NONLINEAR ACOUSTIC WAVE PHENOMENA IN MAGNETIZED COLLISIONLESS RELATIVISTIC PLASMAS

By

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Declaration

I hereby declare that the work contained in this Thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the Thesis contains no material previously published or written by another person except where due reference is cited. Furthermore, the Thesis complies with PLAGIARISM and ACADEMIC INTEGRITY regulation of CUET.

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Dedication

"To All Souls Living On This Earth."

List of Publications

[1] Sagar Barua et al., “Nonlinear propagation of ion-acoustic soliton in a magnetized three component relativistic plasma," *Proceedings of the 2nd International Conference on Nonlinear Dynamics and Applications (ICNDA 2024)*, Springer Proceedings in Physics, Volume 1, (2024), DOI: 10.1007/978-3-031-66874-6_5.

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[3] Sagar Barua et al., “Nonlinear propagation of ion-acoustic soliton with dynamical behaviours around the super-critical values in a magnetized three component relativistic plasma," under submission.

Approval/Declaration by the Supervisor(s)

This is to certify that Sagar Barua has carried out this research work under our supervision, and that he has fulfilled the relevant Academic ordinance of the Chittagong University of Engineering & Technology, so that he is qualified to submit the following thesis in the application for the degree of MASTER OF PHILOSOPHY IN MATHEMATICS. Furthermore, the Thesis complies with PLAGIARISM and ACADEMIC INTEGRITY regulation of CUET.

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Abstract

The thesis investigates the nonlinear propagation of ion-acoustic solitons (IASs) within a magnetized rotating relativistic plasma environment. This environment comprises relativistic ion fluids and electrons following (α, q)-distributions, alongside positrons. Employing the reductive perturbation technique, the study derives the Korteweg-de Vries equation (KdVE) with quadratic nonlinearity. However, when the coefficient associated with this nonlinearity approaches zero, the method encounters limitations. To address this challenge, adjustments are made to the stretching coordinates, leading to a KdVE with cubic nonlinearity, suitable for describing soliton propagation near critical values in these plasma conditions. Furthermore, a KdVE with quartic nonlinearity is derived, relevant for supercritical values of specific plasma parameters in relativistic plasmas.

Prior research has predominantly explored relativistic effects on soliton propagation through expansions of Lorentz relativistic factors up to three terms. In contrast, this thesis extends the consideration to more than ten terms to minimize truncation errors in modeling nonlinear soliton propagation within these plasmas. The investigation reveals that the relativistic streaming factor significantly alters the wave potential functions. Notably, the derived KdVE equations indicate that quadratic nonlinearity supports both compressive and rarefactive soliton propagations, while cubic and quartic nonlinearities exclusively support compressive solitons in these plasma settings.

The study further examines how plasma parameters, with the inclusion of the relativistic Lorentz factor up to eleven terms, influence the amplitude and width of IASs for the first time. It finds that higher-order terms of the relativistic Lorentz factor and obliqueness notably modify the propagation characteristics of IASs within this specific plasma environment.

বিমূর্ত

এই থিসিসটি একটি চৌম্বকীয় ঘূর্ণায়মান আপেক্ষিক প্লাজমা পরিবেশের মধ্যে আয়ন অ্যাকোস্টিক সলিটন-এর (IAS) অরৈখিক প্রসারণের অণুসন্ধান করে। এই পরিবেশটি আপেক্ষিক আয়নের পাশাপাশি (α, q) –বন্টনযুক্ত ইলেকট্রন ও পজিট্রন দ্বারা গঠিত বলে বিবেচনা করা হয়। এই অধ্যয়নে, রিডাক্টিভ পার্টারবেশন পদ্ধতি ব্যবহার করে দ্বিমাত্রিক অরৈখিকতার কর্টেওয়েগ-ডি ভ্রিস সমীকরণটি প্রতিপাদন করা হয়। কিন্তু, যখন এই অরৈখিকতার সাথে যুক্ত সহগটি শূন্যের কাছাকাছি পৌঁছায়, তখন পদ্ধতিটি সীমাবদ্ধতার সম্মুখীন হয়। এই সীমাবদ্ধতা মোকাবেলা করার জন্য, প্রসারণ স্থানাঙ্কের সামঞ্জস্য করা হয়, ফলস্বরূপ ত্রিমাত্রিক অরৈখিকতার একটি KdV সমীকরণ পাওয়া যায়, যা এই প্লাজমা অবস্থার সংকট মানগুলির কাছাকাছি সলিটন প্রসারণের বর্ণনা দেওয়ার জন্য উপযুক্ত। তদ্ব্যতীত, আপেক্ষিক প্লাজমাতে নির্দিষ্ট প্লাজমা পরামিতিগুলির অতি-সংকটপূর্ণ মানগুলির প্রাসঙ্গিক ব্যাখ্যা প্রদানের উদ্দেশ্যে চতুর্মাত্রিক অরৈখিকতা সহ একটি KdV সমীকরণ প্রতিপাদন করা হয়।

পূর্ববর্তী গবেষণাসমূহে মূলত লরেঞ্জ আপেক্ষিক ফ্যাক্টর সম্প্রসারণের তিনটি পদ পর্যন্ত বিবেচনা করে সলিটন প্রসারণের উপর আপেক্ষিকতার প্রভাবগুলি অন্বেষণ করা হয়েছে। বিপরীতে, এই থিসিসে, উপরোক্ত প্লাজমার মধ্যে অরৈখিক সলিটন প্রসারণের মডেলিংয়ে ত্রুটিগুলি হ্রাস করতে দশটিরও বেশি পদের বিবেচনা করা হয়। এই তদন্তটি প্রকাশ করে যে আপেক্ষিক স্ট্রিমিং ফ্যাক্টর তরঙ্গের সম্ভাব্য ক্রিয়াকলাপগুলির উল্লেখযোগ্য পরিবর্তন করে। লক্ষণীয়ভাবে, প্রতিপাদিত KdV সমীকরণগুলি নির্দেশ করে যে, দ্বিমাত্রিক অরৈখিকতা কম্প্রেসিভ এবং রেয়ারফ্যাক্টিভ উভয় সলিটন প্রসারণকেই সমর্থন করে, যেখানে ত্রিমাত্রিক এবং চতুর্মাত্রিক অরৈখিকতা এই প্লাজমা ব্যবস্থায় শুধুমাত্র কম্প্রেসিভ সলিটনগুলিকেই সমর্থন করে।

গবেষণাটি প্রথমবারের জন্য আরও পরীক্ষা করে দেখায় যে, আপেক্ষিক লরেঞ্জ ফ্যাক্টরের এগারো বা তারও অধিক পদের অন্তর্ভুক্তির সাথে প্লাজমা পরামিতিগুলি কীভাবে সলিটনের বিস্তার এবং প্রস্থকে প্রভাবিত করে। এতে পাওয়া যায় যে, আপেক্ষিক লরেঞ্জ ফ্যাক্টরের উচ্চ-ক্রমের পদ এবং তির্যকতার জন্য এই নির্দিষ্ট প্লাজমা পরিবেশে IAS-এর প্রসারণ বৈশিষ্ট্যগুলি উল্লেখযোগ্যভাবে পরিবর্তিত হয়।

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Nomenclature

e-p	electron-positron
e-p-i	electron-positron-ion
IAS	Ion Acoustic Soliton
IAW	Ion Acoustic Wave
KdV	Korteweg-de-Vries
MHD	Magnetic Hydrodynamics
PDE	Partial Differential Equation
mKdV	Modified Korteweg-de-Vries
RLF	Relativistic Lorentz Factor
RPT	Reduction Perturbation Technique

Chapter 1: INTRODUCTION

1.1 HISTORY OF PLASMA

The fourth state of matter, recognised as plasma, is composed of particles with positive and negative charges in an ionized gas. In mid-19th century, Czech physician Jan Purkinje used the word "plasma", to describe the clear fluid that has remained after removing the cellular material from the blood. Later, in 1922, Irving Langmuir (American scientist) suggested that the charged particles (electrons, ions) and neutrals in ionized gas could be considered as corpuscular material suspended in a fluid medium, which he called "plasma" [1–3]. This gaseous medium differs from classical gases which are solely composed of electrically neutral particles, by the nature of interactions among charged particles. Plasma resulting from the ionization of neutral gases comprises myriad positive and negative charges, and their corresponding amounts are inversely proportional to the magnitude of their individual charges. However, not all ionized gases are considered as plasma. When charged and neutral particles behave collectively in a quasineutral gas, it's called plasma with any deviation from charge neutrality being very small. In this state, oppositely charged particles, strongly coupled electrostatically, tend to neutralize each other over large distances [4–6].

The universe is significantly contingent on plasma. Approximately 99% of the material in the observable universe is estimated to be in the plasma state. The recognition of plasma as the fourth state of matter in the physical system was a significant breakthrough exclusive to twentieth-century physics. [6, 7].

Plasma research originated in the mid-18th century with the development of early electrophoretic machines and Leyden flasks, used to demonstrate electrical phenomena and spark discharges to the public. Later in the early 19th century, Volta's invention of galvanic cells led to the discovery of the electric arc. This era was marked as the beginning of plasma physics, influenced by Faraday's electrolysis laws and studies of particle behaviour in low-pressure electric discharge tubes. Faraday and Geisler were the first to observe these discharges and detailed studies began in the 1870s. W. Crookes in his lecture "On Radiant

Matter or the Fourth Aggregate State", presented evidence proving that cathode rays are streams of particles showing their corpuscular nature in 1879.

It took roughly 16 years for electrons to be widely accepted, finally occurred in 1895 when Thompson determined the electron-to-mass ratio for cathode particles. The term "electron" first appeared in physics in 1891 but acquired its modern meaning after 1900, sparking the birth of electronics and plasma physics. Subsequent breakthroughs unfolded rapidly: Planck formulated his radiation equation in 1900, Rutherford described atomic structure in 1911, Bohr proposed his atomic model in 1912, and quantum mechanics emerged in the early 1920s through the work of Heisenberg, De Broglie, Schrödinger, and Dirac, laying the foundation for plasma physics.

Although significant, the progress mainly happened on the fringes of physics advancements. Yet, there wasn't a precise term for the emerging field of gas discharge physics, encompassing phenomena like arcs, glows, and sparks. Until the mid-1920s, only Townsend's research on charged particle mobility in weakly ionized gases was considered pivotal. However, the distinction between plasma physics and discharge physics is often credited to Irvin Langmuir, whose foundational work in the 1920s marked three key achievements. Firstly, Langmuir developed vacuum technology with diffusion pumps for producing high-purity vacuum. Secondly, he improved methods to measure plasma parameters using electrostatic probes, allowing better determination of electron temperature, electrical potential, and density. Thirdly, Langmuir and Tonks conducted theoretical studies, notably discovering specific plasma oscillations known as 'Langmuir' frequency. Finally, Langmuir introduced the term "plasma" to define this state of matter.

Understanding of the plasmas in laboratory experiments, in astrophysics, and in space advanced parallelly over the twentieth century. The 1930s were marked by significant experimental investigations and rapid theoretical development. A few isolated researchers began studying plasma physics, motivated by practical problems. Their research focus was on comprehending the implications of ionospheric plasma on distant shortwave radio transmission and gaseous electron tubes which were used for rectification process, switching, and controlling voltages during the pre-semiconductor period of digital devices. In 1936, L. D. Landau modified the Boltzmann kinetic equation for Coulomb interactions, while B. I. Davydov derived the Boltzmann-Davydov collisional term. In 1938, A. A. Vlasov formulated self-consistent field equations for ionized plasma and later, Landau developed wave-particle interactions in collisionless plasma. Through the decade of the 1950s, it became evident that this collision-free structure of thermal plasmas became an important possessions, emphasizing the collective interactions in plasmas.

