p^3}\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right] = 0, \tag{3.56}$$

$$-\frac{1}{V_p}\frac{\partial\Phi^{(1)}}{\partial T}-V_p\frac{\partial U_{hi}^{(3)}}{\partial X}-\frac{\partial\Phi^{(3)}}{\partial X}-\frac{3}{2V_p^4}\left\{\Phi^{(1)}\right\}^2\frac{\partial\Phi^{(1)}}{\partial X}+\frac{1}{V_p^2}\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right]+\frac{\mu_{hi}}{V_p}\frac{\partial^2\Phi^{(1)}}{\partial X^2}$$
$$=0, \quad (3.57)$$

and

$$N_{r2}\Omega_1\frac{\partial\Phi^{(3)}}{\partial X}+SG\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}+2N_{r2}\Omega_2\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right]+3N_{r2}\Omega_3\left[\Phi^{(1)}\right]^2\frac{\partial\Phi^{(1)}}{\partial X}$$
$$+\delta_{ei}\frac{\partial\Phi^{(3)}}{\partial X}-2\delta_{ei}^2\frac{\partial}{\partial X}\left[\Phi^{(1)}\Phi^{(2)}\right]+N_{r1}\frac{\partial N_{hi}^{(3)}}{\partial X}=0. \quad (3.58)$$

Finally, multiplying Eq. (3.58) by V_p^2 and Eq. (3.51) by N_{r1}, then adding, the following mixed modified BE is derived:

$$\frac{\partial\Phi^{(1)}}{\partial T}+SD\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial X}+B'\left\{\Phi^{(1)}\right\}^2\frac{\partial\Phi^{(1)}}{\partial X}=C\frac{\partial^2\Phi^{(1)}}{\partial X^2}, \quad (3.59)$$

where

$$D=\frac{V_p^3}{2N_{r1}}\left(\frac{\partial C_f}{\partial q}\right)_{q=q_c}, \quad (3.60)$$

One can easily convert mixed modified BE not only to modified BE but also to Burgers equation. Note that Eq. (3.59) is applicable to study DIASWs not only around CVs but also at CVs.

3.5.1 Solution of mixed modified Burgers equation

The solution of Eq. (3.59) is defined as

$$\Phi^{(1)}=\sqrt{\Phi_{mA}\left\{1-\tan h\left(\frac{\xi}{\Phi_{mW}}\right)\right\}}-\frac{SD}{2B'}, \quad (3.61)$$

where

$$\Phi_{mA}=\frac{3}{2B'}\left(V_r+\frac{S^2D^2}{4B'}\right) \quad \text{and} \quad \Phi_{mW}=\frac{C}{V_r+\frac{S^2D^2}{4B'}}. \quad (3.62)$$

where, Φ_{mA} and Φ_{mW} are the amplitude and width of IASWs at the critical values, respectively. The details calculation of finding Eq. (3.61) are displayed in section 2.5.1.

3.6 Results and discussions

An unmagnetized dusty plasma system considered as composed of negatively charged stationary dust, (α, q)-distributed electrons, positively charged Maxwellian light ions, and negatively charged inertial heavy ions. If $\alpha = 0$, then the considered plasma system becomes non-extensive dusty multi-ion plasma, which is in good agreement with the earlier investigation in Ref. 118. The nonlinear propagation of DIASWs phenomena in such plasmas has been investigated by changing the parametric values of the parameters. In addition, the article has mentioned the conditions under which DIASWs phenomena are supported. To do so, the reductive perturbation technique has been employed to formulate Burger equation, modified BE, and mixed modified BE. The outcomes for the nonlinear propagation of electrostatic DIASWs found in this work can be summarized below.

It is clearly observed that the critical values (CVs) can be easily evaluated by putting $C_f = 0$. Figure 3.1 shows the variation of CVs for $q = q_c$ with regard to N_{r1} and δ by considering the other parameters constant. For instance, C_f with the parametric values $\alpha = 0$, $N_{r1} = 0.5$, $N_{r2} = 0.05$ and $\delta = 0.01$ yields the CV for q as $q_c = 5.066562171$. Also, Figure 3.1 shows CVs with the variation of δ and N_{r1}. This figure clearly indicates that CVs are only supported for the case of subthermality, which is in good agreement with the Ref. 118.

Figures 3.2 and 3.3 present the roles of nonthermality, isothermality, superthermality and subthermality by varying α and q on DIASWs described by Burgers equation (3.30). It is clearly observed that the considered plasmas are supported only by rarefactive DIASWs with the presence of nonthermal, isothermal, and superthermal electrons, whereas both compressive and rarefactive DIASWs are supported by the presence of subthermal electrons. It is provided that BE is invalid to study the propagation of DIASWs when $B \to 0$ only in the case of subthermality. As a result, it is only possible to investigate the fundamental characteristics (polarity, amplitude, width, etc.) of DIASWs by depending on the higher-order BEs and the strength of electron nonextensivity ($q > 1$). The influences of δ and N_{r1} (N_{r2}) with the presence of subthermality on the nonlinear propagation of DIASWs are displayed by the solution of Eq. (3.30) in Figures 3.4 and 3.5, respectively. These figures clearly show that the amplitude of electrostatic shock increases (decreases) with the increase in subthermal electron temperature and heavy ion number density (subthermal electron number density). Since the heavy ions contribute to producing the driving force in the plasma system, whereas the lighter species produce the restoring force through their pressure, with the increase of heavy ion number density, the contribution of driving force must be increase. Thus, the driving force (restoring force) dominates rather than the restoring force (driving force) with the increase of heavy ion number density (with the increase of subthermal electron temperature but minimizing its density). Figure 3.6 shows the variation of the width of electrostatic shocks with regards to μ and V_r by the solution of Eq. (3.30). It is observed from Figure 3.6 that the widths of shocks increase with the increase of the viscosity coefficient of heavy-ions, width of shock decreases with the increase of the reference speed, as expected.

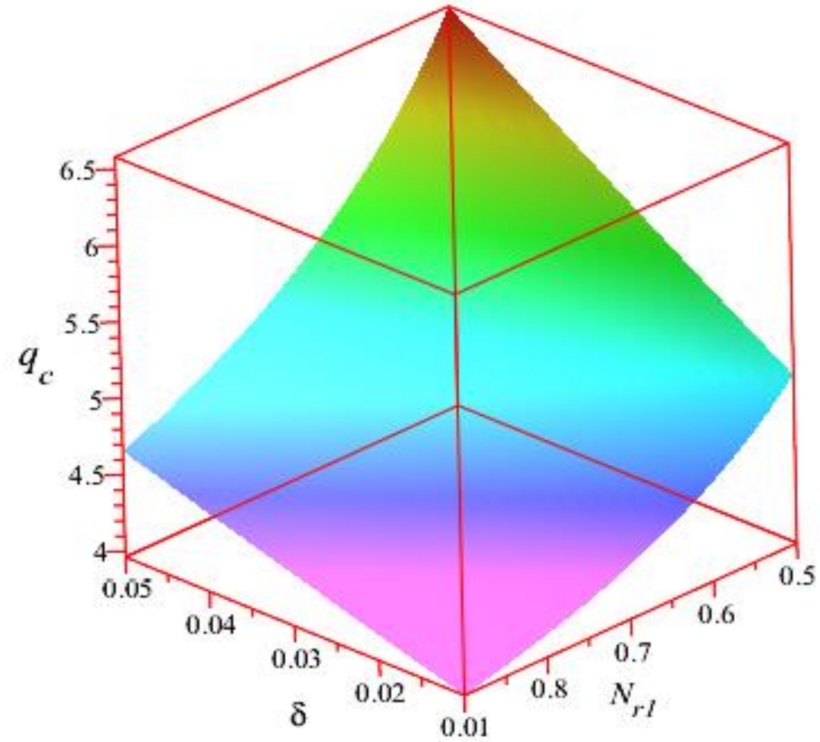
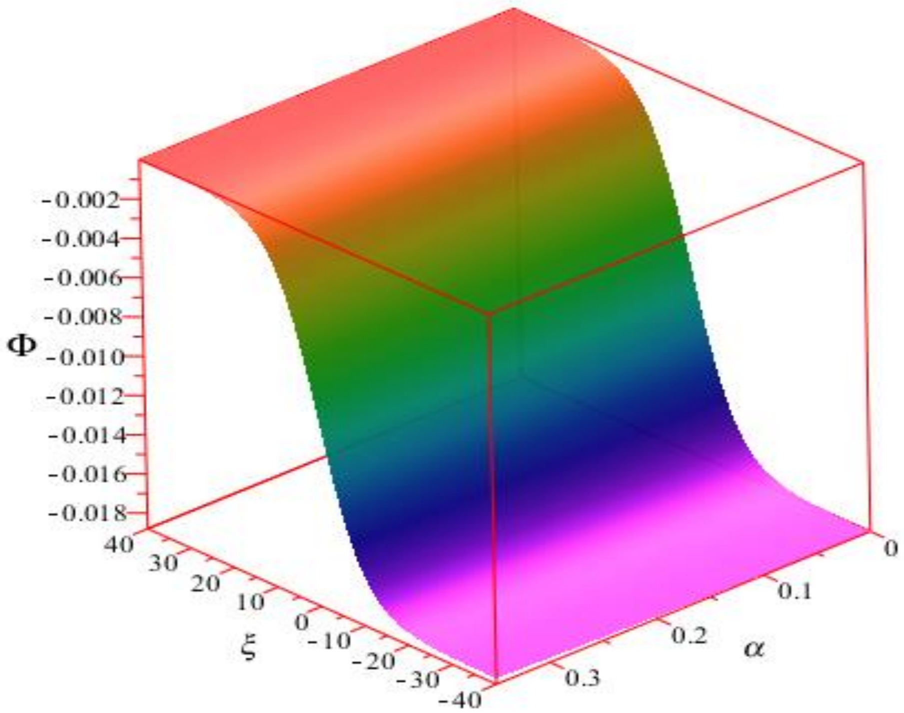
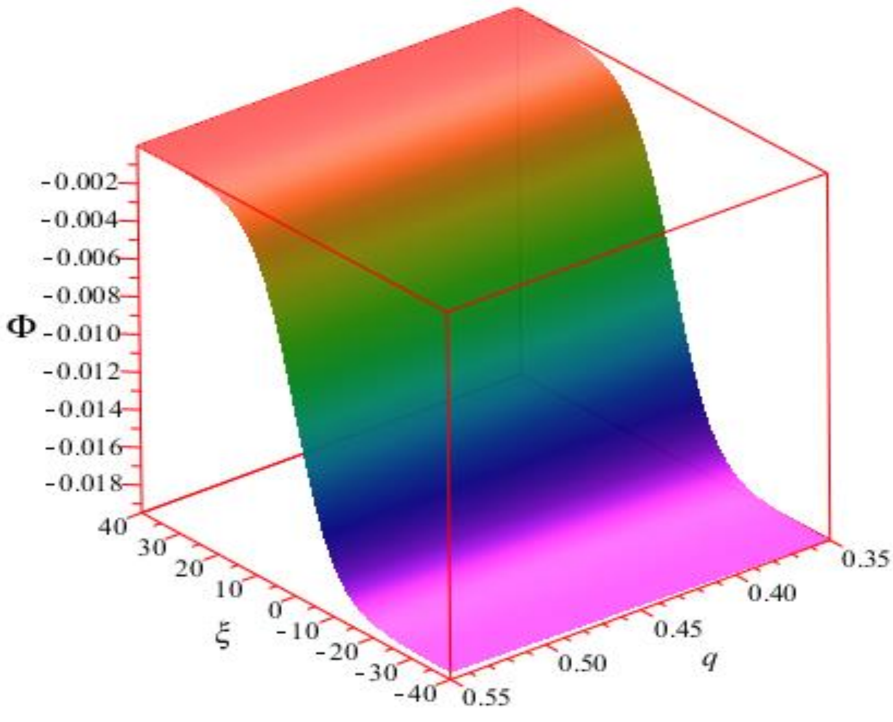


Figure 3.1: Variation of q_c with regard to δ and N_{r1} by taking $B = 0$ in Eq. (3.29). The other parameters are chosen as $\alpha = 0$ and $N_{r2} = 0.05$.