The relatively smooth yet rapid progress in plasma physics during the 1930s was completely interrupted in the 1940s, primarily due to World War II. Nonetheless, around 1930 to 1950, the groundwork of plasma physics was laid, notably as the outcome of research in ionospheric, astrophysical, and solar-terrestrial events. This investigation was driven by various interests such as comprehending radio wave propagation in the ionosphere, the causes of auroral displays and magnetic storms on Earth due to solar activity, as well the significance of magnetic forces in the physics of stars, galaxies, and the interstellar medium. Most notable contributors to this foundational research included H. Alfvén, M. Saha, E. Appleton, S. Chapman, L. Spitzer, S. Chandrasekhar, and T. Cowling, among others.

Modern plasma physics began to develop in the 1950s. This era was marked by two notable events: the Soviet Union's successful launch of the first artificial Earth satellite and the revelation that both the United States and the Soviet Union were exploring thermonuclear fusion for peaceful applications. Initially, this research was classified due to its connection with thermonuclear weapons, but it was later declassified in 1958 when it became evident that controlled fusion research had little military value. Since then, these countries, along with many others, have collaborated on fusion research.

Alongside the prospect of controlled thermonuclear fusion, the late 1940s and early 1950s were notable for another significant event. During this time, the Swedish astrophysicist H. Alfvén published "Cosmic Electrodynamics" [8]. In this dissertation, Alfvén evaluated the self-consistent dynamics of ideally conducting plasma and magnetization, revealing the importance of electromagnetic phenomena in space. His recommended theory of plasma dynamics, known as magneto-hydrodynamics, garnered immediate attention for its novelty and elegance. Consequently, this paved the way for serious theoretical analyses of astrophysical phenomena on large scales, including those occurring in the solar corona, protuberances, and Earth's magnetosphere. Notably, prominent figures such as Fermi and Chandrasekhar quickly contributed to the development of MHD models shortly after Alfvén's publication. MHD found rapid application not only in astrophysics but also in investigations related to controlled thermonuclear fusion. Later, it became apparent that the main obstacles to achieving controlled fusion did not stem from a lack of understanding of nuclear physics, but rather from gaps in our knowledge of plasma physics [9].

Fusion work was developed slowly during the 1960s, but by the end of the decade, the Russian tokamak configuration, established empirically, achieved significantly improved plasma parameters compared to previous decades. Throughout the decade of the 1970s to 1980s, more tokamaks with increasingly improved performance were produced. By the late 20th century, tokamaks were nearing fusion break-even. Alongside tokamaks, various non-tokamak fusion approaches employing magnetic confinement have been pursued with differing degrees of success. Additionally, plasma has been extensively explored for space propulsion, ranging from small ion thrusters for spacecraft maneuvering to more powerful magneto-plasma-dynamic thrusters. Which, with sufficient power, could be enabled for interplanetary missions. Some spacecraft already use plasma thrusters, and there's strong interest in using them for upcoming spacecraft designs.

During the late 1980s, a novel strategy for using plasma physics emerged: plasma processing. This process is vital for manufacturing the intricate integrated circuits found in modern electronic devices and has since become economically significant. In the '90s, research commenced regarding dusty plasmas, where dust grains engrossed in plasma may acquire charged electrically, behaving as extra-charged particles. Due to their comparatively massive size and variable charge, dusty plasmas exhibit both extended and entirely new physical behaviours compared to regular plasmas. Additionally, during the 1980s and 1990s, investigations into non-neutral plasmas has also began. Both dusty and non-neutral plasmas can form peculiar, strongly coupled collective states resembling solids, such as quasicrystalline structures. Non-neutral plasmas also found application in storing large quantities of positrons.

Apart from the aforementioned endeavors, ongoing studies have focused on industrially significant plasmas like arcs, plasma torches, and laser plasmas. Plasma displays are now using in flat panel televisions, and naturally-occurring terrestrial plasmas, such as lightning, also remain as subjects of investigation [1, 7, 10].

Plasma exists throughout the universe on all scales. In essence, much of the physical universe consists of plasma. In the solar system, plasma processes influence the Sun's rotation, magnetic fields, and mass ejections. The solar wind, made of plasma, travels to Earth, becoming turbulent and interacting with Earth's magnetic field to create phenomena like auroras. This system covers a distance of about 10^{-4} light years. On a much larger scale, extra-galactic jets, which start from supermassive black holes surrounded by rotating plasma, span millions of light years and are among the largest plasma structures. These jets are powered by plasma processes that also regulate many other astrophysical systems. Plasma astrophysics studies plasmas beyond Earth's atmosphere, covering both traditional astrophysics (beyond the solar system) and space physics (the Sun, the Heliosphere, and planetary magnetospheres). Through the plasma physics, the usual distinction between astrophysics and space physics fades away [11, 12].

1.2 DEBYE SHIELDING

The ability of plasma to cancel out applied electric potentials is one of its basic characteristics. In their rest frames, plasmas often do not possess a significant electric field. High plasma conductivity could be considered the effect of the plasma concealing an external electric field from its centre. In general terms, a plasma may conduct electricity enough freely to short out any internal electric fields. However, it is more appropriate to think that shielding is a dielectric phenomenon. An external electric field cannot penetrate the plasma medium because of its polarization and the resulting redistribution of space charge. Debye was the first to calculate the Debye length, which is the length scale connected to this type of shielding.

Suppose a quasineutral plasma is sufficiently close to thermal equilibrium such that the number densities of its species (say electron) follow the Maxwell-Boltzmann distribution, then

$$n_e = n_0 e^{\frac{e\phi}{k_B T_e}}, \tag{1.1}$$

where n_e, T_e, n_0 and k_B is the number density of electron, temperature of electron, electron having Maxwellian velocity distribution and the Boltzmann constant, respectively. As the charged particles are assumed to be in thermal equilibrium, the electrostatic potential ϕ can be obtained from the poisson equation as

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0} = \frac{e}{\varepsilon_0}\left(n_i - n_e\right), \tag{1.2}$$

where $\rho(= n_0 - n_e)$ is the charge density, ε_0 is the permittivity of free space, respectively. From the general principle of kinetic theory substituting the value of n_e in the above equation one obtains the nonlinear differential equation,

$$\varepsilon_0\nabla^2\phi = en_0\left(e^{\frac{e\phi}{k_B T_e}} - 1\right). \tag{1.3}$$

Analytically solving the equation (1.3) utilising $\frac{e\phi}{k_B T_e} \ll 1$, and implementing Taylor's expansion, yields the linear differential equation as

$$\nabla^2\phi = \frac{n_0 e^2}{\varepsilon_0 k_B T_e}\phi. \tag{1.4}$$

Since the plasma is isotropic, one can assume the electrostatic potential to be symmetric. Then, equation (1.4) simplifies to

$$\frac{\partial^2(r\phi)}{\partial r^2} - \frac{n_0 e^2}{\varepsilon_0 k_B T_e}(r\phi). \tag{1.5}$$

One obtains the general solution of equation (1.5) as

$$\phi = \frac{A}{r} e^{\frac{-r}{\lambda_D}}, \tag{1.6}$$

where A denotes a constant and r represents the radius. The quantity A can be obtained by requiring that the solution shortens to the Coulomb potential as the radius decreases to zero. Hence, the Debye length, denoted by λ_D, is readily determined by

$$\lambda_D = \left(\frac{\varepsilon_0 k_B T_e}{n e^2} \right)^{\frac{1}{2}}. \tag{1.7}$$

In practical terms, A shorter Debye length means that charges are screened out quickly, and the plasma can effectively shield electric fields within a small distance. A longer Debye length means that the plasma is less effective at shielding electric fields, allowing them to penetrate further. The Debye length is essential for various phenomena in plasmas, such as the behavior of Langmuir waves, plasma oscillations, and the conditions under which a plasma behaves as a collective medium. It is also relevant in many applications, from designing fusion reactors to understanding space plasmas and even in electrolytes in batteries and other devices [1,3,6,13,14].

1.3 PLASMA PARAMETER

The Debye length holds significant importance in plasma physics as it denotes the extent to which the electric field of an individual charged particle propagates within the plasma. It also represents the distance over which fluctuating electric potentials can emerge, reflecting the conversion of thermal kinetic energy into electrostatic potential energy. The Debye shielding effect, a characteristic feature of plasmas, is another critical aspect where the plasma medium screens out external electric fields. However, it's worth noting that not every medium containing charged particles exhibits Debye shielding. For a system to exhibit plasma behaviour, its physical dimensions must be significantly larger than the Debye length (λ_D). If L represents a characteristic dimension of the plasma, a primary criterion for defining a plasma is that L is much greater than λ_D. Therefore,

$$L \gg \lambda_D. \tag{1.8}$$

Since the shielding effect arises from collective particle behavior within a Debye sphere, it's important that the number of electrons within a Debye sphere is substantial. A second criterion for defining a plasma is thus

$$n_e \lambda_D^3 \gg 1. \tag{1.9}$$

This implies that the average distance between electrons must be much smaller than λ_D. Here, $g = \frac{1}{n_e \lambda_D^3}$ is known as the plasma parameter, and the condition $g \ll 1$ defines the plasma approximation. This parameter also signifies the ratio of the average interparticle potential energy to the average plasma kinetic energy. Importantly, the first criterion ensures macroscopic charge neutrality, as deviations from neutrality typically occur only over distances comparable to the Debye length λ_D [6, 14]. Macroscopic neutrality is sometimes considered a third criterion for plasma existence, although it is not independent and can be expressed by the equation,

$$n_e = \sum_i n_i. \tag{1.10}$$

1.4 PLASMA FREQUENCY

When a plasma experiences a sudden disturbance from equilibrium, internal space-charge fields induce collective particle motions to re-establish charge neutrality. These motions occur at the plasma frequency, representing high-frequency collective oscillations. Due to their heavy mass, ions only partially follow the rapid electron motion. Instead, electrons collectively oscillate around ions, with the ion-electron coulomb attraction providing the necessary restoring force. In the plasma's equilibrium state, charged particles are evenly distributed to maintain overall neutrality. The movement of electrons relative to background ions generates an electric field, striving to preserve neutrality by attracting electrons. However, inertia causes electrons to overshoot, creating an opposing electric field that pulls them back towards equilibrium. This results in electrons oscillating at the plasma frequency, with their lighter mass enabling faster oscillations than ions. Consequently, ions are less responsive to the oscillating electron field [2, 3, 15].

To determine the oscillation frequency, consider a plasma consisting of a uniform slab of electrons with number density n_e and a fixed background of positive ions of the same density. If the electron slab is displaced to the right by a small distance Δx and then released, the equation of motion for the electrons is,

$$m_e \frac{d^2 \Delta x}{dt^2} = (-e)E = -\frac{n_e e^2}{\varepsilon_0} \Delta x, \tag{1.11}$$

which implies

$$\frac{d^2 \Delta x}{dt^2} + \left(\frac{n_e e^2}{\varepsilon_0 m_e} \right) \Delta x = 0. \tag{1.12}$$

This corresponds to the harmonic oscillator equation, with the oscillation frequency ω_{pe} derived from the second term of equation (1.12) is

$$\omega_{pe}^2 = \left(\frac{n_e e^2}{\varepsilon_0 m_e} \right). \tag{1.13}$$

If a plasma has several species (s), a plasma frequency can be defined for each species according to the equation

$$\omega_{ps}^2 = \left(\frac{n_s e_s^2}{\varepsilon_0 m_s} \right), \tag{1.14}$$

If the ions are allowed to move in the previously mentioned slab, it can be shown that the oscillation frequency is $\omega_p = \sqrt{\omega_{pe}^2 + \omega_{pi}^2}$, which is approximately ω_{pe} because $\omega_{pi} \ll \omega_{pe}$.

1.5 MAGNETIZED PLASMA

A magnetized plasma is characterized by a surrounding magnetic field **B** that is sufficiently strong to influence the motion of the particles notably. In these plasmas, the behaviour is highly anisotropic, which means they react differently to forces depending on whether the forces are parallel or perpendicular to the direction of $\mathbf{B}$. Additionally, when a magnetized plasma moves with an average velocity $\mathbf{v}$, it generates an electric field $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$. This electric field is unique because it isn't influenced by Debye shielding, which usually screens electric fields in plasmas. Particles in a magnetized plasma move freely along the direction of $\mathbf{B}$ while following circular Larmor orbits, or gyro-orbits, in the plane perpendicular to $\mathbf{B}$ [16]. The typical Larmor radius, or gyroradius, of a charged particle in a magnetic field is given by $\rho = \frac{v_t}{\Omega}$, where $\Omega = \frac{eB}{m}$ is the cyclotron frequency associated with the particle's circular motion. A plasma is considered magnetized if its characteristic length scale L is much larger than the gyroradius. Conversely, if $\rho \gg L$, the particles move in nearly straight lines.

In plasma physics, an electromagnetic electron wave is characterized by a magnetic field component and involves oscillations primarily of the electrons. Adding a magnetic field to plasma waves introduces new phenomena like anisotropy, Alfvén waves, Larmor orbit effects, and other modified or new waves. Since nearly all realistic plasmas found in laboratories, as well as in the ionosphere and astrophysical environments, have magnetic fields, a significant effort is required to study these effects. This effort aims to understand and categorize the wide variety of phenomena arising within these plasmas, which can often be quite complex and varied [17, 18].

1.6 RELATIVISTIC PLASMA

In classical statistical physics, space and time are treated differently, which complicates making the theory relativistic. All physics theories, including those for systems like plasmas, must follow the principles of special relativity. This makes it essential to develop a relativistic theory for statistical systems [19], notably for plasmas. Relativistic plasma theory is particularly important in fields like astrophysics and controlled fusion. Many astrophysical environments, such as the solar corona and the atmospheres of hot stars, feature relativistic plasmas. In controlled thermonuclear reactions, where temperatures can reach the order of mc^2 (with m being the electron mass), at least the electrons need to be treated relativistically [20].

When plasma becomes extremely hot, reaching relativistic temperatures, it's temperature surpasses the remaining mass energy of electrons. In relativistic plasma, the average energy of each particle exceeds the electron's rest mass, leading electron-positron pairs to emerge and influence the plasma characteristics. In Active Galactic Nuclei, plasma near a black hole has ion temperatures around 10^{13} K and electron temperatures around 10^9 K due to rapid cooling. At these high temperatures, electron-positron pair creation and annihilation become important, as noted by Tajima and Taniuti [21]. So, one needs to require the relativistic correction to a particles mass and velocity. Because, the relativistic corrections mainly become significant when a notable number of plasma particle achieve speeds beyond 0.86 times to the speed of light [22]. In addition, the massive particles is required more energy to accelerate to a significant fraction of c in the production of quark–gluon plasma. Bhattacharyya [23] has been confirmed the velocity of ions is approaching to the light speed c by studying the intensity-induced frequency shift and precessional frequency rotationally polarized waves in magnetized plasma.

A thermal relativistic plasma is characterized by its particles following a Maxwell-Boltzmann distribution at a certain temperature T [24] [25]. In the current universe, no astrophysically significant entities are in complete thermal equilibrium at relativistic temperature. Therefore, it was necessary to solve kinetic equations to estimate the equilibrium distributions of protons and particles. Aside from intrinsic theoretical interest, there are some observation of nonthermal relativistic plasma. Relativistic plasma research covers various phenomena and uses, from creating specific structures to controlling strong laser fields and plasma waves. It's essential for progress in fields like astrophysics, laser interactions, and fusion studies.

1.7 DISTRIBUTION FUNCTIONS

From the perspective of classical mechanics, phase space denotes the possibility of potential states within a physical system, where state refers not only the position (q) of the object in the system but also the velocity (v) of the object. The instantaneous dynamic state of every particle can be defined by its position and velocity. Essentially, it's a multidimensional space that has all the information needed to describe the complete behavior of the system. Therefore, to ordain a system's future behaviour, one requires to understand both the system's position and velocity. Let, a particle in motion in a one-dimensional space, whose position and velocity can be expressed by $q = q(t)$ and $v = v(t)$. A two-dimensional graph is an approach to concurrently visualise the q and v trajectories, where $q(t)$ and $v(t)$ represent the horizontal and vertical coordinates, respectively. This $q - v$ plane is called phase space and at any point in time, every particle possesses a particular position and velocity. If there are N particles and each particle has a position and velocity q_i and v_i (where $i = 0, ..., N$), then the total number of coordinates is $6N$. The actual configuration of the system at any given time is expressed by one specific location in the $6N-$dimensional space is $(q_1...q_N, v_1...v_N)$ [26]. A differential element of volume can be visualised as a six-dimensional cube, denoted by

$$f(q,v,t) = \frac{dn(q,v,t)}{dq_x dq_y dq_z dv_x dv_y dv_z} \tag{1.15}$$

is then the number of particles in the volume element $dq_x dq_y dq_z$ at position q and the element $dv_x dv_y dv_z$ in the velocity space with velocity v, at time t. The function that describes the instantaneous density of particles in phase-space is known as the distribution function and is represented by $f(q,v,t)$ [1]. This function typically adheres to the normalization condition $\int f\, dv\, dq = 1$, where the integral is taken over the entire phase space. This condition ensures that the total probability of all possible states is equal to one.

A plasma is a combination of an enormous amount of particles that are not bound together in neutral atoms or molecules but instead exist in a highly ionized and electrically conductive state. Thus, using a statistical method is useful for providing a macroscopic explanation of plasma processes. In plasma physics, distribution function describes the statistical distribution of particle properties within a plasma system. This distribution function is often represented by $f(\mathbf{r}, \mathbf{v}, t)$, gives the probability density of finding a particle with a particular position **r** and velocity **v** at any time t. It describes the statistical behavior of particles in phase space, which is a mathematical space where each point represents the position and velocity of a particle. The form of the distribution function depends on various factors, including the characteristics of the plasma (such as temperature, density, and magnetic field strength), the presence of external forces or fields, and the interactions between particles (such as collisions or collective effects). This function must be continuous in its variables, positive and finite at all times, and tend toward zero as the velocity becomes infinitely large [13, 27].

A plasma is considered inhomogeneous when its distribution function depends on position **r**. Without external forces, an inhomogeneous plasma can reach an equilibrium state through mutual particle interactions, resulting in a homogeneous state where the distribution function no longer depends on **r**. The distribution function can also be "isotropic" or "anisotropic" based on whether it depends on the orientation or only the magnitude of the velocity vector **v**. Describing plasmas requires the use of inhomogeneous or homogeneous, as well as anisotropic or isotropic distribution functions. A plasma in thermal equilibrium is characterized by a homogeneous, isotropic, and time-dependent distribution function. One main challenge in kinetic theory is determining the distribution function for a given system, which is governed by the Boltzmann equation, describing its temporal and spatial variations [13].

1.7.1 MAXWELLIAN DISTRIBUTION

The distribution function describing the system under investigation must be known to determine the average value of the physical properties of the particles. A useful way to describe plasma dynamics is to consider that plasma particle motions are influenced by both external fields and the averaged internal fields, smoothed over space and time. This approach results in the Vlasov equation, which is derived from the Boltzmann equation. The Maxwellian velocity distribution is the most probable distribution that meets the macroscopic conditions of a system and occurs when particles are in thermal equilibrium [13, 27]. For a gas where particles move in only one dimension, the one-dimensional Maxwellian distribution

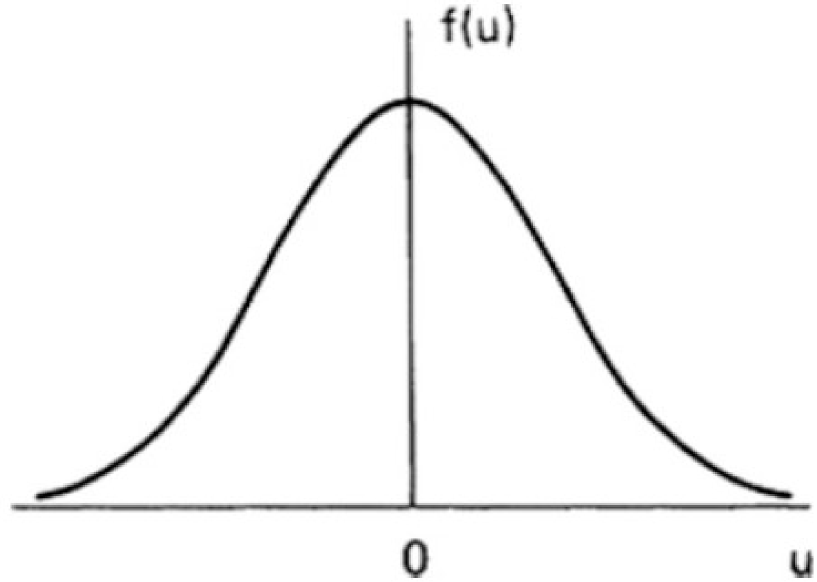

Figure 1.1: A Maxwellian distribution function [6]

for particle species h is given by,

$$f_h(v) = Ae^{-\frac{m_h v^2}{2k_B T_h}}. \tag{1.16}$$

This is homogeneous and isotopic, where $A = n_h \left(\frac{m_h}{2\pi k_B T_h}\right)^{1/2}$, $f_h dv$ is the number of particles per cubic meter with velocity between v and $v + dv$, k_B is the Boltzmann constant ($k_B = 1.39 \times 10^{23} JK^{-1}$) and T_h is the particles temperature. The density n_h or number of particles per cubic meter can be written as

$$n_h = \int_{-\infty}^{\infty} f_h(v) dv. \tag{1.17}$$

If we define $v_{th} = \left(\frac{2k_b T_h}{m}\right)^{1/2}$, then the function f_h describes a three-dimensional distribution of velocities can be described as

$$f_h(v) = \frac{n}{\left(\sqrt{\pi} v_{th}\right)^3} e^{-\frac{\mathbf{v}^2}{v_{th}^2}}. \tag{1.18}$$

By differentiating, it is possible to illustrate the maximal of this distribution is at $v = v_{th}$. For a Maxwellian distribution, several key characteristic velocities can be associated with temperature. In three dimensions, the average particle kinetic energy can be calculated as $E_{av} = \frac{3}{2}k_B T$ [28]. Plasma can exhibit multiple temperatures simultaneously, with ions and electrons often having distinct Maxwellian distributions and different temperatures (T_i and T_e) due to the higher collision rates within each species. Even one species say ions, may possess multiple temperatures in a field of magnetic force because of the Lorentz effect, which causes the forces along the magnetic field **B** to vary from those perpendicular to it [6].

1.7.2 NON-MAXWELLIAN DISTRIBUTION

In plasma, particles frequently collide and move at varying velocities, exchanging kinetic energy and momentum to reach thermal equilibrium. The Maxwellian distribution effectively describes a plasma system where particles are in thermal equilibrium. However, the particles are deviated from the Maxwellian case in describing long-range interacting systems (like coulomb and gravitational). It is experimentally confirmed such deviations for electrons due to the temperature gradient of electrons are dormant. Non-Maxwellian distributions are common in various space plasma phenomena, such as the interstellar medium [29], thermosphere [30], ionosphere [31], solar wind [32], planetary magnetospheres [33], and magnetosheaths. Vasylinuas [34] was the first to analyze the energy spectra of electrons within the plasma sheet. In 1955, Renyi [35] introduced a generalization of Boltzmann-Gibbs statistics, which was later expanded by Tsallis [36] in 1988 with the addition of a non-extensive parameter q. Observations made by the Freja satellite [37] and Viking spacecraft [38] in 1995 studied the rarefaction of ion number density, known as cavitons. Cairns et al. [39] proposed the Cairns distribution, a type of non-thermal distribution, and investigated cavitons in non-thermal equilibrium plasma systems. Subsequently, Qureshi [40] introduced another generalized distribution function with spectral indices r and q, called the (r, q)-distribution function, which describes energetic particles and superthermality in the velocity distribution curve of plasma species. The Maxwellian distribution and plasma dispersion function are commonly used to model tenuous and collisionless plasmas in space and astrophysics, even though these plasmas are often far from equilibrium [27, 41, 42]. Many space and astrophysical plasmas, such as those found in the solar wind [32], planetary magnetospheres [33], and the interstellar medium [29] do not conform to Maxwellian distributions. These environments often exhibit conditions far from thermal equilibrium, making non-Maxwellian distributions more accurate for representing particle velocities and energies. Observations frequently show the presence of high-energy particles that deviate from the Maxwellian distribution. These particles often follow a power-law distribution, which can better account for phenomena such as superthermal particles and energetic tails in the velocity distribution [43, 44]. Hence, by considering non-Maxwellian distributions, scientists can achieve a more accurate and comprehensive understanding of plasma behavior, leading to better theoretical models, more precise experimental interpretations, and improved predictions of plasma phenomena in both space and laboratory settings.