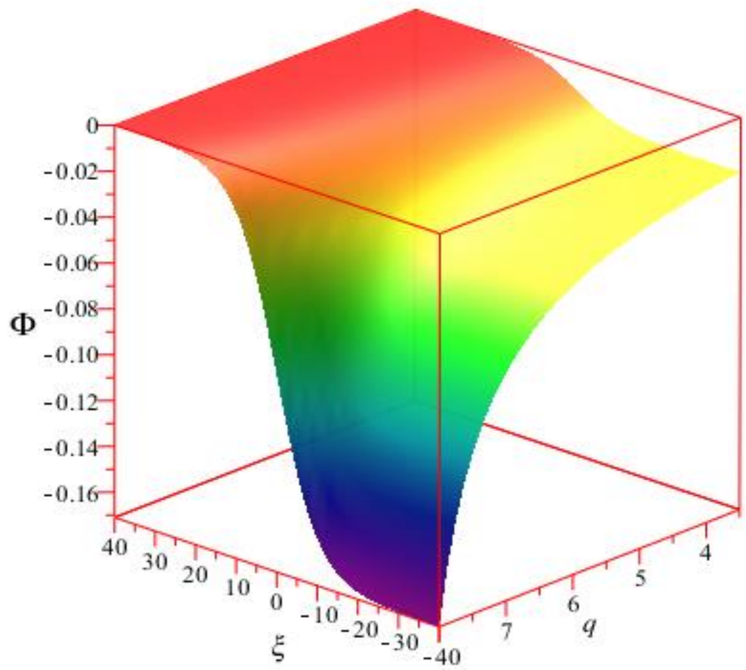


(a)

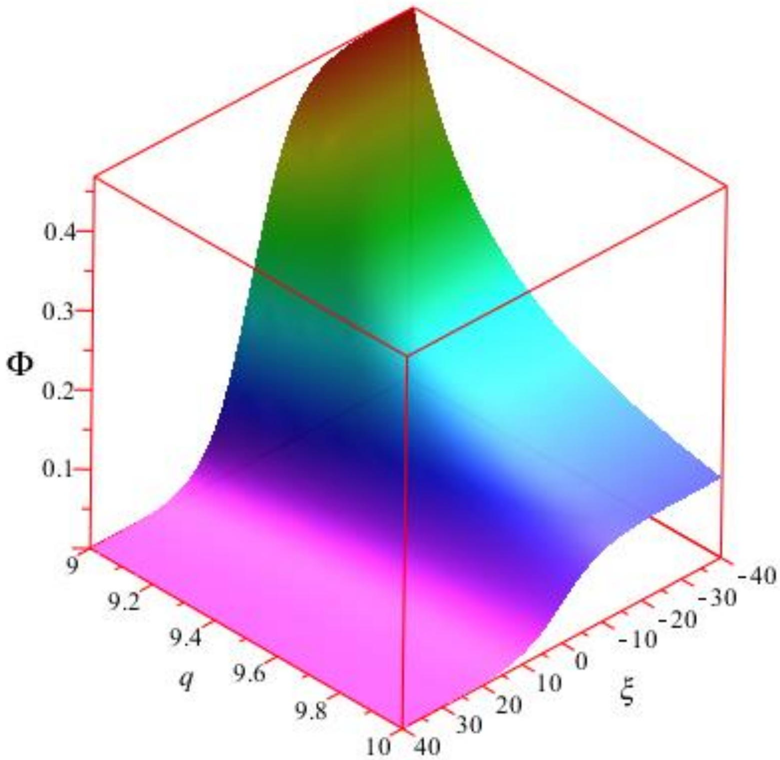


(b)

Figure 3.2: Effect of (a) population of nonthermal electrons ($q = 1$) and (b) superthermal electrons ($\alpha = 0$) on DIASW profiles. The other fixed parametric values are $N_{r1} = 0.5$, $N_{r2} = 0.05$, $\mu = 0.1$, $\delta = 0.1$ and $V_r = 0.01$.

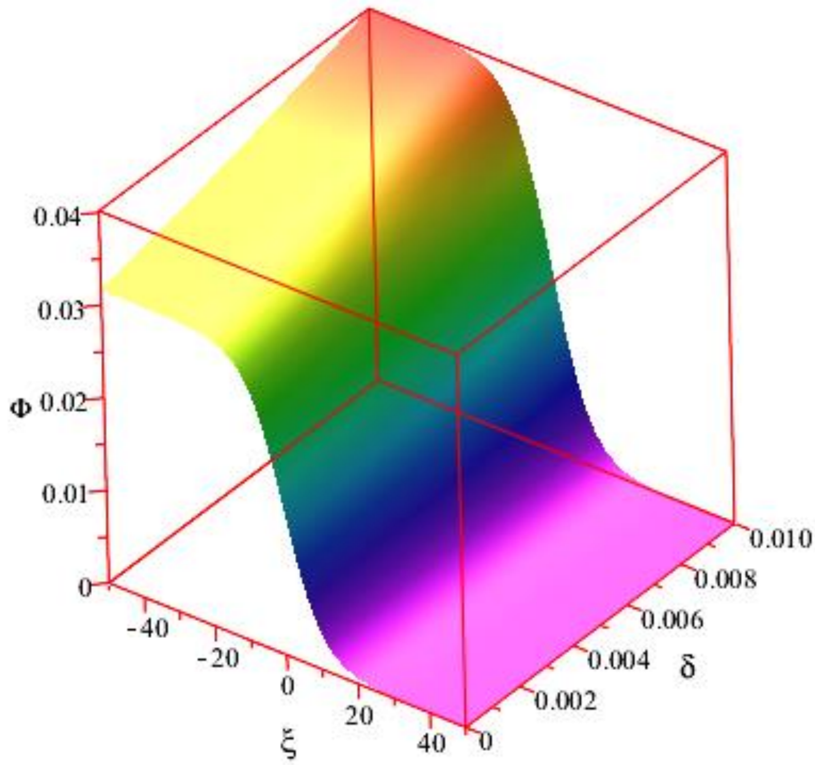


(a)

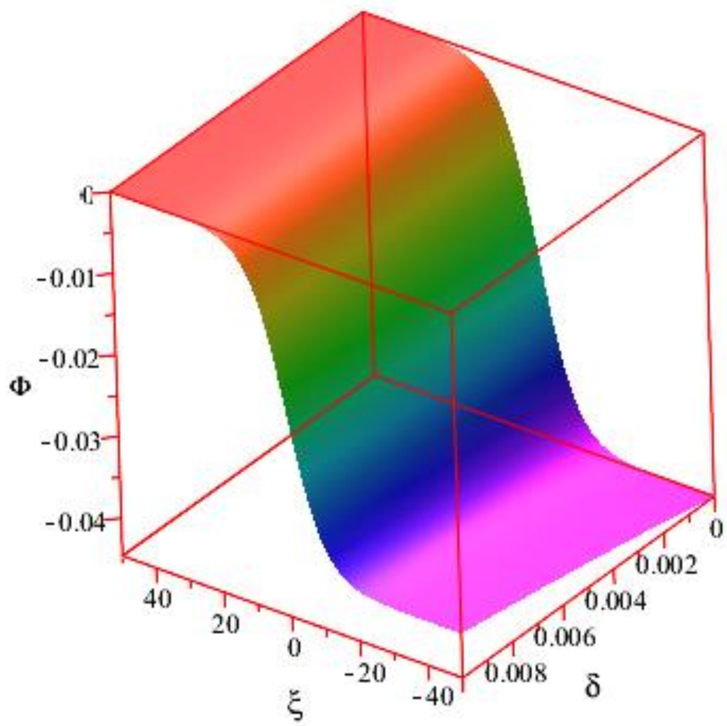


(b)

Figure 3.3: Effect of subthermal electrons ($\alpha = 0$) on (a) compressive DIASW and (b) rarefactive DIASW profiles. The remaining parameters are selected as $\alpha = 0$, $N_{r1} = 0.5$, $N_{r2} = 0.05$, $\mu = 0.1$, $\delta = 0.1$ and $V_r = 0.01$.

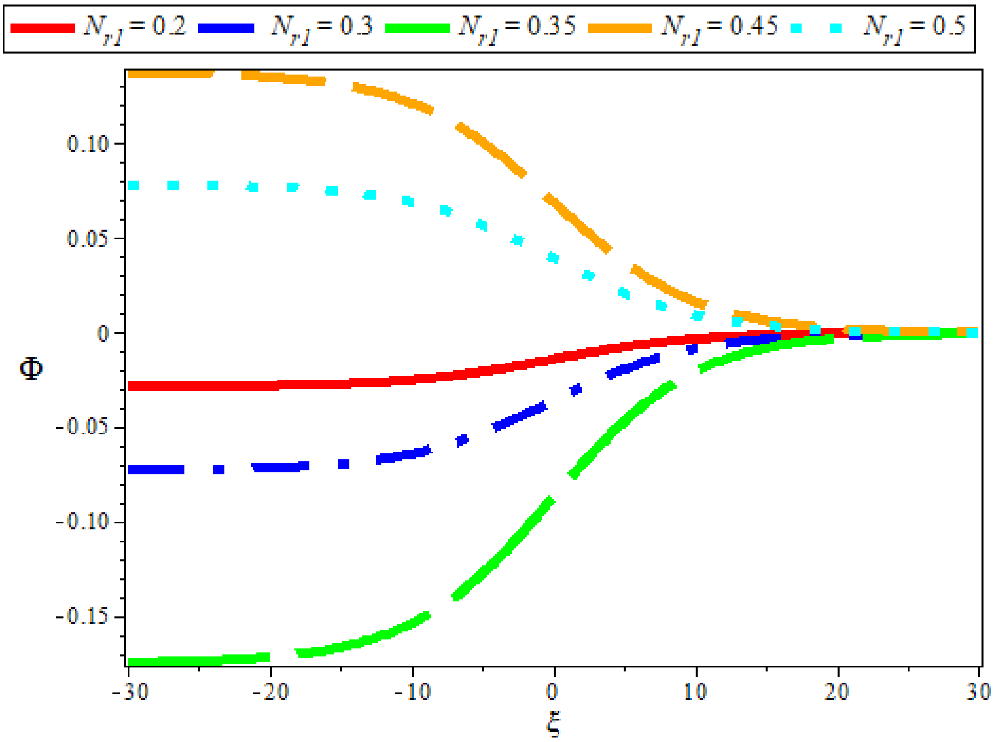


(a)

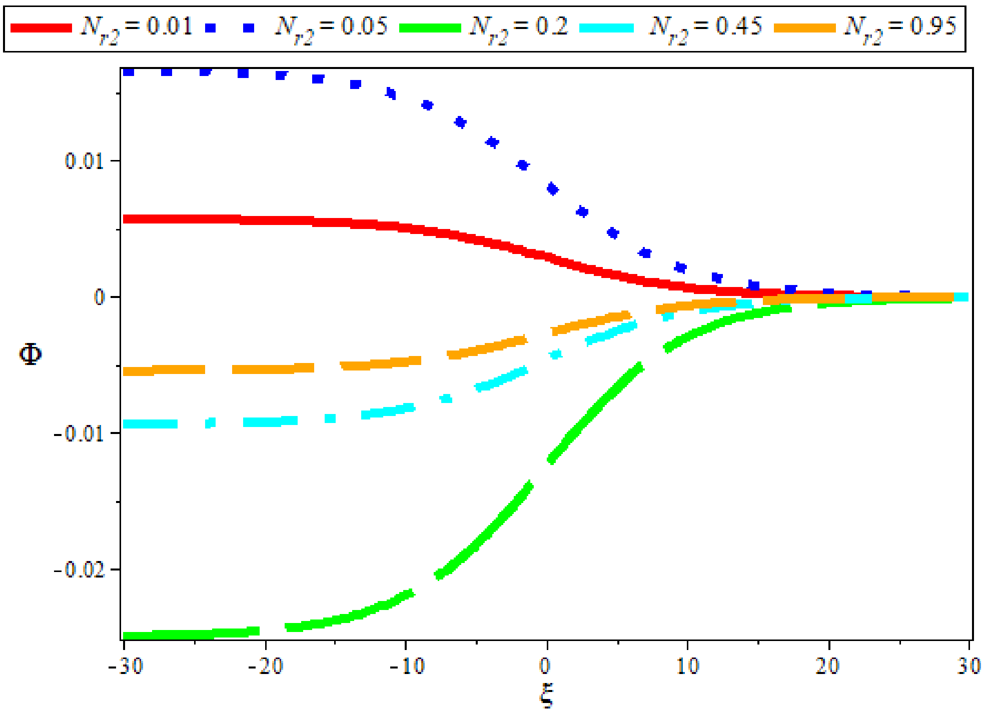


(b)

Figure 3.4: Variation of positive (negative) electrostatic shocks with regards to ξ and δ for (a) $q = 7$, and (b) $q = 3.5$. The other parameters are chosen as $\alpha = 0$, $N_{r1} = 0.5$, $N_{r2} = 0.05$, $\mu = 0.1$, and $V_r = 0.01$.



(a)



(b)

Figure 3.5: Electrostatic DIA shocks for different values of (a) N_{r1} with $\alpha = 0$, $\delta = 0.01$, $\mu = 0.1$, $q = 6$, $N_{r2} = 0.05$ and $V_r = 0.01$, and (b) N_{r2} with $\alpha = 0$, $\delta = 0.01$, $\mu = 0.1$, $q = 9$, $N_{r1} = 0.6$ and $V_r = 0.01$.