$q-$ DISTRIBUTION

In many astrophysical Environments space plasmas clearly indicate the particles are deviated from the Maxwellian velocity distribution [45]. For this reason, A generalized Boltzmann-Gibbs statistics which was introduced by Renyi in 1955 [35], later, Proposed by Tsallis in 1988 [36] by adding an additional nonextensive parameter q. This new statistical approach has been successfully applied to various systems with long-range interactions. The parameter q, which bases the Tsallis's generalized entropy, is associated with the system's fundamental dynamics and quantifies the degree of nonextensivity. The q-nonextensive distribution of electrons and positrons, introducing the plasma nonthermality, is described by the following distribution function,

$$f_e(v) = C_q \left\{ 1 + (q-1)\frac{m_e v^2}{2T_e} - \frac{e\phi}{T_e} \right\}^{\frac{1}{q-1}}. \tag{1.19}$$

Here, C_q is the normalization constant, and the parameter q represents the degree of nonextensivity. It is essential to note here that for $q < -1$, the q-distribution cannot be normalized. In the extensive limit, as q to approaches 1, this distribution simplifies to the well-known Maxwellian distribution. By integrating the function in equation (1.19), one obtains the number density of electrons and positrons (N_h) as,

$$N_h = N_{h0}\left[1 \pm (q-1)\frac{e\phi}{k_B T_e}\right]^{\frac{1+q}{2(q-1)}}. \tag{1.20}$$

The condition $q < 1$ refers to the superextensive case [46–48], while $q > 1$ refers to the subextensive case [42]. Recently, Verheest [49] suggested that only the range $1/3 < q < 1$ should be used for the superextensive case and $q > 1$ for the subextensive case to explain the effects of nonextensivity on the characteristics of nonlinear structures. The q-distribution better represents these non-equilibrium conditions compared to the traditional Maxwellian distribution. It provides a more comprehensive and accurate framework for describing plasmas under non-equilibrium conditions, with long-range interactions, and in the presence of high-energy particles, making it an essential tool in plasma physics research.

$(\alpha, q)-$ **DISTRIBUTION**

The $(\alpha, q)-$velocity distribution functions are very useful to describe the energy of electrons in all cases of thermality since the energies of the electrons may be isothermal, nonthermal or have a smaller (subthermal) or superior (superthermal) amount of isothermality. The (α, q) distribution is defined by the composition of Tsallis [36] and Cairns [39] velocity distribution functions [50] as

$$y(v_x) = k\left(1 + \alpha\frac{v_x^4}{v_t^4}\right) \times \left\{1 - (q-1))\frac{v_x^2}{2v_t^2}\right\}, \tag{1.21}$$

where $v_t = (k_B T_e/m_e)^{1/2}$ is the electron thermal velocity, $v_x = (2e\phi/m_e)^{1/2}$ is the velocity vector, q is the nonextensivity strength, α represents the population of faster electrons, k_B is defined as Boltzmann constant and k is the normalized constant, respectively [50]. When

$q > 1$, a thermal cut-off on the maximum value allowed for electrons velocity is defined [50] as $v_{max} = \sqrt{2k_B T_e/m_e(q-1)}$ beyond which no probable states exist. Hence, the electron density function (N_e) can be written by integrating the above equation over velocity space in which includes an additional potential term of interacting electrons $\left(v_x = \sqrt{\frac{2k_B e\phi}{m_e}}\right)$ [51] that is

$$N_e = \begin{cases} \int_{\infty}^{\infty} y(v_x) dv, -1 < q < 1 \\ \int_{v_{max}}^{-v_{max}} y(v_x) dv_x, q > 1 \end{cases}, \tag{1.22}$$

where ϕ, T_e and e are the electrostatic potential function, electron temperature and magnitude of electron charge, respectively. Note that the above equation is applicable if nonthermality and nonextensivity may act concurrently on the nature (rarefactive or compressive) of the acoustic wave mode [50]. Later, the proper ranges of α and q are defined in Refs. [52, 53] based on the physical cut-off obligatory by $q \geq 5/7$, and $\alpha_{Max} = (2q-1)/4$ as (i) $q = 1, 0 < \alpha < 0.35$ (nonthermality case), (ii) $q = 1, \alpha = 0$ (isothermality case), (iii) $0.33 < q < 1, \alpha = 0$ (superthermality case), and (iv) $q > 1, \alpha = 0$ (subthermality case), respectively.

1.8 ELECTRON-POSITRON-ION PLASMA

Electron-positron (e-p) plasma is commonly defined as a completely ionised gas comprised of electrons and positrons of equal mass and opposing charge. Theoretical studies of e-p plasma and its electromagnetic wave dispersion properties are of significant interest due to their relevance in various scenarios. These include astrophysical and cosmological contexts and laboratory experiments involving ultra-intense laser pulses interacting with matter. The e-p plasmas may be expected to be present in pulsar magnetospheres [54, 55], in Active Galactic Nuclei [56, 57], around black holes, in the early universe [58–60], and at the centre of this galaxy [61]. Additionally, e-p plasmas are argued to be present during the MeV era of the very early universe. According to the conventional cosmological model, temperatures in the MeV range $(T \approx 10^{10} K)$ prevail up to one second after the Big Bang [62].
While electron-positron pairs make up the majority of the astrophysical and cosmic plasmas mentioned earlier, a small number of heavy ions are also likely present. For instance,

during the MeV epoch of the early universe, there were approximately 10^9 to 10^{10} protons and neutrons for every light particle (such as electrons, neutrinos, and photons), based on the current understanding of baryon asymmetry. Earlier, at times less than 100 seconds after the Big Bang, the primordial plasma contained π mesons, K mesons, and pairs of protons-antiprotons and neutrons-antineutrons. Consequently, environments with protons or other ions suggest the natural occurrence of a three-component electron-positron-ion (e-p-i) plasma.

Cosmic-ray measurements of electrons and positrons provide insights into the nature and distribution of galactic sources and the characteristics of cosmic ray propagation in the galactic disk and halo [63]. Research referenced in [63] has shown significant discrepancies between cosmic-ray positrons and electrons, especially at energies above 10 GeV. The PAMELA satellite later confirmed the abundance of positrons in cosmic radiation with energies ranging from 1.5 to 100 GeV [64]. The composite system gains intriguing features from the presence of the minority ion population. Due to their long lifespan, the positrons may be utilised to investigate particle transport in tokamaks, transforming the two-component e-i plasma into a e-p-i one [65, 66]. This three-component plasmas have been studied in various contexts, including pulsar magnetospheres [67, 68]. Additionally, theoretical investigations have explored relativistic collisionless shock waves in e-p-i plasmas, which are relevant to astrophysical sources of synchrotron radiation [69].

1.9 WAVES IN PLASMA

Waves in plasmas are a connected system of particles and fields that propagate periodically. Plasma is a conductive, quasineutral fluid composed of electrons and positive ions, and may also include negative ions, positrons, or dust particles. Because it has a collective nature, plasma interacts with electric and magnetic fields, which give rise to various wave phenomena. Studying these waves is beneficial for plasma diagnostics as they depend on plasma properties. Waves of plasma could be categorised as electromagnetic or electrostatic following the existence of an oscillating magnetic field. Different types of waves in plasma are influenced by the oscillating species within it. Typically, electron temperatures exceed ion temperatures, reflecting the much lighter mass of electrons and their consequent faster movement. Modes associated with electron motion are affected by electron mass, while ions are often considered stationary due to their larger inertia. In some cases, such as the lower hybrid wave, modes may involve both electron and ion inertia. Waves can also be categorized based on their orientation relative to the magnetic field. Plasma's complexity allows it to support nonlinear waves, resulting in phenomena like solitons, double layers, and vortices, observed in laboratories, space, and astrophysical plasmas.

Plasma waves are of mainly two types; linear waves with very small amplitude and nonlinear waves with large amplitudes. When we study linear plasma waves, we consider and write variable quantities as perturbed and unperturbed state of plasma. This method allows us to neglect nonlinear terms and write a basic linear equation. One must remember that only small wave amplitude is considered for linear theory of plasma waves. In case, if we consider large wave amplitude then nonlinearities are taken into account. Solitary waves are very important example while studying nonlinear plasma waves [27, 70].

1.9.1 SOLITARY WAVE AND SOLITON

A common theme in the development of science is that of a important discovery which is not widely recognised as such when it is first reported. Most often this comes about not through the ignorance or indifference of the scientific community, but because the current state of knowledge of the field is insufficiently developed for the full significance of the result to be realized.

The first conscious observation of what was termed a solitary wave in 1834 was not appreciated until its significance as an important stable state of some nonlinear system was realized in the mid-1960's. The solitary wave emerged as a mathematical fascination approximately a century ago. Still, it continues to spread across multiple practical mathematics and physics domains, including meteorology, basic physics for particles, plasma research, and laser physics.

Scottish scientist and engineer John Scott Russell published the first recognized observation of the solitary wave in 1834. A new wave on the canal's surface attracted Scott Russell's attention while he observed a canal barge. This observation was not purely chance counter. He was studying canal barge designs for the Union Canal Society of Edinburgh on an unpaid basis. The first encounter of the solitary wave had been near the "Hermiston Experimental Station" on the canal, approximately six miles from the heart of the Scottish capital [71,72]. Since this particular kind of wave exists independently of other oscillatory motion of waveforms, Scott Russell himself developed the term "solitary wave" [73–75]. He carried out investigations in which he would place weights at one end of a lengthy, shallow water canal. Over the next ten years, he continued this experiment to examine the single wave in tanks and canals and discovered that it was an autonomous dynamic entity travelling at a consistent speed and form. He illustrated four aspects utilising a wave tank [72]:

- The basic structure of solitary waves is $hsech^2[k(x-vt)]$;
- A sufficiently large initial quantity of water generates two or more separate solitary waves;
- Solitary waves that are solely cross one another "without change of any kind";
- The velocity of a wave with height h and depth d in a channel may be expressed as $v=\sqrt{g(d+h)}$, where d represents the greatest amplitude above the water's surface, h denotes a limited depth, and g represents the acceleration of gravity.

According to the correlation above, a solitary wave with a high amplitude travels quicker than one with a low amplitude. Accordingly, a solitary wave is one that, when observed in the reference frame travelling at the wave's group velocity, propagates without experiencing no significant evolution in its overall dimensions or structure. This results from the nonlinear and dispersive effects being balanced [76].

However, the mathematical community refused to embrace the empirical findings of Russell. In 1845 publishing a theory of long waves in his work "Tides and Waves", Airy further identified a relationship between a wave's height and amplitude and its speed. Based on his findings, Airy assessed that a single wave could not exist, which led to a verbal confrontation between Russell and Airy. After Korteweg and de Vries developed their well-known equation (referred to as the KdV equation) in 1895, this controversy was eventually resolved and the notion of solitary waves was confirmed analytically and this equation demonstrates the solitary character of shallow water waves [77].

Scientists have undertaken a massive amount of research to examine the topic of solitary waves since its identification. Also the propagation of plasma waves in a dispersive medium can be explained by the KdV equation.

1.10 LAYOUT OF THE THESIS

The focus of this thesis is to investigate nonlinear ion-acoustic solitons (IASs) generated by completely ionized, collisionless, magnetized, obliquely propagated e-p-i relativistic plasmas observed in astrophysical, space, and laboratory environments. Intending to get an in-depth understanding of the relevant physics, the purpose of the study is to investigate the production of nonlinear IASs in e-p-i plasmas for highly relativistic regimes.

This chapter addressed the basic phenomena of plasma physics as well as a brief history of its development. There has also been discussion of wave phenomena, Maxwellian, Non-Maxwellian, and plasma in the relativistic region.

In Chapter-2, The fluid behaviour of plasma and its model equations have been discussed. This chapter additionally discusses the widely used Reductive perturbation approach.

In Chapter-3, The nonlinear obliquely propagating IASs by proposing a magnetized rotating relativistic plasma environment having relativistic ion fluids, and non-extensive electrons as well as positions have been investigated. KdV equation involving the potential function has been derived by using the conventional reductive perturbation method for analyzing such wave phenomena. The effect of plasma parameters on the amplitude and width of IASs has been discussed with the consideration of the relativistic Lorentz factor (RLF) up to eleven terms for the first time. It is observed that the RLF up to eleven terms and obliqueness significantly modify the propagation characteristics of IASs in the considered plasma environment.

In Chapter-4, the study focused on the nonlinear propagation of IASs for strongly relativistic plasmas involving relativistic ions and (α, q)-distributed electrons and positions. Leveraging the renowned reductive perturbation approach, the mKdV equation involving the potential function has been developed to analyse such wave phenomena. The effect of plasma parameters on the amplitude and width of IASs has been discussed with the consideration of the RLF up to twenty terms for their critical values. The RLF up to twenty terms and obliqueness is observed, which significantly modify the propagation characteristics of IASs in the considered plasma environment.

In Chapter-5, The KDV equation with quartic nonlinearity is developed to investigate the properties of nonlinear propagation of IASs in strongly relativistic plasma having relativistic ions and $(\alpha, q)-$ distribution of electrons and positrons applying the conventional reductive perturbation approach. The effect of plasma parameters on the amplitude and width of IASs has been discussed with the consideration of the RLF up to twenty terms for their super-critical values. The effect of the RLF up to twenty terms, obliqueness and magnitude of the rotational frequency on the propagation characteristics of IASs in the considered plasma environment is observed.

Finally, in Chapter 6, this work concludes with the remarks and the potential future works for further investigations are discussed.

Chapter 2: THEORETICAL MODEL EQUATIONS AND METHODOLOGY

2.1 INTRODUCTION

Plasma, a state of matter consisting of charged particles, is a subject of extensive research due to its unique properties. While plasma exhibits complex behaviour across a wide range of scales and environments, it can often be treated as fluid on macroscopic scales. Studies on plasma have provided insight into its complex fluid dynamics, including viscoelastic and non-Newtonian behaviours. For several fields of study, such as astrophysics, fusion research, space science, and industrial applications, a comprehension of the fluid behaviour of plasma is significant.

When the phase velocity of a wave excited in plasma exceeds the thermal velocity, most of the plasma particles are not in resonance with the wave. Consequently, only a small number of particles exchange energy with the wave. A fluid state of the plasma can be implemented in this scenario. In the plasma fluid, the electron and ion fluids are supposed to interact with each other through the electromagnetic field and exchange momentum and energy through collisions. While kinetic theory explains transport coefficients based on collisions, those linked to plasma waves can be understood by using fluid theory [78].

One can explore the fundamental principles underlying the fluid behaviour of plasma from its collective motion and MHD phenomena. Considering plasma as a fluid, one may utilise established concepts from classical fluid dynamics to describe and predict its behavior in diverse scenarios. Through this exploration, one can provide insights into the rich and dynamic nature of plasma as fluids, highlighting both their fundamental properties and their practical implications in scientific research and technological applications [79].

2.2 MAXWELL'S EQUATION

Maxwell's equations [6] are fundamental in describing electromagnetism. They unify electric and magnetic fields and predict electromagnetic wave phenomena across various frequencies [80]. These equations have been approached from various perspectives, including an axiomatic approach [81], an inverse scattering problem for the time-dependent Maxwell equations [82], and an examination of their relationship with the continuity equation and charge conservation [83]. The study of Maxwell's equations has also extended to applications in different fields, such as geophysics, optics [84], plasma physics and etc.

In plasma physics these equations are foundational, governing the behaviour of electromagnetic fields in plasmas. These equation narrates how electric and magnetic fields are generated by charges, currents and change of each other. In plasma simulations, these equations are essential for comprehending the dynamics of plasma particles and fields. Specifically, the Maxwell-Faraday equation, a component of Maxwell's equations, can be adjusted using extended stencils to improve its numerical treatment [85]. These equations are as given below

$$\begin{aligned} \varepsilon_0 \overline{\nabla} \cdot \mathbf{E} &= \rho, \\ \overline{\nabla} \cdot \mathbf{B} &= 0, \\ \overline{\nabla} \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\ \overline{\nabla} \times \mathbf{H} &= \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \end{aligned}$$

where $\varepsilon_0, \rho, \mathbf{E}, \mathbf{B}, \mathbf{H}, \mathbf{J}$ is the permittivity of vacuum, charge density, the electric field, the magnetic induction, magnetic field, and the current density, respectively.

2.3 EQUATION OF CONTINUITY

The equation of continuity [6,79] for plasma describes the conservation of mass and charge within the plasma, expressing how the density and velocity of charged particles change over time. It states that the rate of change of particle density within a given volume is equal to the net flux of particles across the boundary of that volume. To maintain matter conservation, let the total number of particles N within a volume V changes only if there's a net flow of particles across its boundary surface S. This flow, represented by $n_j \mathbf{U}_j$, is determined using the divergence theorem,

$$\oint_v \frac{\partial n_j}{\partial t} dV = \oint_v (n_j \mathbf{U}_j) \cdot dS. \tag{2.1}$$

Since this rule applies to any volume V, the expressions being integrated must be equal,

$$\frac{\partial n_j}{\partial t} + \overline{\nabla} \cdot \left(n_j \mathbf{U}_j \right) = 0. \tag{2.2}$$

For each particle species in plasma, there's an equation of continuity, ensuring mass conservation. Any sources or sinks of particles are accounted for on the right-hand side of the equation. This equation ensures that mass and charge are conserved within the plasma, with any changes in density being balanced by corresponding fluxes across the plasma boundary. The equation of continuity is a cornerstone of plasma physics, giving essential insights into the complex dynamics and behaviors of plasma across a broad spectrum of scales and environmental conditions.

2.4 EQUATION OF MOMENTUM

In the fluid approximation, plasma appears as a mixture of distinct fluid components (such as ions, electrons, positrons etc.) and each representing a different species within the plasma. As the nature of particles in a plasma system differs within particular species, they respond in different ways in the presence of electric and magnetic fields. Yet, in the majority of instrumental uses, the plasma's collective behaviour i.e., the prevalence of particle collision effects and the plasma's more continuum-like behaviour is significant. In the most basic scenario, the equations of motion are required for the negatively charged electron fluid and the positively charged ion fluid. One can also consider an expression for the fluid of neutral particles in a partly ionized gas. Via the collisional interaction the neutral particles could operate with the ions and electrons. For any specific application, an alternative may be available for the fundamental equations like multiple dimensions; multicomponent species; scalar, vector, or tensor forms. Also when there are no collisions, the **E** and **B** fields that the ion and electron flows produce will lead them to interact with one another. Maxwell's equations offer insight into how the electric and magnetic forces influence the plasma's current state [70, 86]. Now, one can express the law of motion for a single charged particle having an electromagnetic field as,

$$m\frac{d\mathbf{v_j}}{dt} = q\left(\mathbf{E} + \mathbf{v}_j \times \mathbf{B}\right), \tag{2.3}$$

where, m is the mass of a plasma particle travelling at $\mathbf{v}_j$ velocity with carrying charge q. In the absence of thermal motions and collisions, all particle will move together in a plasma system. So the average velocity $\mathbf{U}_j$ of the system will be equal to the $\mathbf{v}_j$. Hence, the equation of motion for n_j plasma particles will be

$$mn_j\frac{d\mathbf{U}_j}{dt} = qn_j\left(\mathbf{E} + \mathbf{U}_j \times \mathbf{B}\right). \tag{2.4}$$

For the conversion of components within a particular frame, let $\mathbf{U}_j(\mathbf{r},t)$ be any characteristic of fluidity in space with three dimensions. Hence, the time-dependent variation in $\mathbf{U}_j$ in a fluid-moving frame is

$$\frac{d\mathbf{U}_j(\mathbf{r},t)}{dt} = \frac{\partial \mathbf{U}_j}{\partial t} + \frac{\partial \mathbf{U}_j}{\partial x}\frac{dx}{dt} + \frac{\partial \mathbf{U}_j}{\partial y}\frac{dy}{dt} + \frac{\partial \mathbf{U}_j}{\partial z}\frac{dz}{dt},$$

$$\text{Or, } \frac{d\mathbf{U}_j(\mathbf{r},t)}{dt} = \frac{\partial \mathbf{U}_j}{\partial t} + \left(\mathbf{U}_j \cdot \overline{\nabla}\right)\mathbf{U}_j. \tag{2.5}$$

Using equation (2.4) and (2.5) one can write

$$mn_j\left\{\frac{\partial \mathbf{U}_j}{\partial t} + \left(\mathbf{U}_j \cdot \overline{\nabla}\right)\mathbf{U}_j\right\} = qn_j\left(\mathbf{E} + \mathbf{U}_j \times \mathbf{B}\right). \tag{2.6}$$

The addition of an external pressure is necessary on the opposite side of equation (2.6) when the thermal motion is considered. This force is generated by the erratic movement of particles in and out of a fluid component and isn't represented in the calculation for a single particle. Adding the electromagnetic force and the pressure gradient force and expanding to multiple dimensions, one can write the fluid formula as

$$mn_j \left\{ \frac{\partial \mathbf{U}_j}{\partial t} + \left(\mathbf{U}_j \cdot \overline{\nabla} \right) \mathbf{U}_j \right\} = qn_j \left(\mathbf{E} + \mathbf{U}_j \times \mathbf{B} \right) - \overline{\nabla} p. \tag{2.7}$$

For natural gases, charged particles provide momentum by collisional interactions. The momentum loss of each encounter corresponds with the relative velocity $\mathbf{U}_j - \mathbf{U}_0$, wherein $\mathbf{U}_0$ denotes the flow rate of the natural fluid. Assuming τ is the mean free time among collisions, the equation of motion for anisotropic pressure and neutral interactions can be expressed as,

$$mn_j \left\{ \frac{\partial \mathbf{U}_j}{\partial t} + \left(\mathbf{U}_j \cdot \overline{\nabla} \right) \mathbf{U}_j \right\} = qn_j \left(\mathbf{E} + \mathbf{U}_j \times \mathbf{B} \right) - \overline{\nabla} p - \frac{mn_j \left(\mathbf{U}_j - \mathbf{U}_0 \right)}{\tau}. \tag{2.8}$$

For any anisotropic fluid, $\overline{\nabla} p$ is replaced by $\overline{\nabla} \cdot \mathbf{P}$, where $\mathbf{P}$ is the stress tensor,

$$mn_j \left\{ \frac{\partial \mathbf{U}_j}{\partial t} + \left(\mathbf{U}_j \cdot \overline{\nabla} \right) \mathbf{U}_j \right\} = qn_j \left(\mathbf{E} + \mathbf{U}_j \times \mathbf{B} \right) - \overline{\nabla} \cdot \mathbf{P} - \frac{mn_j \left(\mathbf{U}_j - \mathbf{U}_0 \right)}{\tau}. \tag{2.9}$$

However, in the absence of the electric ($\mathbf{E}$) and magnetic ($\mathbf{B}$) field, the law of motion becomes

$$mn_j \left\{ \frac{\partial \mathbf{U}_j}{\partial t} + \left(\mathbf{U}_j \cdot \overline{\nabla} \right) \mathbf{U}_j \right\} = -\overline{\nabla} \cdot \mathbf{P}. \tag{2.10}$$

Ordinary fluids follow the Navier-Stokes equation and often collide with their component particles, hence

$$\rho \left\{ \frac{\partial \mathbf{U}_j}{\partial t} + \left(\mathbf{U}_j \cdot \overline{\nabla} \right) \mathbf{U}_j \right\} = -\overline{\nabla} p + \rho v \nabla^2 \mathbf{U}_j. \tag{2.11}$$