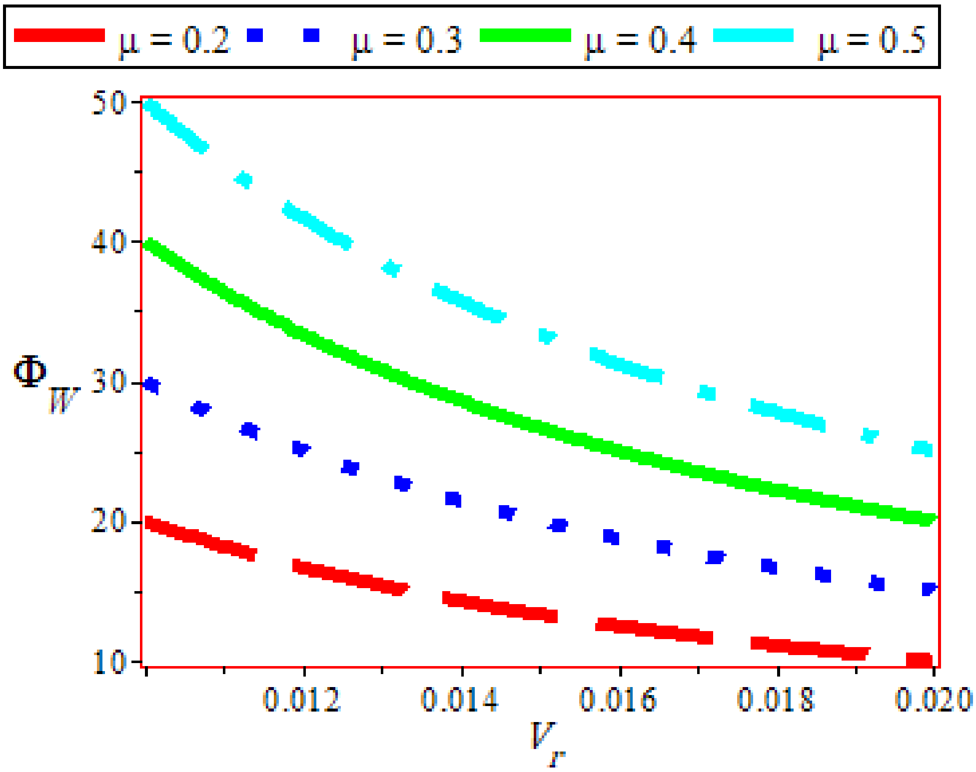
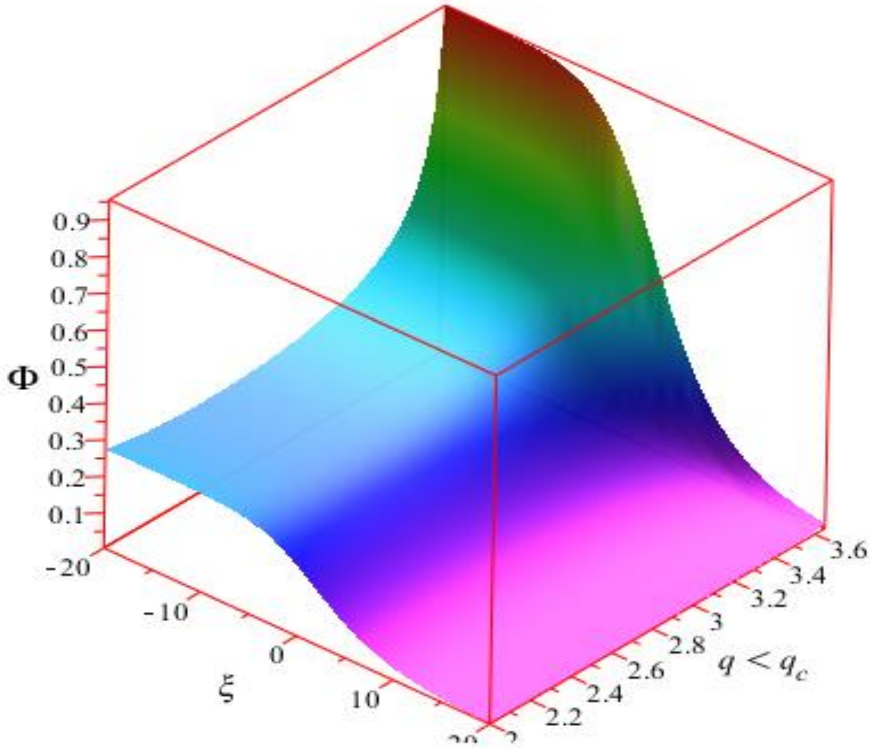
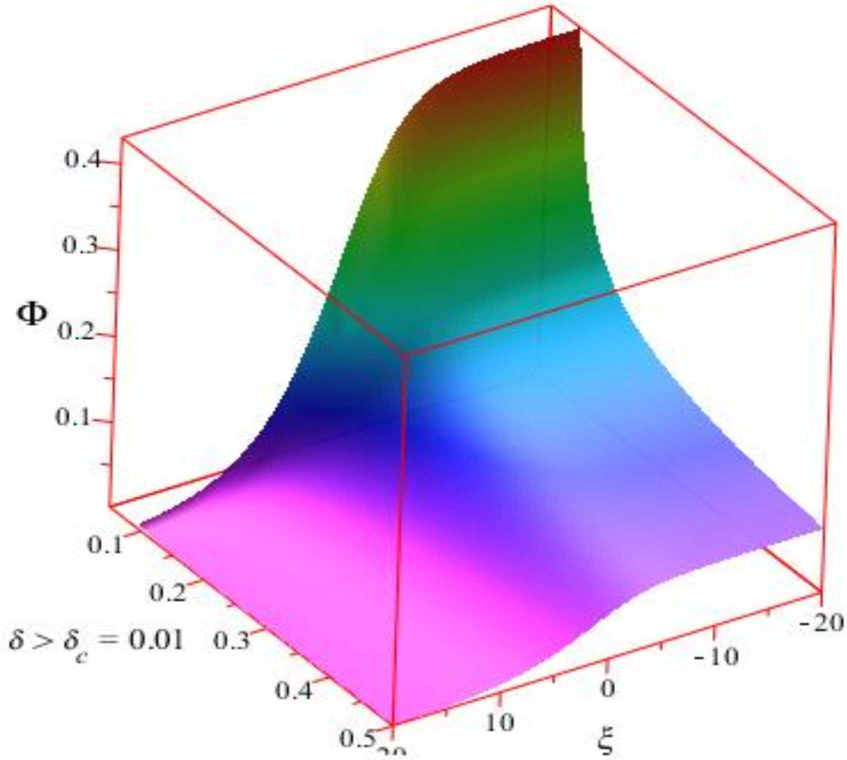


Figure 3.6: Electrostatic DIA shocks width with regards to V_r and the variation of μ. The remaining parameters are considered as $\alpha = 0$, $N_{r1} = 0.01$, $N_{r2} = 0.05$ and $q = 7$.

Figures 3.7(a) and 3.7(b) (Figures 3.8(a) and 3.8(b)) show the electrostatic DIASWs with regards to ξ and q, and ξ and δ (ξ and N_{r1}, and ξ and N_{r2}), respectively around the CVs. It is found from these Figures that the positive formation of DIASWs is developed in the considered plasmas for the cases of (i) q less than from its CVs (q_c), (ii) δ greater than from its CVs (δ_c), (iii) N_{r1} less than from its CVs (N_{r1C}), and (iv) N_{r2} greater than from its CVs (N_{r2C}). Otherwise, it is not possible for predicting what happens with the electrostatic DIASWs in the considered plasma system because the amplitude of DIASWs around CVs becomes complex. That is why, the mixed modified BE is needed to overcome such complexity. It is also found from Figures 3.7 and 3.8 that the maximum amplitude of shocks are occurred very closed to the CVs.

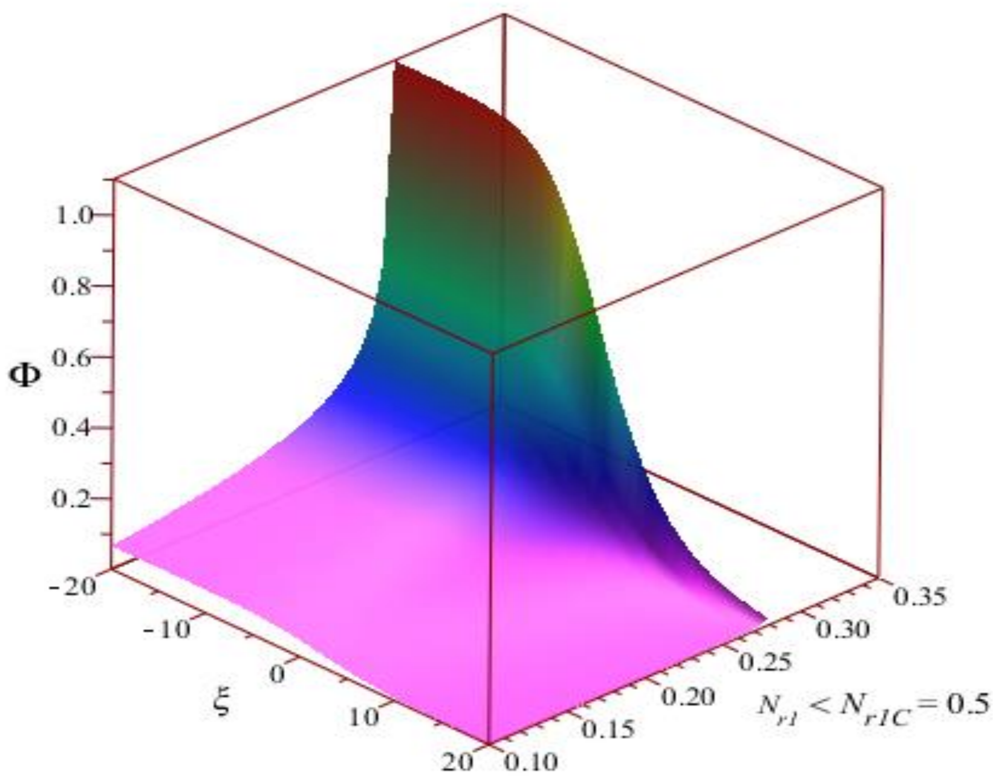


(a)

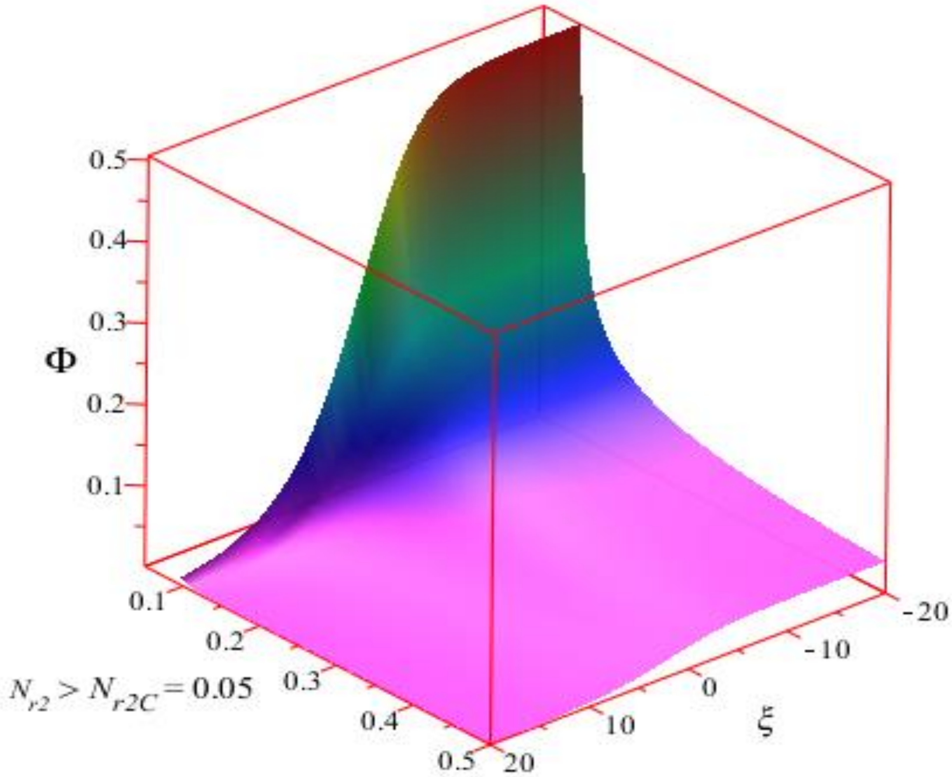


(b)

Figure 3.7: Variation of mB shock profiles with regards to (a) ξ and q around CV (q_c =5.066562171), and (b) ξ and δ around CV (δ_c).