The above equation is identical to equation (2.9), except for lacking electric and magnetic force and collisional interactions within species, where ρ signifies the mass density and $\rho v \nabla^2 \mathbf{U}_j$ represents the viscosity term. A relativistic term (γ) may be included in the equation of motion of plasma species, resulting in a relativistic modification of the equation. This addition enhances the understanding of ion movement and finite temperature effects on strong relativistic plasma waves.

$$mn_j \left\{ \frac{\partial \left(\mathbf{U}_j \gamma\right)}{\partial t} + \left(\mathbf{U}_j \cdot \overline{\nabla}\right) \mathbf{U}_j \gamma \right\} = qn_j \left(\mathbf{E} + \mathbf{U}_j \gamma \times \mathbf{B}\right). \tag{2.12}$$

In deriving equation (2.9), one could implicitly assume numerous collisions, which arises from assuming a Maxwellian velocity distribution. The Maxwellian distribution, typically resulting from frequent collisions, is commonly assumed in fluid theory. While other distributions with the same average yield similar outcomes, deviations from the Maxwellian distribution can be crucial in certain cases, necessitating the use of kinetic theory.
The other reason the fluid model is effective for plasmas is the presence of a **B** which can influence the collisions in certain regions. Particles acceleration by an **E** increases the velocity continuously and allow it to stream freely. And because of the frequent collisions, the particles appear to a constrained velocity corresponding to the **E**. The presence of a **B** restricts free-streaming by causing particles to gyrate in Larmor orbits, resulting in collisional plasma acting like a collisional fluid. Hence, one can take the fluid theory as a valid one for a plasma system. And the equation of motion for plasma is a complex yet crucial aspect of plasma physics, with contributions from various studies shedding light on different facets of plasma behaviour under diverse conditions [6, 79].

2.5 EQUATION OF STATE

Where the transfer of energy via external sources is prohibited in a fluid plasma [6, 79], the fluid's density ρ and pressure p have a relation by

$$p = C\rho^{\lambda}, \tag{2.13}$$

where C is a constant, $\lambda = C_p/C_v$ represents the specific temperature ratio, C_p denotes the specific temperature at a constant pressure, and C_v denotes the specific temperature at a constant volume. So, ∇p can be obtained as

$$\frac{\nabla p}{p} = \lambda \frac{\nabla n_j}{n_j}. \tag{2.14}$$

Considering the isothermal compression, we can write

$$\nabla p = \nabla (nKT) = KT \nabla n_j,$$

evidently, $\lambda = 1$. KT changes during adiabatic compression, resulting in a greater value for λ. If N represents the number of degrees of freedom, λ can be obtained as

$$\lambda = 1 + \frac{2}{N}. \tag{2.15}$$

For the equation of state to be valid, there must be very little heat movement or a weak thermal conductivity. Despite this, most fundamental occurrences may be effectively expressed through the simple assumption of equation (2.13).

2.6 POISSON'S EQUATION

The Poisson equation [6,79,87] serves as an indispensable element in elucidating the electric field dynamics inherent to plasma systems, delineating the intricate interplay between charged particles and electromagnetic fields within such complex environments. It is significant in describing the self-consistent electric field in plasma, along with the Vlasov equation that characterizes the distribution function of electrons and ions. It provides a mathematical framework to determine the electric potential distribution within a plasma, which arises from the collective behavior of charged particles. In the equation, $\nabla^2 \phi$ represents the Laplacian of the electric potential ϕ, which describes how the potential varies spatially. The right-hand side of the equation $\left(\frac{\rho}{\varepsilon_0}\right)$, accounts for the charge density ρ in the plasma divided by the vacuum permittivity. This term quantifies how the distribution of charges influences the electric potential.

$$\varepsilon_0 \overline{\nabla} \cdot \mathbf{E} = \rho = e(n_i - n_e),$$

$$\varepsilon_0 \overline{\nabla} \cdot \left(-\overline{\nabla}\phi\right) = e(n_i - n_e),$$

$$\varepsilon_0 \nabla^2 \phi = e(n_i - n_e), \tag{2.16}$$

where, $\mathbf{E} = -\overline{\nabla}\phi; e, n_i$ and n_e is the charge of particles and the number density of ions and electrons, respectively. Solving Poisson's equation allows us to understand how the electric potential responds to the distribution of charges in the plasma. This, in turn, provides insights into various plasma phenomena, such as the formation of electric fields, the confinement of charged particles, and the behavior of plasma instabilities. Furthermore, the Vlasov-Maxwell equations and its electrostatic equivalent, the Vlasov-Poisson system, contribute to understanding the microscopic dynamics of collisionless and the magnetic plasmas.

2.7 REDUCTIVE PERTURBATION METHOD

In today's physics research, there has been considerable concentration on nonlinear phenomena, since numerous physical events intrinsically exhibit nonlinearity. Nevertheless, the mathematical approaches utilised to analyze these problems remained mostly linear for a considerable period. Initially, two quantities were considered to be dependent on each other only when they were proportional; nonlinear dependency were neglected. This situation arose due to the limitations of mathematics during that period. Where physics responded by consistently resorting to approximations, which enabled the substitution of the original unsolvable nonlinear mathematical problems with linear, solvable ones. From this perspective, the multiscale evaluation is absolutely justifiable: it operates as a perturbative technique with a linear approximation as its initial order. This not only enables the derivation of minor modifications applicable for the same evolution time as in the linear approximation but also is responsible for the cumulative impact over an extended period of evolution [88].

In mathematics and applied mathematics, perturbation theory comprises techniques aimed at approximating a solution to a problem by initially solving a related, simpler problem.

The solution is typically represented as a power series in a small parameter, denoted as ε, within perturbation theory. The initial term of this series is recognized as the solution to the solvable problem. Subsequent terms in the series, involving higher powers of ε, typically diminish in magnitude. An approximate “perturbation solution” is derived by truncating the series, often retaining only the initial two terms: the solution to the known problem and the first-order's perturbation correction. The underlying principle of perturbation theory is to break down a challenging problem into an infinite sequence of comparatively simpler ones. Consequently, the perturbation technique proves to be advantageous because the initial few phases suggest the major aspects of the solution and the subsequent ones provide minor corrections [89].

A nonlinear evolution equation characterising any system of particles may be defined in terms of a single variable and it seems to have a somewhat simple structure. Nonetheless, the initial equations of that system of particles can usually be complex and involve several dependent variables. We require a methodical process that separates each of these sets of equations into easier-to-comprehend forms. Typically, these processes include perturbations; one such technique is the reductive perturbation theory (RPT).

The reductive perturbation technique is a valuable method used to derive simplified models that describe nonlinear wave propagation and interaction [88]. This method has been successfully applied in various fields such as physics, including the study of nonlinear ion-acoustic waves [90], optical solitons [91], dusty plasmas [92], electron-ion plasmas [93], and trapped gases of bosons [94]. The reductive perturbation method involves introducing small disturbances into a stretched coordinate system to analyze nonlinear wave propagation [95]. Through the application of this method, researchers have been able to derive fundamental equations like the KdV equation and the NLSE to explain the evolution of waves in different mediums [96].

In RPT one can express nonlinear evolution equation in terms of a single dependent variable and its partial derivatives with respect to space variables and time. It rescales both time and space variables in the given system and introduces new space variables and time for describing the nonlinear wave phenomena with long wave length. It should be emphasised that this method has shortcomings because it depends on the individual's expertise in choosing the appropriate scales [97]. Since the variety of scale expansion supports the main concepts of the RPT, we stretch the dependent variable utilizing the very small perturbation parameter ε. As instances

$$\begin{aligned} n &= n_0 + \varepsilon n_1 + \varepsilon^2 n_2 + \varepsilon^3 n_3, \\ v &= \varepsilon v_1 + \varepsilon^2 v_2, \qquad . \\ \phi &= \varepsilon \phi_1 + \varepsilon^2 \phi_2. \end{aligned} \tag{2.17}$$

The boundary conditions often indicate whether a first term exists or not. For instance, in the majority of situations, the density is usually disturbed from its equilibrium value; hence, $n \to n_0$ as $x \to \pm\infty$, but in the event of background flow, $v \to 0$ and $\phi \to 0$. The decision-making process is heavily influenced by the external environment. To obtain the dispersion relation for the plasma waves, it is required to consider the linearised version of the model equations. Any oscillatory physical quantity can be expressed as $e^{i\theta}$, where $\theta = kx - \omega(k)t$ and the function $\omega(k)$ (frequency) satisfies the dispersion relation in term of k (the wave number). For propagation of long waves, one can consider $k = e^p K$, where K denotes a new wave number of order one and p is a unrevealed constant to be obtained later. Hence,

$$\theta(x,t) = \varepsilon^p Kx - \omega(\varepsilon^p K)t. \tag{2.18}$$

Since one is dealing with purely dispersive wave, the Taylor series expansion of $\omega(\varepsilon^p K)$ will contain only even or only odd powers K [6]. For nonlinear wave with long wave length only odd powers of K will materialize in the expansion. Therefore the initial two terms in a Taylor expansion for $\omega(k)$ provides $\omega(k) = \omega^{'}(0)\varepsilon^p K + \omega^{'''}(0)\varepsilon^{3p} K^3$. Then equation (2.18) can be expressed as

$$\theta(x,t) = \varepsilon^p K(x - \omega^{'}(0)t) - \varepsilon^{3p} K^3 \omega^{'''}(0). \tag{2.19}$$

Based on the relation (2.19), one can choose a suitable scaling for the independent variables x and t as

$$\xi = \varepsilon^{p}(x-\lambda t); \tau = \varepsilon^{3p} t. \tag{2.20}$$

Stretched variables, notably ξ and τ, are new variables that must undergo significant changes in x and t before they exhibit any noticeable change. It takes an established reasoning to determine the value of p. An excellent option for determining the value of p generally emerges when the fundamental set of equations expands in powers of ε and space and time are similarly rescaled as in equation (2.20). It often turns out that once the KdV and mKdV equation develops then p typically attains the value of 1/2 and 1. To explain the RPT, One can suppose a linear dispersion law for an electrostatic mode as

$$\sum_j \frac{\omega_{pj}^2}{\omega^2 - k^2 v_{thj}^2} = 1. \tag{2.21}$$

For small values of ω and k, equation (2.21) can be approximated as

$$\omega = \lambda k - \frac{1}{A} k^3, \tag{2.22}$$

where $A = 2\lambda \sum_j \left[\frac{\omega_{pj}^2}{\left(\lambda^2 - v_{thj}^2\right)^2} \right]$ and $\frac{1}{A}$ represent the dispersive term's coefficients in KdV or mKdV equation. Also ω_{pi} and ω_{pe} are ion plasma frequency and electron plasma frequency,v_{pi} and v_{pe} are the ion-thermal and electron-thermal speeds, respectively. Here, will be coefficient of dispersion term in the KdV or mKdV equation. The phase for the nonlinear wave with long wave length is given by

$$kx - \omega t = k(x-\lambda t) + \frac{1}{A} k^3 t + ..., \tag{2.23}$$

and leads to the following standard stretching of the KdV equation as

$$\xi = \varepsilon^{1/2}(x-\lambda t); \tau = \varepsilon^{3/2} t. \tag{2.24}$$

Using the relation (2.24) with expansions (2.17), one can obtain the KdV equation for the considered plasma system. Furthermore, the reductive perturbation method has been extended and modified to address specific challenges in different systems. For example, the extended reductive perturbation method has been utilized to study weakly nonlinear waves in collisionless cold plasmas. Additionally, the modified reductive perturbation method has been employed to investigate higher-order terms in perturbation expansions, balancing nonlinearities with dispersive effects to prevent secularities in solutions [98–100]. The reductive perturbation method is a crucial tool for simplifying the analysis of nonlinear wave phenomena across various disciplines, offering valuable insights into the behavior of waves in complex systems.

Chapter 3: SOLITON PROPAGATION IN MAGNETIZED PLASMA SYSTEM

3.1 INTRODUCTION

Over the past few decades, many researchers [41, 42, 46–48, 101–113] have focused their attention on the e-p-i plasmas to investigate the linear and nonlinear acoustic wave propagation by considering various types of plasma environment. For instance, the ion-acoustic wave (IAW) phenomena have extensively studied by many research scholars [42, 46, 101, 105–113] with the existence of nonlinear coherent structures. In most of the studies, the fundamental properties of nonlinear IAWs are studied via the equilibrium state of charged particles in which the total energy is considered to be extensive. However, the particles are deviated from the extensive case in describing long-range (like Coulomb and gravitational) interacting systems [34]. It is experimentally confirmed that such deviation for electrons due to the temperature gradients of electrons are dormant [114]. This discrepancy recommended the extensive distribution function is inadequate for forecasting various waves and instabilities [115]. Whereas, the non-Maxwellian distribution function is commonly used for space plasma phenomena, like, interstellar medium [29], ionosphere [31], solar wind [32], planetary magnetosphere [33], and so on. The non-extensive (α, q)-distribution is one of the most usable as a non-Maxwellian distribution that was proposed in Ref. [36]. Due to the wide application of (α, q)-distribution in many astrophysical and cosmological circumstances, such as proto-neutron stars [116], stellar polytropes [117], quark-gluon plasma [118], and dark-matter halos [119], etc., one can still now consider the non-extensive plasma to describe the underlying physics.

On the other hand, the advanced Satellite for Cosmology and Astrophysics is confirmed that the production of e-p pair with the existence of ions in the peculiar environment. The existence of e-p pair along with the positive ions background are also supported in many astrophysical and space environments due to the oscillatory electron overtake energy $2m_0c^2$

(c is the light speed). Additionally, the energetic heavy ions of energies 0.1 to 100 MeV in the presence of highly energetic e-p pair are existed in interstellar space, solar atmosphere, etc. [120, 121] and formed the e-p-i relativistic plasmas [105–113]. In such plasmas, one needs to require the relativistic correction to a particles mass and velocity. Because, the relativistic corrections mainly become significant when a notable number of plasma particle achieve speeds beyond 0.86 times to the speed of light [22,25]. In addition, the massive particles is required more energy to accelerate to a significant fraction of c in the production of quark–gluon plasma. Bhattacharyya [23] has been confirmed the velocity of ions is approaching to the light speed c by studying the intensity-induced frequency shift and precessional frequency rotationally polarized waves in magnetized plasma. However, researchers [105–113] have only focused their concentration to study the impact of relativistic ions on nonlinear IAWs in e-p-i unmagnetized plasmas by considering either two-terms or three-terms expansions of relativistic Lorentz factor (RLF). Additionally, Hafez [110, 111] has been clearly mentioned that one needs to consider more than three-terms expansion of RLF for improving accuracy of the IAWs propagation characteristics in relativistic plasmas. Further, a few authors [27, 105] have investigated the nonlinear propagation characteristics of IAWs in magnetized relativistic plasmas by considering only two-terms expansion RLF. Malik [105] has been investigated the features of IAWs in weakly relativistic magnetized warm plasma and focused exclusively on the Maxwellian distribution system. Mushtaq and Shah [27] have examined the oblique two-dimensional IAWs described Zakharov-Kuznetsov equation in a weakly relativistic, rotating magnetized e-p-i plasma having Maxwellian distributed electrons and positrons. To the best of author's knowledge, no research work has been made previously in exploring the nonlinear propagation of IAWs in magnetized plasma having (α, q)-distributed electrons and positions by taking RLF up to 11 terms. Thus, the presented work explores the propagation of ion-acoustic solitons (IASs) by consider an magnetized plasma environment consisting of worm ions fluids, (α, q)-distributed electrons and (α, q)-distributed positions, where the ion fluid velocity is comparable to the speed of light. The effect of parameter on the propagation characteristics of IASs is investigated by deriving the Korteweg de-Vries equation from the proposed theoretical model equations.

3.2 THEORETICAL MODEL EQUATIONS

Let us consider a three component collision-less e-p-i plasma with the presence of relativistic ion-fluids and having nonthermal (α, q)- distributed charged particles. The considered plasma system is presumed to be magnetized, relativistic, rotating and propagates in the (x, y) plane, where the external magnetic field is directed to the x axis, i.e. $\mathbf{B_0} = B_0\hat{x}$. In this scenario, the normalized theoretical equations governing the nonlinear IASs in a rotating magnetized relativistic plasmas can be defined as:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) + \frac{\partial}{\partial y}(n_i v_i) = 0 \tag{3.1}$$

$$\frac{\partial}{\partial t}(\gamma u_i) + u_i \frac{\partial}{\partial x}(\gamma u_i) + v_i \frac{\partial}{\partial y}(\gamma u_i) + \frac{\partial \phi}{\partial x} + \frac{\sigma}{n_i}\frac{\partial n_i}{\partial x} = 0 \tag{3.2}$$

$$\frac{\partial v_i}{\partial t} + u_i \frac{\partial v_i}{\partial x} + v_i \frac{\partial v_i}{\partial y} + \frac{\partial \phi}{\partial y} + \frac{\sigma}{n_i}\frac{\partial n_i}{\partial y} - \Omega_c w_i = 0 \tag{3.3}$$

$$\frac{\partial w_i}{\partial t} + u_i \frac{\partial w_i}{\partial x} + v_i \frac{\partial w_i}{\partial y} + \frac{\partial \phi}{\partial x} + \Omega_c v_i = 0 \tag{3.4}$$

Here u_i, v_i and w_i represents the ion fluids velocity along the x, y and z axis, respectively. These quantities are normalized by $c_{si} = (T_e/m_i)$ and n_i is the i species particle density normalized by their unperturbed density n_{r0} ($r = i$ for ions, e for electrons and p for positrons, respectively). We have taken $\mathbf{E} = -\nabla\phi$, where $\mathbf{E}$ is the electric field and ϕ is the wave potential function normalized by (T_i/e). The angular velocity along the x axis is $\Omega = \Omega_0\hat{x}$ (where $\hat{x}$ is the unit vector and Ω_0 is the magnitude of rotational frequency) and $\omega_{ci} = \mathbf{eB_0}/m_i c$ is the ion gyro-frequency (where m_i is the mass of ion and e is the magnitude of electron charge) and $\Omega_c = \omega_{ci} + \Omega_0$. The spacetime co-ordinates are normalized by the Debye length $\lambda_D = \sqrt{T_e/4\pi n_i e^2}$ (the ion Plasma period) and $\sigma = T_i/T_e$ is the ratio of ion to electron temperature. Also the relativistic Lorentz factor $\gamma = 1/\sqrt{(1-\gamma_0^2)}$ and $\gamma_0 = \frac{u_i}{c}$.

In plasma physics, the charge distribution is necessary to describe the statistical distribution of charged particles (such as electrons and ions) for studying plasma dynamics, collisions, transport phenomena as well as for developing models and simulations of plasma behavior in various applications such as fusion research, space physics and semiconductor processing. Maxwellian velocity distribution represents the most probable distribution function satisfying the macroscopic conditions imposed on the system and occurs when particles are in thermal equilibrium. However, the velocity distribution of particles in space plasmas has a non-Maxwellian superthermal tail and decreases generally as a power law of the velocity. Since the inertialess charged particles are nonthermality extensive, we assume the (α, q)-distribution for their thermal pressure. Hence, based on this (α, q)-distribution function the electron density (N_e) and the positron density (N_p) can be written as

$$\left.\begin{aligned} N_e &= \rho\left[1+(q-1)\left(\frac{e\phi}{T_e}\right)\right]^{\frac{q+1}{2(q-1)}} \times \left[1-B_1\left(\frac{e\phi}{T_e}\right)+B_2\left(\frac{e\phi}{T_e}\right)^2\right], \\ N_p &= (1-\rho)\left[1-(q-1)\left(\frac{e\delta\phi}{T_p}\right)\right]^{\frac{q+1}{2(q-1)}} \times \left[1+B_1\left(\frac{e\delta\phi}{T_p}\right)+B_2\left(\frac{e\delta\phi}{T_p}\right)^2\right], \end{aligned}\right\} \tag{3.5}$$

where $B_1 = \frac{16\alpha q}{3-14q+15q^2+12\alpha}$ and $B_2 = \frac{16\alpha q(2q-1)}{3-14q+15q^2+12\alpha}$. And e, T_e and T_p are the magnitude of the electron charge, electron temperature and positron temperature, respectively. $\rho = \frac{1}{1-p}$, where $p = \frac{N_{p0}}{N_{e0}}$ is the ratio of positron number density and electron number density and $\delta = \frac{T_e}{T_p}$ is the temperature ratio of electron to positron. Here, Eq. (3.5) is applicable if nonthermality and nonextensivity may act concurrently on nature (rarefactive and compressive) of the acoustic wave. Later the proper ranges of α and q are defined in Refs. [52,53] based on the physical cut-off obligatory by $q \geq 5/7$, and $\alpha_{Max} = (2q-1)/4$ as for superthermal case $0.33 > q > 1, \alpha = 0$; for isothermal case $q = 1, \alpha = 0$; for nonthermal case $q = 1, 0 < \alpha < 0.35$; for subthermal case $q > 1, \alpha = 0.35$. With the consideration of charge neutrality condition, the plasma environment is closed by the following normalized

Poisson's equation:

$$\nabla^2\phi = 1 + k_1\phi + k_2\phi^2 + k_3\phi^3 + k_4\phi^4 + ... - n_i \tag{3.6}$$

where

$$k_1 = [\rho - (1-\rho)\delta] \times \left(\frac{q+1}{2} - B_1\right),$$

$$k_2 = \left[\rho + (1-\rho)\delta^2\right] \times \left(B_2 - \frac{(q+1)B_1}{2} + \frac{(q+1)(3-q)}{8}\right),$$

$$k_3 = \left[\rho - (1-\rho)\delta^3\right] \times \left(\frac{(q+1)B_2}{2} - \frac{(q+1)(3-q)B_1}{8} + \frac{(q+1)(3-q)(5-3q)}{48}\right),$$

$$k_4 = \left[\rho + (1-\rho)\delta^4\right] \times \left(\frac{(q+1)(3-q)B_2}{8} - \frac{(q+1)(3-q)(5-3q)B_1}{48} + \frac{(q+1)(3-q)(5-3q)(7-5q)}{384}\right).$$

It is noted that the considered plasma equations are equivalent with the Ref. [27] if $q = 1$ and $\alpha = 0$.