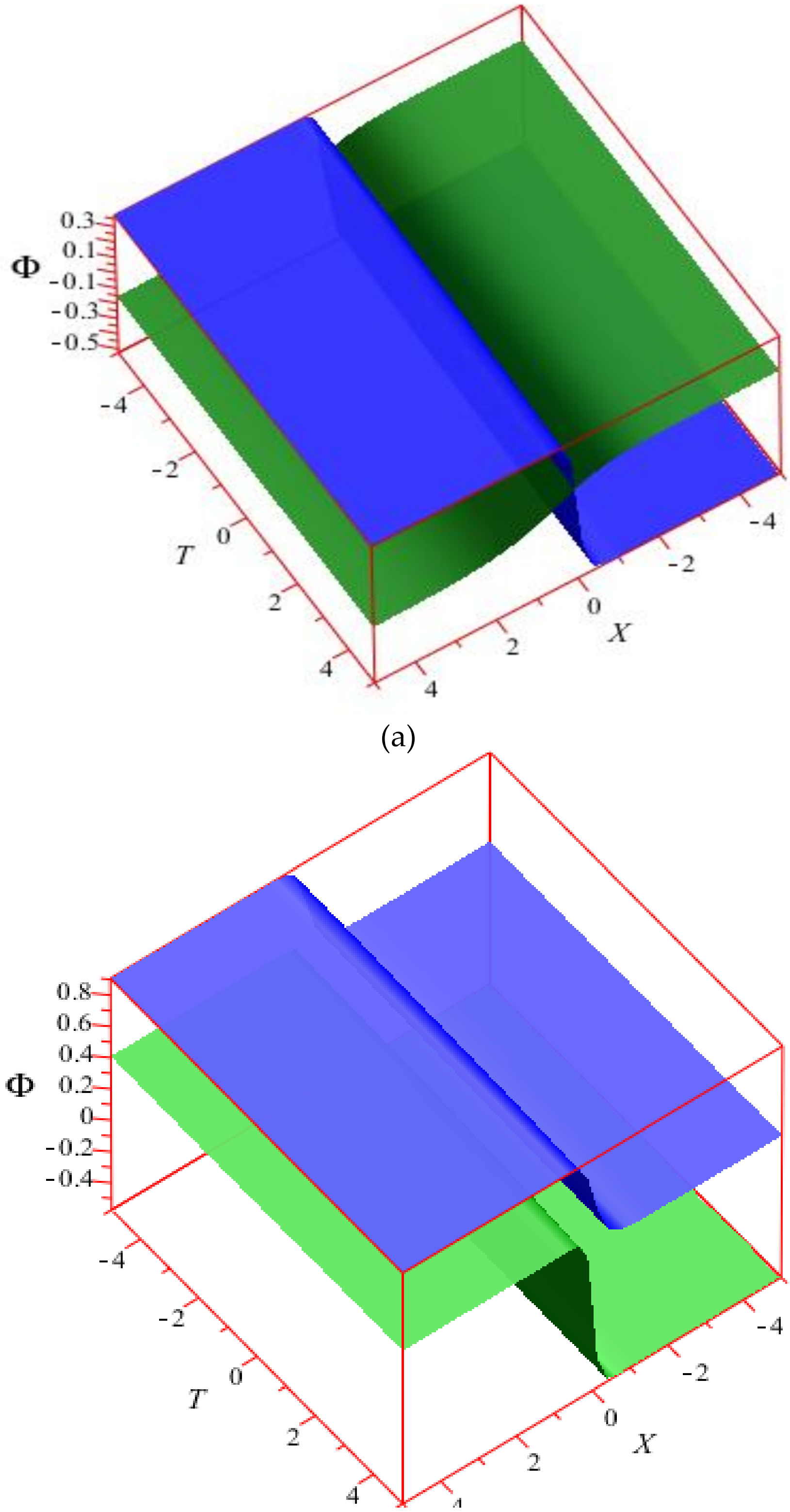
(a)



(b)

Figure 3.8: Variation of mB shock profiles with regards to (a) ξ and N_{r1} around CV (N_{r1C}), and (b) ξ and N_{r2} around CV (N_{r2C}).

Figure 3.9 displays the electrostatic shocks and double layer very close to the critical composition. It is found that the considered plasma system is supported electrostatic shocks very close to the critical composition and at the critical composition, but the double layer is only produced for the case of $\delta = 0.195 > \delta_c$. It is also found that the electrostatic shocks and double layer described by the mixed modified BE are supported very close to the critical composition and at the critical composition by depending on all the related parameters, except the kinematic viscosity coefficient (μ) and reference speed (V_r). Since $C_f = N_{r2}\Omega_2 - \frac{1}{2}\delta_{ei}^2 + \frac{3N_{r1}}{2V_p^4}$ is independent of μ and V_r. However, both the amplitude and width of IASWs described by the mixed modified Burgers equation are strongly dependent on μ and V_r. That is why the influence of μ and V_r on the electrostatic shocks and double layer is displayed in Figure 3.10. It is interesting to found that the thickness of monotonically shocks and double layer are increased, but the amplitude remains unchanged with the increase of μ. In addition, both of amplitude and thickness of monotonically shocks and double layer are remarkably effected by the variation of V_r. Because, both of amplitude and thickness of electrostatic shocks (double layer) are decreased (increased) with the increase of V_r with the presence of subthermal electrons.



(a)

(b)

Figure 3.9: Electrostatic **DIA** shock profile with X and T for (a) $\delta = 0.009 < \delta_c = 0.01$ (blue surface) and $\delta = 0.2 > \delta_c = 0.01$ with $\alpha = 0$, $q = 5.066562171$, $N_{r1} = 0.5$, $N_{r2} = 0.05$, $\mu = 0.1$, and $V_r = 0.01$ and (b) $q = 5 < q = 5.066562171$ (green surface) and $q = 5.5 > q = 5.066562171$ with $\alpha = 0$, $\delta = 0.01$, $N_{r1} = 0.5$, $N_{r2} = 0.05$, $\mu = 0.1$, and $V_r = 0.01$.

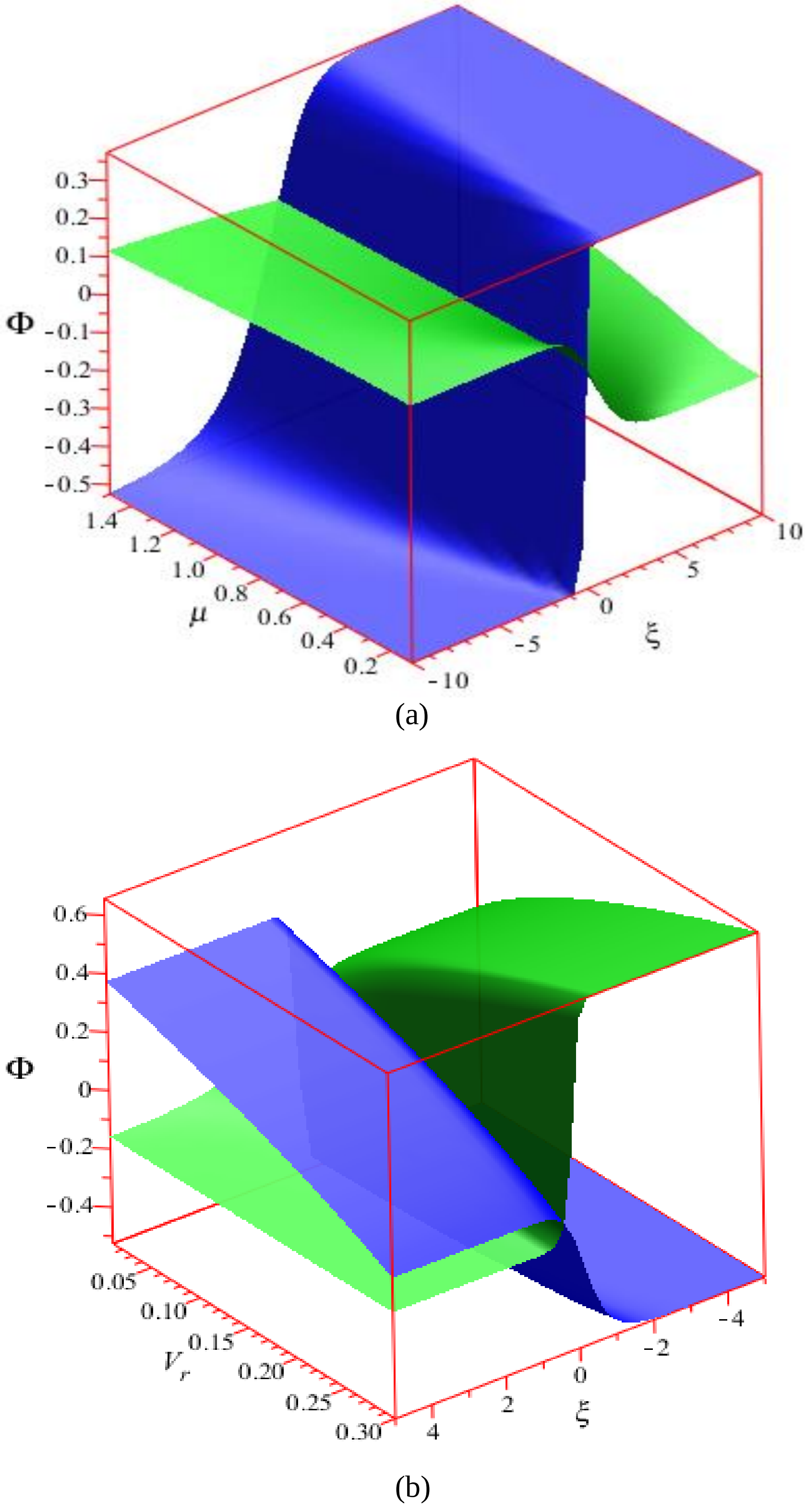


Figure 3.10: Effect of (a) viscosity coefficient ($\mu = 0.1$) and (b) reference speed ($V_r = 0.01$) on the electrostatic DIA shock profile by considering $\delta = 0.009 < \delta_c = 0.01$ (blue surface) and double layer by considering $\delta = 0.2 > \delta_c = 0.01$. The remaining parametric values are $\alpha = 0$, $q = 5.066562171$, $N_{r1} = 0.5$ and $N_{r2} = 0.05$.

3.7 Concluding remarks

It is concluded that the outcome of this study would be helpful for comprehending nonthermality and nonextensivity effects in laboratory plasmas as well as interstellar and space plasmas (particularly in protoneutron stars, dark-matter halos, stellar polytropes, hadronic matter, quark-gluon plasma, and other objects). It is remarkable to be noted that the reinvestigating outcomes with the presence of subthermal electrons are represented the actual scenarios of the dynamics of electrostatic shocks and double layers not only around the CVs but also at the critical compositions CVs. It is also provided in this article that the appropriate solutions of higher-order BEs are essential to be predicted not only the nature of electrostatic shocks and double layers but also the propagation of electrostatic shocks and double layers in further laboratory verification. Thus, it may be suggested to conduct a laboratory experiment that will be able to distinguish the unique new features of the DIASWs propagating in the dusty multi ion plasmas with the presence of all electrons energy cases.

Chapter 4

Dust-ion-acoustic shock wave excitations at super-critical points with quartic nonlinearity

4.1 Introduction

The presence of dust particles along with other charged particles is well confirmed in the majority of astrophysical and space plasma systems (ASPSs), e.g. the Earth's ionosphere, planetary environments, interstellar media, protostellar disks, molecular clouds, asteroid areas, comet tails, and nebula [3-6]. So, it is possible to study the dust-ion acoustic (DIA) or DA wave features in ASPSs. Moreover, the basic features of arbitrary and small but finite amplitudes propagation of DIA waves are still mostly focused issues for better understanding the mechanism of dusty plasmas (DPs) [6, 62, 93, 97, 120]. Shukla and Silin [6] originally proposed a theoretical explanation for the presence of low-frequency DIA waves in ASPSs. In their foundational investigation, they have mentioned that probably the presence of dense, massive, and immobile charged dust particles in electron-ion plasmas has a substantial impact on the motion of the waves. It is mainly occurred during a period of time that is likewise significantly shorter than the dust plasma period by developing the low-frequency DIA waves [100]. But so far, according to the work of Shukla and Mamun [2], the stability of quasineutrality is impacted by the charged dust particles' effect, which is thought to be stationary. Also, a great deal of focus is still now to be paid for the investigation of acoustic waves in dusty multi-ion plasmas by astrophysics experts. Because both ASPSs [10, 101, 102, 104, 106] and plasma laboratories [67, 69, 107] have now successfully verified the existence of multi-ions, specifically positive ions (PIs) and negative ions (NIs). In many plasma environments, such as the Earth's ionosphere and cometary comae, the plasma is produced by a combination of PIs and NIs in response to electrons [106]. The resources for the establishment and implementation of PIs and NIs plasmas include neutral beam sources [67], plasma processing reactors [107], and laboratory studies [69] etc. Also, it was shown that negatively charged ions significantly improved in plasma etching than positively charged ions. As a result, the significance of negative ion plasmas improves to the field of plasma studies gradually.

On the other hand, the fundamental characteristics of ASPSs are impacted by the related plasma species velocity distribution (VD). The most commonly associated distribution in collisionless plasma is the Maxwellian velocity distribution (MVD). The VD of plasma particles in lab settings and ASPSs, however, is different from the MVD. The origin of high energy charged particles generally exhibit nonextensive-, kappa-, or non-thermal distributions which depart from MVD [15, 121, 122]. In accordance with the concept of traditional statistical dynamics, the interplay between the charged particles is therefore primarily short-range. However, many plasma grains interact with one another in surroundings across extended distances, or through long-range Coulomb interactions, where the extensible property is typically lost. For instance, the ring of Saturn provides a physical representation for the presence of the previously discussed complex plasma system [4]. It is obviously found from such work that various types of number densities and temperature are existed in Saturn ring, that is, (i) the ion number densities are denoted as $\rho_{N_{i0}}$ have the values for E ring, F ring and spokes are $10^1 cm^{-3}$, $10^1 - 10^2 cm^{-3}$, $0.1 - 10^2 cm^{-3}$ respectively, (ii) the dust number densities are denoted as $\rho_{N_{d0}}$ have the value for E ring, F ring and spokes are $10^{-7} - 10^{-8} cm^{-3}$, $< 30\ cm^{-3}$, $1\ cm^{-3}$ respectively and (iii) the temperature T have the value for E ring, F ring and spokes are $10^5 - 10^6 K$, $10^5 - 10^6 K$, 2×10^4 respectively [3]. Due to the above evidence, Ema et al. [118] proposed a very importance model by the mixture of heavy NIs having influence of viscosity, stationary dusts ions (SDIs) as well as inertialess electrons and PIs, where the electrons and PIs are assumed to follow nonextensive VD and MVD. They have studied the DIA shock wave excitations (SWEs) not only around critical but also at critical values by formulating the higher-order Burgers equations. But, they have overlooked what happens with the existence SWEs for the super-critical values (SCVs), which is also essential to require further laboratory studies. To the best of author's knowledge, no theoretical research work has also been up to that time made for describing the nature of SWEs with the existence of SCVs in any plasma environments.

Thus, the goal of the presented work is to explore the extension of DIA SWEs for SCVs from the proposed model in Ref. 118 by deriving new evolution equation (i.e. modified Burgers-type equation having quartic nonlinearity). Exacting questions to be answered are, e.g., (i) how to determine whether or not SCVs of any specific parameters exist for the considered model, (ii) which evolution equation applicable to study SWEs at this situation, (iii) what happens with the nature of SWEs not only around SCVs but also at the SCVs, (iv) how the existence regions of the SWEs affect by the plasma parameters, and (v) whether the temperature and density ratio's can modify the trajectories of SWEs. The results from this research are expected to contribute to the in-depth understanding of the nonlinear SWEs that may appear in the interplanetary as well as astrophysical plasmas in general and laboratory plasmas.

4.2 Theoretical model equations with plasma assumptions

To meet our objectives, the (1+1)-dimensional collisionless dusty multi-ion plasma system composing of heavy NIs having influence of viscosity, SDIs, Maxwellian PIs and nonextensive VD electrons is considered. In account of this, the equilibrium charge neutrality requirement is maintained by

$$\rho_{N_{i0}} - Z_{hi}\rho_{N_{hi0}} - \rho_{N_{e0}} - Z_d\rho_{N_{d0}} = 0, \tag{4.1}$$

where the species unperturbed number densities denote by $\rho_{N_{l0}}$, $l = i$ (Maxwellian PIs), hi (heavy NIs), e(nonextensive electrons) and Z_d signify how many electrons are available on the dusty particle surface. Based on the above consideration, the following dimensionless model equations [118] are considered to investigate whether or not DIA SWEs supports not only around SCVs but also at the SCVs:

$$\frac{\partial \rho_{N_{hi}}}{\partial t} + \frac{\partial(\rho_{N_{hi}} W_{hi})}{\partial x} = 0, \tag{4.2}$$

$$\frac{\partial W_{hi}}{\partial t} + W_{hi}\frac{\partial W_{hi}}{\partial x} = \frac{\partial \Psi}{\partial x} + \nu_{hi}\frac{\partial^2 W_{hi}}{\partial x^2}, \tag{4.3}$$

$$\frac{\partial^2 \Psi}{\partial x^2} = \rho_{N_{r2}}\left\{[1 + (q-1)]^{\frac{q+1}{2(q-1)}\Psi}\right\} + \rho_{N_{r1}}\rho_{N_{hi}} - e^{-\tau_{ei}\Psi} + \rho_{N_{r3}}, \tag{4.4}$$

Eqs. (4.2)-(4.4) are normalized by introducing

$$\left.\begin{aligned} &\frac{\rho_{N_{hi}}}{\rho_{N_{hi,0}}} \to \rho_{N_{hi}}, \frac{W_{hi}}{C_{hi}} \to W_{hi}, \frac{\Psi}{(k_B T_e/e)} \to \Psi, C_{hi} = \left(\frac{k_B T_e}{m_{hi}}\right)^{1/2}, \frac{x}{\lambda_{Dhi}} \to x, \frac{t}{\omega_{phi}^{-1}} \to t \\ &\frac{\nu_{hi}}{m_{hi}\rho_{N_{hi,0}}\omega_{phi}\lambda_{Dhi}^2} \to \nu_{hi}, \lambda_{dhi} = \left(\frac{k_B T_e}{4\pi e^2 \rho_{N_{hi,0}}}\right)^{1/2}, \omega_{phi}^{-1} = \left(\frac{m_{hi}}{4\pi e^2 \rho_{N_{hi,0}}}\right)^{1/2} \end{aligned}\right\}. \tag{4.5}$$

Due to the introducing of Eq. (4.5) in Eqs. (4.2)-(4.4), the density (temperature) ratio's are to be formed as $\rho_{N_{r1}} = \rho_{N_{e0}}/\rho_{N_{i0}}$ and $\rho_{N_{r2}} = Z_{hi}\rho_{N_{hi0}}/\rho_{N_{i0}}$ ($\tau_{ei} = T_e/T_i$). Here, $\rho_{N_{hi}}$ (W_{hi}, C_{hi}, ν_{hi}), Ψ, T_e (T_i), k_B (e) and λ_{dhi} (ω_{phi}^{-1}) denotes the heavy NIs density (fluid velocity, acoustic speed, viscosity coefficient), electrostatic wave potential, temperature of electrons (PIs), familiar Boltzmann constant (magnitude of the electron charge) and plasma Debye length (frequency). Additionally, the proper indexing values of q is measured as (i) superthermality (-1< $q < 1$), (ii) subthermal ($q > 1$) and (iii) isothermality ($q = 1$) for electrons, respectively.

4.3 Formation of modified Burgers-type equation having quartic nonlinearity

To derive a new evolution equation, the space and time variables can be stretched as

$$\zeta = \varepsilon^3\big(x - V_{ph}t\big),\ \ \eta = \varepsilon^6 t, \qquad 0 < \varepsilon < 1, \tag{4.6}$$

where V_{ph} is the propagation speed to be evaluated later. The above expressions yield

$$\frac{\partial}{\partial t} = \varepsilon^6\frac{\partial}{\partial \eta} - \varepsilon^3 V_{ph}\frac{\partial}{\partial \zeta}\ ,\qquad \frac{\partial}{\partial x} = \varepsilon^3\frac{\partial}{\partial \eta}\,. \tag{4.7}$$

For a small deviation from equilibrium state, one yields $\rho_{N}{}_{hi}^{(0)} = 1$ and $W_{hi}^{(0)} = \Psi^{(0)} = 0$. As a result, the growth of the perturbed quantities are considered [118] as

$$\begin{bmatrix}\rho_{N}{}_{hi}\\ W_{hi}\\ \Psi\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix} + \sum_{i}^{\infty}\varepsilon^i\begin{bmatrix}\rho_{N}{}_{hi}^{(i)}\\ W_{hi}^{(i)}\\ \Psi^{(i)}\end{bmatrix}. \tag{4.8}$$

Apply Eq. (4.7) and Taylor series expansion as in Eq. (4.8) into Eqs. (4.2)-(4.4), one can be converted Eqs. (4.2)-(4.4) by various orders of ε, namely $O(\varepsilon^r)$, $r = 4,5,6,\ldots\ldots\ldots$. The lowest order equations, that is, for $O(\varepsilon^4)$ equations yields [118] as follows

$$\rho_{N}{}_{hi}^{(1)} = -\frac{1}{V_{ph}^2}\Psi^{(1)}, W_{hi}^{(1)} = -\frac{1}{V_{ph}}\Psi^{(1)}, V_{ph} = \sqrt{\frac{\rho_{N}{}_{r1}}{\rho_{N}{}_{r2}\Lambda_1 + \tau_{ei}}}, \tag{4.9}$$

where V_{ph} is the linear phase velocity and $\Lambda_1 = (q+1)/2$. Again, the simplification of $O(\varepsilon^5)$ equations (ignored for simplicity) provides

$$\rho_{N}{}_{hi}^{(2)} = \frac{1}{V_{ph}^2}\left[\frac{3}{2V_{ph}^2}\{\Psi^{(1)}\}^2 - \Psi^{(2)}\right], W_{hi}^{(2)} = \frac{1}{V_{ph}}\left[\frac{1}{2V_{ph}^2}\{\Psi^{(1)}\}^2 - \Psi^{(2)}\right], \tag{4.10}$$

and

$$-\left[\frac{\rho_{N}{}_{r2}(q+1)(3-q)}{8} - \frac{1}{2}\tau_{ei}^2 + \frac{3\rho_{N}{}_{r1}}{2V_{ph}^4}\right]\{\Psi^{(1)}\}^2 = 0. \tag{4.11}$$

Eq. (4.11) yields

$$P_f = \rho_{N_{r2}}\Lambda_2 - \frac{1}{2}\tau_{ei}^2 + \frac{3\rho_{N_{r1}}}{2V_{ph}^4} = 0, \Lambda_2 = \frac{\rho_{N_{r2}}(q+1)(3-q)}{8}. \qquad (4.12)$$

It should be noted that Eq. (4.12) lists the critical values (CVs) of any plasma parameter which indicates the Burgers equation that formulated in Ref. 118 fails to adequately characterize the fundamental property of SWEs in the context of plasmas. As a result, one needs to consider higher order of ε equations for overcome such difficulty.

For $O(\varepsilon^6)$:

$$-V_{ph}\frac{\partial \rho_{N_{hi}}^{(3)}}{\partial \zeta} + \frac{\partial W_{hi}^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}\left(\rho_{N_{hi}}^{(1)} W_{hi}^{(2)}\right) + \frac{\partial}{\partial \zeta}\left(\rho_{N_{hi}}^{(2)} W_{hi}^{(1)}\right) = 0, \qquad (4.13)$$

$$-V_{ph}\frac{\partial W_{hi}^{(3)}}{\partial \zeta} + W_{hi}^{(1)}\frac{\partial W_{hi}^{(2)}}{\partial \zeta} + W_{hi}^{(2)}\frac{\partial W_{hi}^{(1)}}{\partial \zeta} - \frac{\partial \Psi^{(3)}}{\partial \zeta} = 0, \qquad (4.14)$$

and

$$\rho_{N_{r2}}\Lambda_1\Psi^{(3)} + 2\rho_{N_{r2}}\Lambda_2\Psi^{(1)}\Psi^{(2)} + \rho_{N_{r2}}\Lambda_3\left[\Psi^{(1)}\right]^3 + \tau_{ei}^3\left[\Psi^{(1)}\right]^3 + \tau_{ei}\Psi^{(3)} - 2\tau_{ei}^2\Psi^{(1)}\Psi^{(2)} + \rho_{N_{r1}}\rho_{N_{hi}}^{(3)} = 0, \qquad (4.15)$$

where

$$\Lambda_3 = \frac{(q+1)(3-q)(5-3q)}{48}.$$

For $O(\varepsilon^7)$:

$$\frac{\partial \rho_{N_{hi}}^{(1)}}{\partial \eta} - V_{ph}\frac{\partial \rho_{N_{hi}}^{(4)}}{\partial \zeta} + \frac{\partial \rho_{N_{hi}}^{(4)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}\left(\rho_{N_{hi}}^{(2)} W_{hi}^{(2)}\right) + \frac{\partial}{\partial \zeta}\left(\rho_{N_{hi}}^{(1)} W_{hi}^{(3)}\right) + \frac{\partial}{\partial \zeta}\left(\rho_{N_{hi}}^{(3)} W_{hi}^{(1)}\right) = 0, \qquad (4.16)$$

$$\frac{\partial W_{hi}^{(1)}}{\partial \eta} - V_{ph}\frac{\partial W_{hi}^{(4)}}{\partial \zeta} + W_{hi}^{(1)}\frac{\partial W_{hi}^{(3)}}{\partial \zeta} + W_{hi}^{(3)}\frac{\partial W_{hi}^{(1)}}{\partial \zeta} + W_{hi}^{(2)}\frac{\partial W_{hi}^{(2)}}{\partial \zeta} - \frac{\partial \Psi^{(4)}}{\partial \zeta} - \nu_{hi}\frac{\partial^2 W_{hi}^{(1)}}{\partial \zeta^2} = 0, \qquad (4.17)$$

and

$$(\rho_{N_{r2}}\Lambda_1+\tau_{ei})\Psi^{(4)}+(\rho_{N_{r2}}\Lambda_2-\tau_{ei}^2)\{\Psi^{(2)}\}^2+2(\rho_{N_{r2}}\Lambda_2-\tau_{ei}^2)\Psi^{(1)}\Psi^{(3)}$$
$$+3(\rho_{N_{r2}}\Lambda_3+\tau_{ei}^3)\{\Psi^{(1)}\}^2\Psi^{(2)}+(\tau_{ei}^4+\rho_{N_{r2}}\Lambda_4)\{\Psi^{(1)}\}^4+\rho_{N_{r1}}\rho_{N_{hi}}^{(4)}$$
$$=0, \tag{4.18}$$

where

$$\Lambda_4=\frac{(q+1)(3-q)(5-3q)(7-5q)}{384}.$$

Now, it is formulated the following solutions along with a remarkable equation from the $O(\varepsilon^6)$-order equations:

$$\rho_{N_{hi}}^{(3)}=\frac{1}{V_{ph}^2}\left[\frac{3}{V_{ph}^2}\{\Psi^{(1)}\Psi^{(2)}\}-\frac{5}{2V_{ph}^4}\{\Psi^{(1)}\}^3-\Psi^{(3)}\right], \tag{4.19}$$

$$W_{hi}^{(3)}=\frac{1}{V_{ph}}\left[\frac{1}{V_{ph}^2}\{\Psi^{(1)}\Psi^{(2)}\}-\frac{1}{2V_{ph}^4}\{\Psi^{(1)}\}^3-\Psi^{(3)}\right], \tag{4.20}$$

and

$$-P_f\Psi^{(3)}+2P_f\{\Psi^{(1)}\Psi^{(2)}\}+\left[\rho_{N_{r2}}\Lambda_3+\frac{1}{6}\tau_{ei}^3-\frac{5\rho_{N_{r1}}}{2V_{ph}^6}\right]\{\Psi^{(1)}\}^3=0. \tag{4.21}$$

Eq. (4.21) yields

$$-Q_f\{\Psi^{(1)}\}^3=0. \tag{4.22}$$

It is clearly found from Eq. (4.22) that

$$Q_f=\rho_{N_{r2}}\Lambda_3+\frac{1}{6}\tau_{ei}^3-\frac{5\rho_{N_{r1}}}{2V_{ph}^6}=0. \tag{4.23}$$

It's noteworthy to note that the $O(\varepsilon^6)$-order equations is solvable only if $Q_f=0$ and CVs are supported by setting $P_f=0$. As a result, one can be evaluated the super critical values (SCVs) for any given parameter by setting $Q_f=0$.

Finally, substituting Eqs. (4.9), (4.10), (4.19), and (4.20) into the $O(\varepsilon^7)$-order equations yields

$$-\frac{1}{V_{ph}^2}\frac{\partial\Psi^{(1)}}{\partial\eta}-V_{ph}\frac{\partial\rho_{N_{hi}}^{(4)}}{\partial\zeta}+\frac{\partial W_{hi}^{(4)}}{\partial\zeta}+\frac{1}{V_{ph}^3}\frac{\partial}{\partial\zeta}\{\Psi^{(2)}\}^2-\frac{6}{V_{ph}^5}\frac{\partial}{\partial\zeta}\left[\{\Psi^{(1)}\}^2\Psi^{(2)}\right]$$
$$+\frac{15}{4V_{ph}^7}\frac{\partial}{\partial\zeta}\{\Psi^{(1)}\}^4+\frac{2}{V_{ph}^3}\frac{\partial}{\partial\zeta}\left[\Psi^{(1)}\Psi^{(3)}\right]=0, \tag{4.24}$$

$$-\frac{1}{V_{ph}}\frac{\partial\Psi^{(1)}}{\partial\eta}-V_{ph}\frac{\partial W_{hi}^{(4)}}{\partial\zeta}-\frac{\partial\Psi^{(4)}}{\partial\zeta}+\frac{1}{2V_{ph}^2}\frac{\partial}{\partial\zeta}\{\Psi^{(2)}\}^2-\frac{3}{2V_{ph}^4}\frac{\partial}{\partial\zeta}\left[\{\Psi^{(1)}\}^2\Psi^{(2)}\right]$$

$$+\frac{1}{V_{ph}^2}\frac{\partial}{\partial\zeta}\left[\Psi^{(1)}\Psi^{(3)}\right]+\frac{5}{8V_{ph}^6}\frac{\partial}{\partial\zeta}\{\Psi^{(1)}\}^4+\frac{\nu_{hi}}{V_{ph}}\frac{\partial^2\Psi^{(1)}}{\partial\zeta^2}=0, \qquad (4.25)$$

and

$$\left(\rho_{N_{r2}}\Lambda_1+\tau_{ei}\right)\frac{\partial\Psi^{(4)}}{\partial\zeta}+\left(\rho_{N_{r2}}\Lambda_2-\tau_{ei}^2\right)\frac{\partial}{\partial\zeta}\{\Psi^{(2)}\}^2+2\left(\rho_{N_{r2}}\Lambda_2-\tau_{ei}^2\right)\frac{\partial}{\partial\zeta}\left[\Psi^{(1)}\Psi^{(3)}\right]$$

$$+3\left(\rho_{N_{r2}}\Lambda_3+\tau_{ei}^3\right)\frac{\partial}{\partial\zeta}\left[\{\Psi^{(1)}\}^2\Psi^{(2)}\right]+\left(\tau_{ei}^4+\rho_{N_{r2}}\Lambda_4\right)\frac{\partial}{\partial\zeta}\left[\{\Psi^{(1)}\}^4\right]$$

$$+\rho_{N_{r1}}\frac{\partial\rho_{N_{hi}}^{(4)}}{\partial\zeta}=0. \qquad (4.26)$$

By eliminating the fourth order quantities with the help of Eqs.(4.12) and (4.23), one arises with

$$\frac{\partial\Psi^{(1)}}{\partial\eta}+G\{\Psi^{(1)}\}^3\frac{\partial\Psi^{(1)}}{\partial\zeta}=H\frac{\partial^2\Psi^{(1)}}{\partial\zeta^2}, \qquad (4.27)$$

The coefficients of Eq. (4.27) that are nonlinear and dissipative are obtained as

$$G=\frac{V_{ph}^3}{2\rho_{N_{r1}}}\left(\frac{1}{6}\tau_{ei}^4-4\rho_{N_{r2}}\Lambda_4-\frac{35\rho_{N_{r1}}}{2V_{ph}^8}\right), H=\frac{\nu_{hi}}{2}. \qquad (4.28)$$

Eq. (4.27) is so-called the modified Burgers-type equation having quartic nonlinearity. It is remarkable to note that Eq. (4.27) is formulated for the first time. As a result, the useful solution of Eq. (4.27) must therefore be required in order to analyze the propagation of DIASWEs around the SCVs in the plasmas. The next section provides a detailed derivation of the solution to Eq. (4.27).

4.4 Analytical solution of modified Burgers-type equation having quartic nonlinearity

To determine the stationary shock wave solution of Eq. (4.27), one can be converted Eq. (4.27) by considering $\Psi^{(1)}(\zeta,\eta)=\Omega(\chi)$ with $\chi=\zeta-V_{rh}\eta$ (V_{rh} is the constant speed of reference frame) with the boundary conditions $\Omega\to 0$, $\Omega^{'}\to 0$, $\Omega^{''}\to 0, \ldots\ldots.$ as $\chi\to\pm\infty$ to the following form:

$$-V_{rh}\frac{d\Omega}{\partial\chi}+G\Omega^3\frac{d\Omega}{\partial\chi}=H\frac{d^2\Omega}{d\chi^2}. \tag{4.29}$$

Integrating Eq. (4.29) one times with regards to ξ, yields

$$-V_{rh}\Omega+\frac{1}{4}G\Omega^4=H\frac{d\Omega}{d\chi}, \tag{4.30}$$

The integral form of Eq. (4.30) is obtained as

$$\frac{H}{3V_{rh}}\int\left[\frac{3\Omega^2}{\Omega^3-\frac{4V_{rh}}{G}}-\frac{3}{\Omega}\right]d\Omega=\int d\chi,$$

which gives

$$\Omega^3=\frac{2V_{rh}}{G}\left[1-\frac{exp\left(\frac{3V_{rh}}{2H}\chi\right)+exp\left(\frac{-3V_{rh}}{2H}\chi\right)}{exp\left(\frac{3V_{rh}}{2H}\chi\right)-exp\left(\frac{-3V_{rh}}{2H}\chi\right)}\right].$$

Hence, the useful stationary shock wave solution of Eq. (4.27) is formulated as

$$\Psi=\left[\Psi_A\left\{1-tan\,h\left(\frac{\chi}{\Psi_W}\right)\right\}\right]^{1/3}, \tag{4.31}$$

where $\Psi^{(1)}\sim\Psi$, $\Psi_A=(2V_{rh}/G)$ and $\Psi_W=(2H/3V_{rh})$ are the amplitude and thickness of DIA SWEs around SCVs. The details calculation to derive stationary shock wave solution of equation (4.27) are listed in appendix B.

4.5 Results and discussions

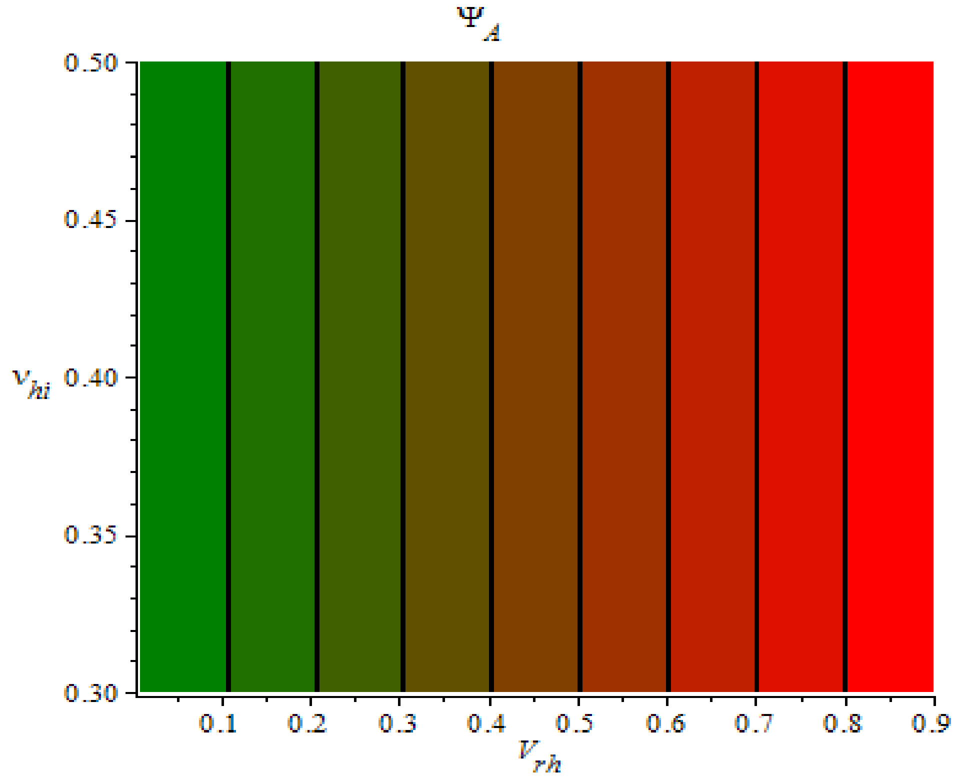
To examine the nature of electrostatic DIA SWEs by the consideration of SCVs, a nonlinear evolution equation as in Eq. (4.27) has been formulated form the considered plasma model. But, it is fascinating and essential to determine the exact shock wave solution for Eq. (4.27). By directly integrating, the exact solution of Eq. (4.27) has already been constructed as in Eq. (4.31). Based on the effective exact solution, the nonlinear propagation characteristics of DIA SWEs related to physical parameters including $\rho_{N_{r1}}$ (electrons to PIs density ratio), $\rho_{N_{r2}}$ (heavy NIs to PIs density ratio), τ_{ei} (electrons to PIs temperature ratio) and

ν_{hi} (viscosity coefficient of heavy NIs) is described in Figures below. In the presented discussion, the typical ranges of plasma parameters are considered based on the Ref. 118. It is considered just for the purpose of qualitative analysis of SWEs.

Figures 4.1 show the variation of amplitude and thickness of DIA SWEs with regards to V_{rh} and ν_{hi} since not only CVs but also SCVs are independent of V_{rh} and ν_{hi}. It is found from Figures 4.1 that the amplitudes (thickness) of DIA SWEs are increased (decreased) with the increased of V_{rh}, whereas only the thickness is increased but amplitude remained unchanged with the increase of ν_{hi}, as it is expected. The 3D shape of SWEs with the influence of V_{rh} and ν_{hi} is also displayed in Figure 4.2. It is found from Figure 4.2 that the monotonically shocks are supported around SCVs, as it expected because the viscosity coefficient is only response to the formation of shocks in such plasmas.

Figures 4.3(a) and 4.3(b) demonstrate the electrostatic DIA SWEs with the variation of χ and q, and χ and τ_{ei} around SCVs as well as at the SCVs, respectively. Whereas, Figures 4.4(a) and 4.4(b) demonstrate the electrostatic DIA SWEs with the variation of χ and $\rho_{N_{r1}}$, and χ and $\rho_{N_{r2}}$ around SCVs as well as at the SCVs, respectively. It is observed from Figures. 4.3 and 4.4 that the compressive electrostatic SWEs are only supported for the cases of (i) τ_{ei} is less or equal its SCVs (τ_{eiC}), (ii) q is greater or equal its SCVs (q_C), (iii) $\rho_{N_{r1}}$ is greater or equal its SCVs ($\rho_{N_{r1C}}$) and (iv) $\rho_{N_{r2}}$ is less or equal its SCVs ($\rho_{N_{r2C}}$). Additionally, the electrostatic SWEs are increased (decreased) with the increase of τ_{ei} and $\rho_{N_{r2}}$ (q and $\rho_{N_{r1}}$) from its SCVs.

In the physical sense, the electrostatic potential fall transversely the double layer enhances, and then more plasma particles may accelerate in the considered plasmas with the increase of shock amplitudes. Such mechanism is actually made with the increase of subthermal electrons energy and heave NIs density. This means that the restoring force plays vital role with the increase of temperature of electrons, whereas the driving leads to leading role with the increase of heave NIs density. From the above discussion, it can be concluded that the presented research work must be useful to cheek the laboratory experiments when the supercritical points of any parameters are suddenly appeared in the plasmas



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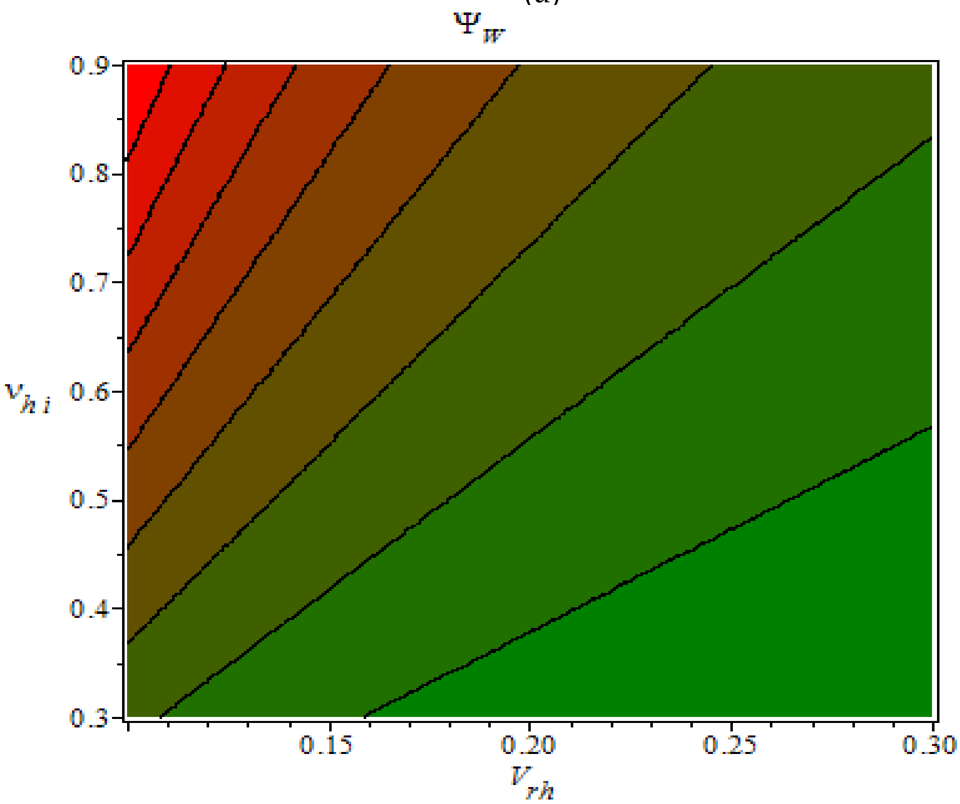
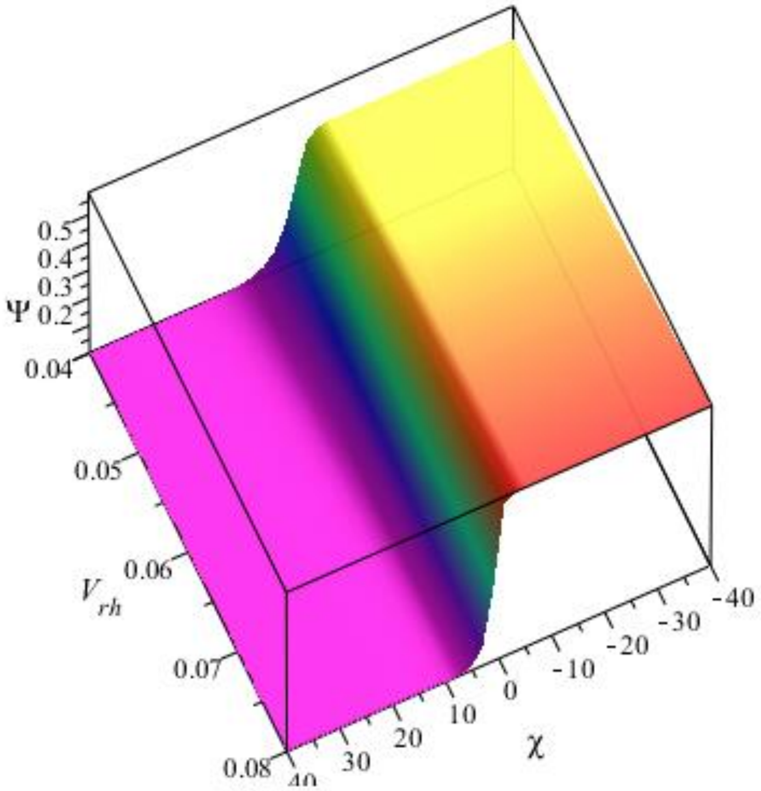


(b)

Figure 4.1: The contour of the shock wave (a) amplitude and (b) thickness in the (ν_{hi}, V_{rh}) plane with $\rho_{N_{r1}} = 0.5$, $\rho_{N_{r2}} = 0.05$, $\tau_{ei} = 0.1$ and $q = 5$.



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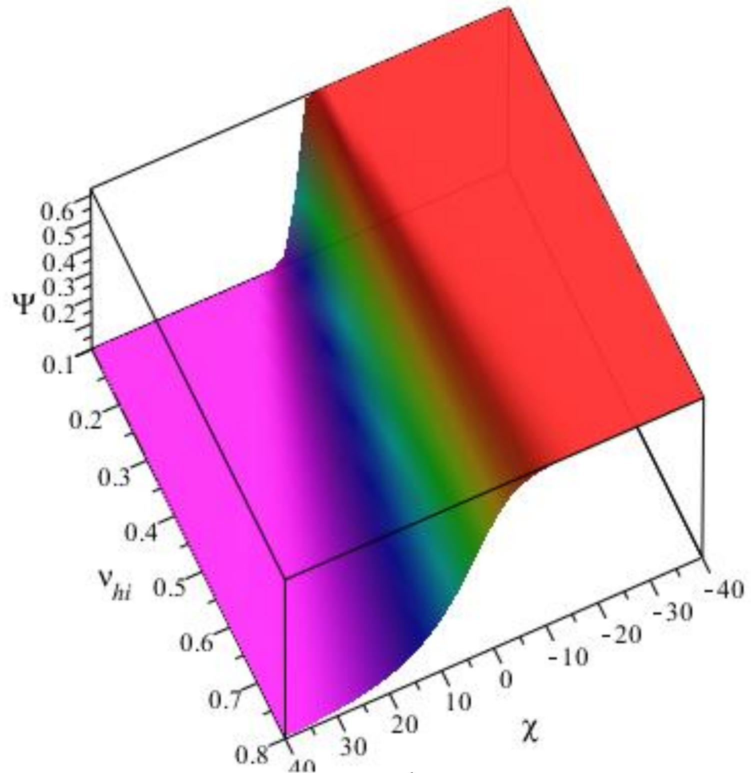
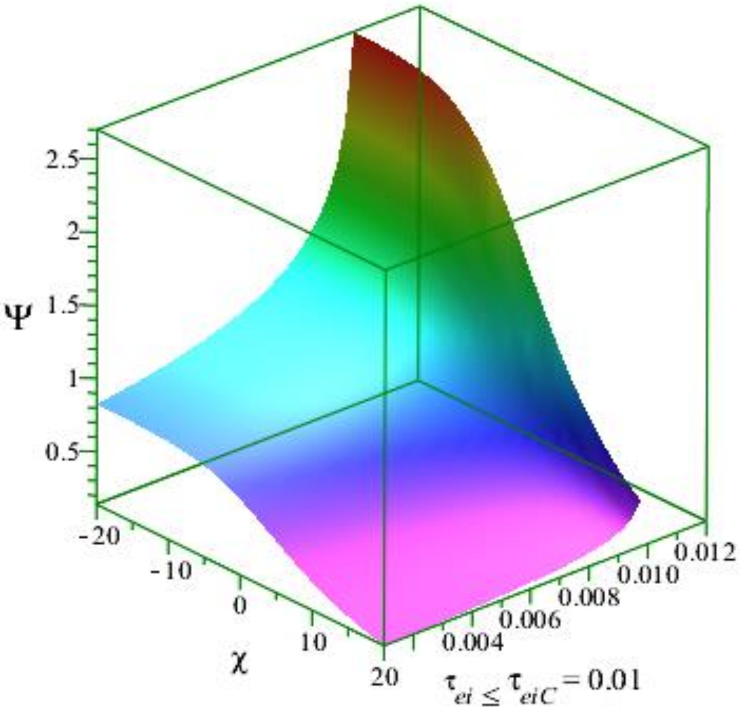


(b)

Figure 4.2: The variation of shock profile with regards (a) χ and V_{rh} ($\nu_{hi} = 0.1$) and (b) χ and ν_{hi} ($V_{rh} = 0.1$) with $\rho_{N_{r1}} = 0.5$, $\rho_{N_{r2}} = 0.05$ and $q = 5$.



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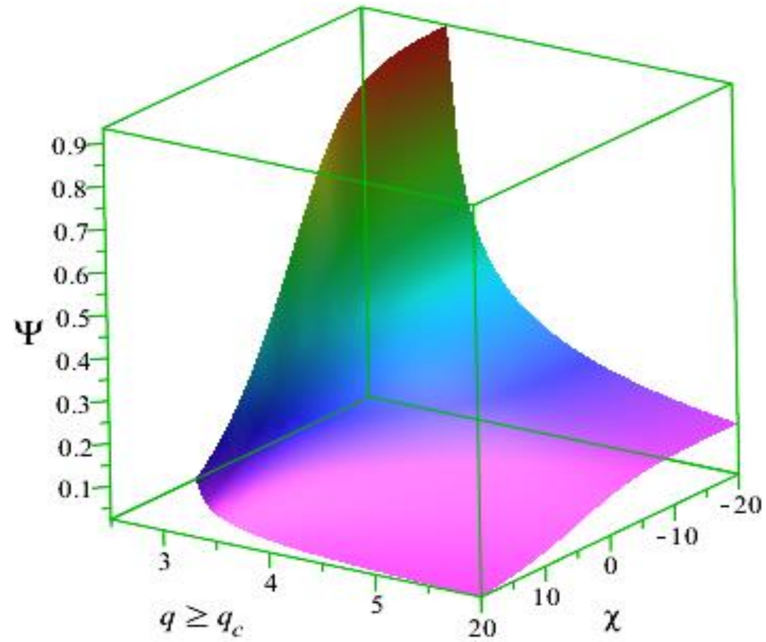
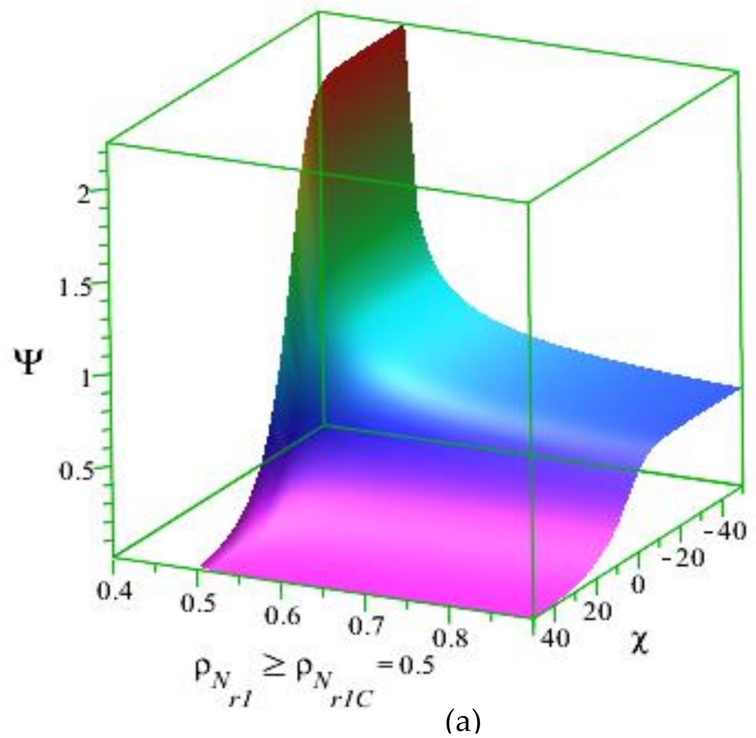


(b)

Figure 4.3: The variation of shock profiles with regards to (a) χ and τ_{ei} around and at SCV (τ_{eiC}) with $\rho_{N_{r1}} = 0.5$, $\rho_{N_{r2}} = 0.05$, $\nu_{hi} = 0.1$, $V_{rh} = 0.01$ and $q = 3.26128286$, and (b) χ and q around and at SCV ($q_c = 3.26128286$) with $\rho_{N_{r1}} = 0.5$, $\rho_{N_{r2}} = 0.05$, $\nu_{hi} = 0.1$, $V_{rh} = 0.01$ and $\tau_{ei} = 0.1$.



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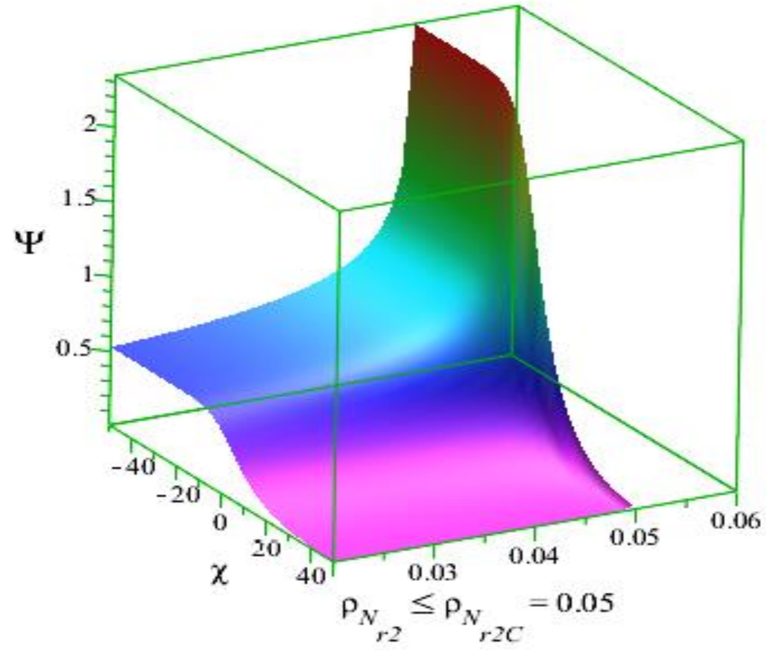

(b)

Figure 4.4: The variation of shock profiles with regards to (a) χ and $\rho_{N_{r_1}}$ around and at SCV ($\rho_{N_{r1C}}$) with $\tau_{ei} = 0.1$, $\rho_{N_{r_2}} = 0.05$, $\nu_{hi} = 0.1$, $V_{rh} = 0.01$ and $q = 3.26128286$, and (b) χ and $\rho_{N_{r_2}}$ around and at SCV ($\rho_{N_{r2C}}$) with $\rho_{N_{r_1}} = 0.5$, $q = 3.26128286$, $\nu_{hi} = 0.1$, $V_{rh} = 0.01$ and $\tau_{ei} = 0.1$.

4.6 Concluding remarks

An investigation of the nonlinear DIA shock waves not only around SCVs but also at the SCVs propagating in a collisionless complex multi-ions plasma with subthermal electrons has been presented. The compressive SWEs have only found around SCVs but also at the SCVs by formulating new evolution equation, so called the modified Burgers-type equation having quartic nonlinearity. It is noted that the new evolution equation is only applicable to study SWEs for the SCVs. By directly integrating, the exact solution of Burgers-type equation having quartic nonlinearity has been determined for the first time. The existence regions of SWEs based on the SCVs have also been provided. Thus, the theoretical investigation in this work would be provided a better understanding of SWEs in dusty multi-ions plasma with nonextensive electrons in further laboratory verification.

Chapter 5

Summary and future direction

The critical findings that have been found from the previous chapters are summarized as below.

Chapter 1 has been dedicated for introductory discussions including fundamental of plasmas, the existence of shock waves and how to derive nonlinear evolution equations (NLEEs) via the mathematical techniques.

In Chapter 2, the nonlinear propagation of ion acoustic shock waves (IASWs) in an unmagnetized collisionless pair-ions plasma with (α, q)-distributed electrons have been investigated. To do so, Burger equations having various kinds of nonlinearity have been derived by implementing the reductive perturbation technique. The appropriate solutions of Burger equations having various kinds of nonlinearity have been determined. The effects of parameters on the nonlinear electrostatic IASWs described by Burger equations have been reported. In addition, the electrostatic IASWs and normalized electric fields have been investigated around the critical values with the changes of viscosity coefficients of positive and negative ions. The outcomes obtained from this chapter might be very useful to understand the behavior of shocks and behavior of shocks around the critical values in the F- and D-regions of Earth's ionosphere, and the later experimental verification in plasma laboratory.

In Chapter 3, the nonlinear propagation of DIASWs in a collisionless four-component unmagnetized dusty multi-ion plasma have investigated. To do so, several nonlinear evolution equations have formulated by implementing the reductive perturbation technique. This chapter has also been reinvestigated the electrostatic DIASWs in dusty multi-ion nonextensive plasma because of the misleading information about nonlinear coefficients and analytical solutions of Burger equations involving higher-order nonlinearities in Ref. [118]. The effects of parameters on the DIASWs have been reported. It has been found that the rarefactive electrostatic shocks are only supported with the presence of superthermality, isothermality and nonthernality electrons, but both of compressive and rarefactive shocks are supported with the presence of subthermality electrons. It has also been investigated the nature of electrostatic shocks and double layer around the critical values and at the critical values of any specific plasma parameters. The outcomes from this work would be helpful for better understanding the dynamics of DIASWs in space environments and plasma laboratory.

In chapter 4, progress in understanding the propagation of DIASWs for the super critical values of any specific parameter which accompany an unmagnetized collisionless four-component dusty multi-ion nonextensive plasma has been presented. To accomplish this goal, the formation of modified Burgers-type equation having quartic nonlinearity via the reductive perturbation method and its analytical solution has been yielded for the first time. It is found that the compressive electrostatic shocks are supported not only around the super critical values but also at the super critical values of the specific parameters. The presented results would be applicable to comprehend wave propagation in interplanetary plasmas and laboratory plasmas.

It can be concluded that this thesis serves as an illustration for creating nonlinear shock wave structures with the variation of plasma parameters by deriving realistic NLEEs. Such NLEEs are obtained by stretching only the space and time variables and only the expansion of perturb quantities. But still there are so many possibilities to carry further investigation on nonlinear analysis of shock wave excitations by considering any other plasma circumstances or deriving any other NLEEs from the previously proposed models. In addition, one can study the dynamical analysis of shock wave excitations for the super critical values of any specific parameter in the plasmas. Further, one may study the nonlinear shock wave phenomena by NLEEs involving spherical or cylindrical coordinates via the numerical techniques. These are also the problems of great importance in plasma physics for better understanding the characteristics of shock waves as observed in many astrophysical and space plasmas, but beyond the scope of this thesis. Finally, the structure and properties of parametric effects on the shock structures are still not clear and an extensive development in this direction can be achieved by undertaking a systematic study.

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Appendices

Appendix A: The verification of the solution of modified Burger equation (2.77)

$$> \xi := X - V_r T$$

$$> \Phi_A := (3V_r A/2B)$$

$$> \Phi_W := (C/V_r B)$$

$$> \Phi^{(1)} := \sqrt{\Phi_A \left\{1 - \tan h\left(\frac{\zeta}{\Phi_W}\right)\right\}}$$

$$> diff(\Phi^{(1)}, T)$$

$$> diff(\Phi^{(1)}, X)$$

$$> diff(\Phi^{(1)}, X, X)$$

$$> A * diff(\Phi^{(1)}, T) + B * (\Phi^{(1)})^2 * diff(\Phi^{(1)}, X) + C * diff(\Phi^{(1)}, X, X)$$

$$> 0$$

It is noted that the stationary solution

$$> \Phi^{(1)} = \sqrt{\left(\frac{3u_0}{2B}\right)\left\{1 - \tan h\left(\frac{\zeta - u_0\tau}{\sqrt{\frac{C}{u_0}}}\right)\right\}}$$

provided in the previous literature in Refs. [101-103] does not satisfy the following modified Burgers equation:

$$A\frac{\partial \Phi^{(1)}}{\partial \tau} + B\left\{\Phi^{(1)}\right\}^2 \frac{\partial \Phi^{(1)}}{\partial \zeta} = C\frac{\partial^2 v^{(1)}}{\partial \zeta^2}$$

which clearly indicates that the above solution of modified Burgers equation is not useful for later experimental verification.

Appendix B: The details calculation to derive stationary shock wave solution of equation (4.27)

$$-V_{rh}\frac{d\Omega}{\partial\chi}+G\Omega^3\frac{d\Omega}{\partial\chi}=H\frac{d^2\Omega}{d\chi^2}$$

$$\Rightarrow -V_{rh}\Omega+\frac{1}{4}G\Omega^4=H\frac{d\Omega}{d\chi}$$

$$\Rightarrow \frac{Hd\Omega}{\frac{1}{4}G\Omega^4-V_{rh}\Omega}=d\chi$$

$$\Rightarrow \frac{4H}{G}\frac{d\Omega}{\Omega\left(\Omega^3-\frac{4V_{rh}}{G}\right)}=d\chi$$

$$\Rightarrow \frac{H}{3V_{rh}}\left[\frac{3\Omega^2}{\Omega^3-\frac{4V_{rh}}{G}}-\frac{3}{\Omega}\right]d\Omega=d\chi$$

$$\Rightarrow ln\left(\Omega^3-\frac{4V_{rh}}{G}\right)-ln\,\Omega^3=\frac{3V_{rh}}{H}\chi$$

$$\Rightarrow \Omega^3-\frac{4V_{rh}}{G}=\Omega^3\,exp\left(\frac{3V_{rh}}{H}\chi\right)$$

$$\Rightarrow \Omega^3\left[exp\left(\frac{-3V_{rh}}{C}\chi\right)-1\right]=\frac{4V_{rh}}{G}exp\left(\frac{-3V_{rh}}{H}\chi\right)$$

$$\Rightarrow \Omega^3=-\frac{4V_{rh}}{G}\frac{exp\left(\frac{-3V_{rh}}{H}\chi\right)}{1-exp\left(\frac{-3V_{rh}}{H}\chi\right)}$$

$$\Rightarrow \Omega^3=-\frac{4V_{rh}}{G}\frac{exp\left(\frac{-3V_{rh}}{2H}\chi\right)}{exp\left(\frac{3V_{rh}}{2H}\chi\right)-exp\left(\frac{-3V_{rh}}{2H}\chi\right)}$$

$$\Rightarrow \Omega^3=\frac{2V_r}{G}\left[1-\frac{exp\left(\frac{3V_{rh}}{2H}\chi\right)+exp\left(\frac{-3V_{rh}}{2H}\chi\right)}{exp\left(\frac{3V_{rh}}{2H}\chi\right)-exp\left(\frac{-3V_{rh}}{2H}\chi\right)}\right]$$

$$\therefore \Omega = \left[\frac{2V_{rh}}{G}\left\{1 - tan\,h\left(\frac{3V_{rh}}{2H}\chi\right)\right\}\right]^{1/3}$$

the stationary shock wave solution of Eq. (4.27) is obtained as

$$\Psi = \left[\Psi_A\left\{1 - tan\,h\left(\frac{\chi}{\Psi_W}\right)\right\}\right]^{1/3}$$

where, $\Psi_A = (2V_{rh}/G)$ and $\Psi_W = (2H/3V_{rh})$ are the amplitude and thickness of DIA SWEs around SCVs. The above solution is determined for the first time.

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