3.3 FORMATION OF KDV EQUATION

To study the propagation of the ion acoustic wave, the basic equations are simplified by using conventional reductive perturbation technique. For this reason, the following stretching coordinates are introduced

$$\xi = \varepsilon^{1/2}(lx + my - v_p t), \tau = \varepsilon^{3/2} t, \tag{3.7}$$

where v_p is the normalized phase velocity of IASs in (ξ, τ) plane, $l = cos\theta$, $m = sin\theta$ and $l^2 + m^2 = 1$ and θ is the angle. From equation (3.7), the operators are defined as

$$\left.\begin{array}{c}\frac{\partial\tau}{\partial t}=\varepsilon^{3/2},\\ \frac{\partial\xi}{\partial t}=-\sqrt{\varepsilon}v_p,\\ \frac{\partial\xi}{\partial x}=\sqrt{\varepsilon}l,\\ \frac{\partial\xi}{\partial y}=\sqrt{\varepsilon}m,\\ \frac{\partial\xi}{\partial z}=0.\end{array}\right\} \tag{3.8}$$

Hence, Equations. (3.1)-(3.4) and (3.6) are then converts into

$$\varepsilon^{3/2}\frac{\partial n_i}{\partial\tau}-\sqrt{\varepsilon}v_p\frac{\partial n_i}{\partial\xi}+\sqrt{\varepsilon}l\frac{\partial}{\partial\xi}(n_iu_i)+\sqrt{\varepsilon}m\frac{\partial}{\partial\xi}(n_iv_i)=0, \tag{3.9}$$

$$\varepsilon^{3/2}\frac{\partial}{\partial\tau}(\gamma u_i)-\sqrt{\varepsilon}v_p\frac{\partial}{\partial\tau}(\gamma u_i)+\sqrt{\varepsilon}lu_i\frac{\partial}{\partial\xi}(\gamma u_i)+\sqrt{\varepsilon}mv_i\frac{\partial}{\partial\xi}(\gamma u_i)+\sqrt{\varepsilon}l\frac{\partial\phi}{\partial\xi}+\sqrt{\varepsilon}l\frac{\sigma}{n_i}\frac{\partial n_i}{\partial\xi}=0, \tag{3.10}$$

$$\varepsilon^{3/2}\frac{\partial v_i}{\partial\tau}-\sqrt{\varepsilon}v_p\frac{\partial v_i}{\partial\tau}+\sqrt{\varepsilon}lu_i\frac{\partial v_i}{\partial\xi}+\sqrt{\varepsilon}mv_i\frac{\partial v_i}{\partial\xi}+\sqrt{\varepsilon}m\frac{\partial\phi}{\partial\xi}+\sqrt{\varepsilon}m\frac{\sigma}{n_i}\frac{\partial n_i}{\partial\xi}-\Omega_cw_i=0, \tag{3.11}$$

$$\varepsilon^{3/2}\frac{\partial w_i}{\partial\tau}-\sqrt{\varepsilon}v_p\frac{\partial w_i}{\partial\tau}+\sqrt{\varepsilon}lu_i\frac{\partial w_i}{\partial\xi}+\sqrt{\varepsilon}mv_i\frac{\partial w_i}{\partial\xi}+\sqrt{\varepsilon}l\frac{\partial\phi}{\partial\xi}+\Omega_cv_i=0, \tag{3.12}$$

$$\varepsilon\left(l^2+m^2\right)\left(\frac{\partial^2\phi}{\partial\xi^2}\right)=1+k_1\phi+k_2\phi^2+k_3\phi^3+k_4\phi^4+...-n_i, \tag{3.13}$$

where ε is a dimensionless expansion parameter. The expansion of dependent variables are defined as:

$$\left.\begin{array}{c}n_i=1+\varepsilon n_1+\varepsilon^2n_2+\varepsilon^3n_3+\varepsilon^4n_4+...\\ u_i=u_0+\varepsilon u_1+\varepsilon^2u_2+\varepsilon^3u_3+\varepsilon^4u_4+...\\ v_i=\varepsilon^2v_1+\varepsilon^3v_2+\varepsilon^4v_3+\varepsilon^5v_4+...\\ w_i=\varepsilon^{3/2}w_1+\varepsilon^{5/2}w_2+\varepsilon^{7/2}w_3+\varepsilon^{9/2}w_4+...\\ \phi=\varepsilon\phi_1+\varepsilon^2\phi_2+\varepsilon^3\phi_3+\varepsilon^4\phi_4+...\end{array}\right\}. \tag{3.14}$$

Substituting equation (3.7) and (3.14) into equation (3.9)-(3.13), one can obtain the nonlinear partial differential equations (PDEs) by collecting the order of ε. Now, the lowest order of ε yields,

$$(lu_0 - v_p)\frac{\partial n_1}{\partial \xi} + l\frac{\partial u_1}{\partial \xi} = 0, \tag{3.15}$$

$$(lu_0 - v_p)\gamma_1\frac{\partial u_1}{\partial \xi} + l\frac{\partial \phi_1}{\partial \xi} + l\sigma\frac{\partial n_1}{\partial \xi} = 0, \tag{3.16}$$

$$m\frac{\partial \phi_1}{\partial \xi} + \sigma m\frac{\partial n_1}{\partial \xi} - \Omega_c w_1 = 0, \tag{3.17}$$

$$(lu_0 - v_p)\frac{\partial w_1}{\partial \xi} + \Omega_c v_1 = 0, \tag{3.18}$$

$$n_1 = k_1\phi_1. \tag{3.19}$$

Simplifying equations (3.15)-(3.19), we obtain

$$\left.\begin{aligned} n_1 &= \zeta_1 u_1, \\ u_1 &= \zeta_2 \phi_1, \\ w_1 &= -\tfrac{m\gamma_1\zeta_2}{\Omega_c\zeta_1}\tfrac{\partial \phi_1}{\partial \xi}, \\ v_1 &= \zeta_3\tfrac{\partial^2 \phi_1}{\partial \xi^2}, \end{aligned}\right\} \tag{3.20}$$

where

$$\zeta_1 = -\frac{l}{(lu_0 - v_p)},$$

$$\zeta_2 = \frac{l(lu_0 - v_p)}{-\gamma_1(lu_0 - v_p)^2 + l^2\sigma},$$

$$\zeta_3 = \frac{lm\zeta_2\gamma_1}{\Omega_c\zeta_1},$$

$$\gamma_1 = \sum_{r=0}^{\infty}\frac{(-1)^r\left(-\frac{1}{2}\right)\Gamma\left(-\frac{1}{2}\right)}{r!\left(-\frac{1}{2}-r\right)\Gamma\left(-\frac{1}{2}-r\right)}(2r+1)\,\beta^{2r}.$$

Here $\beta = \frac{u_0}{c}$ is the relativistic streaming factor, w_1 is the $\mathbf{E}\times\mathbf{B}$ drift along z axis and v_1 is the polarization drift along y axis. The linear phase velocity for the ion acoustic wave can be obtained using equation (3.15) to (3.18) as

$$v_p = \left[u_0 + \left(\frac{1}{\gamma_1 k_1} + \frac{\sigma}{\gamma_1}\right)^{\frac{1}{2}}\right] l. \tag{3.21}$$

Finally, from the next order of ε the following PDEs are obtained

$$(lu_0 - v_p)\frac{\partial n_2}{\partial \xi} + l\frac{\partial u_2}{\partial \xi} + l\frac{\partial}{\partial \xi}(n_1 u_1) + \frac{\partial n_1}{\partial \tau} + m\frac{\partial v_1}{\partial \xi} = 0, \tag{3.22}$$

$$(lu_0 - v_p)\gamma_1\frac{\partial u_2}{\partial \xi} + l\sigma\frac{\partial n_2}{\partial \xi} + l\frac{\partial \phi_2}{\partial \xi} + \gamma_1\frac{\partial u_1}{\partial \tau} + (lu_0\gamma_2 + l\gamma_1 - v_p\gamma_2)u_1\frac{\partial u_1}{\partial \xi} - l\sigma n_1\frac{\partial n_1}{\partial \xi} = 0, \tag{3.23}$$

$$(lu_0 - v_p)\frac{\partial v_1}{\partial \xi} - m\sigma n_1\frac{\partial n_1}{\partial \xi} + m\sigma\frac{\partial n_2}{\partial \xi} - \Omega_c w_2 + m\frac{\partial \phi_2}{\partial \xi} = 0, \tag{3.24}$$

$$(lu_0 - v_p)\frac{\partial w_2}{\partial \xi} + lu_1\frac{\partial w_1}{\partial \xi} + \frac{\partial w_1}{\partial \tau} + \Omega_c v_2 = 0, \tag{3.25}$$

$$(l^2 + m^2)\frac{\partial^2 \phi_2}{\partial \xi^2} - k_2\phi_1^2 - k_1\phi_2 + n_2 = 0, \tag{3.26}$$

where

$$\gamma_2 = \sum_{r=1}^{\infty}\frac{(2r+1)!}{(2r-1)!}\frac{(-1)^r\left(-\frac{1}{2}\right)\Gamma\left(-\frac{1}{2}\right)}{r!\left(-\frac{1}{2}-r\right)\Gamma\left(-\frac{1}{2}-r\right)}\frac{\beta^{2r}}{u_o}.$$

Simplifying the above equations, we obtain the following nonlinear KdV equation for the first-order electrostatic potential as,

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0, \tag{3.27}$$

where,

$$A = \frac{l}{2}\left[\left(3 - \frac{\gamma_2}{\zeta_1\gamma_1} - \frac{\zeta_1^2\sigma}{\gamma_1}\right)\zeta_2 - \left(\frac{2k_2}{\zeta_1\zeta_2^2\gamma_1}\right)\right],$$

$$B = \frac{1}{2\zeta_1\zeta_2}\left(\frac{l}{\gamma_1\zeta_2} + m\zeta_3\right),$$

which are the coefficients of the nonlinear and dispersion term, respectively.

3.4 SOLUTION OF KDV EQUATION

The KdV equation is a non-linear one-dimensional equation which describes small amplitude non-linear waves in plasmas. One can use the following traveling wave transformation to find the analytical wave solutions of the KdV equation (3.27):

$$\chi = \xi - U_0 \tau, \tag{3.28}$$

where U_0 stands for the constant reference speed. Using the transformation (3.27) in the KdV equation (3.28), one can obtain

$$-U_0 \frac{d\phi_1}{d\chi} + A\phi_1 \frac{d\phi_1}{d\chi} + B\frac{d^3\phi_1}{d\chi^3} = 0. \tag{3.29}$$

Now integrating equation (3.29) with respect to χ using boundary conditions, $\phi_1 \to 0, \frac{\partial \phi_1}{\partial \chi} \to 0, \ldots$ as $\chi \to \pm\infty$, one can obtain

$$-U_0\phi_1 + \frac{A}{2}\phi_1^2 + B\frac{d^2\phi_1}{d\chi^2} = 0, \tag{3.30}$$

$$\text{or, } \frac{d^2\phi_1}{d\chi^2} = \frac{U_0}{B}\phi_1 - \frac{A}{2B}\phi_1^2. \tag{3.31}$$

Equation (3.31) can be represent in planar dynamical system as

$$\begin{cases} \frac{d\phi_1}{d\chi} = z, \\ \frac{dz}{d\chi} = \frac{U_0}{B}\phi_1 - \frac{A}{2B}\phi_1^2. \end{cases} \tag{3.32}$$

The dynamical system (3.32) can be represented a Hamiltonian system with Hamiltonian function

$$H(\phi_1, z) = \frac{z^2}{2} - \frac{U_0}{2B}\phi_1^2 + \frac{A}{6B}\phi_1^3 = h(say). \tag{3.33}$$

For any homoclinic orbit of the dynamical system (3.32) at $(0,0)$, one can have $H(\phi_1,z) = 0$, which gives

$$\frac{z^2}{2} - \frac{U_0}{2B}\phi_1^2 + \frac{A}{6B}\phi_1^3 = 0,$$

$$\text{or, } z = \pm\sqrt{\frac{U_0}{B}}\phi_1\sqrt{1-\frac{A}{3U_0}\phi_1},$$

$$\text{or, } \frac{d\phi_1}{d\chi} = \pm\sqrt{\frac{U_0}{B}}\phi_1\sqrt{1-\frac{A}{3U_0}\phi_1},$$

$$\text{or, } \frac{d\phi_1}{\phi_1\sqrt{1-\frac{A}{3U_0}\phi_1}} = \pm\sqrt{\frac{U_0}{B}}d\chi. \tag{3.34}$$

Let $\frac{A}{3U_0}\phi_1 = f^2$, applying this equation into equation (3.34) and integrating, we obtain

$$\int \frac{df}{f\sqrt{1-f^2}} = \pm\int \frac{1}{2}\sqrt{\frac{U_0}{B}}d\chi,$$

$$\text{or, } sech^{-1}f = \pm\frac{1}{2}\sqrt{\frac{U_0}{B}}\chi,$$

$$\text{or, } f = sech\left(\pm\frac{1}{2}\sqrt{\frac{U_0}{B}}\chi\right). \tag{3.35}$$

Using f in equation (3.35), one can obtain

$$\phi_1 = \phi_0 sech^2\left\{\frac{\chi}{W}\right\}. \tag{3.36}$$

Equation (3.36) represents the solitary wave solution of the KdV equation (3.27) where $\phi_0 = \frac{3U_0}{A}$ and $W = \sqrt{\frac{4B}{U_0}}$ are the amplitude and width of the soliton, respectively.

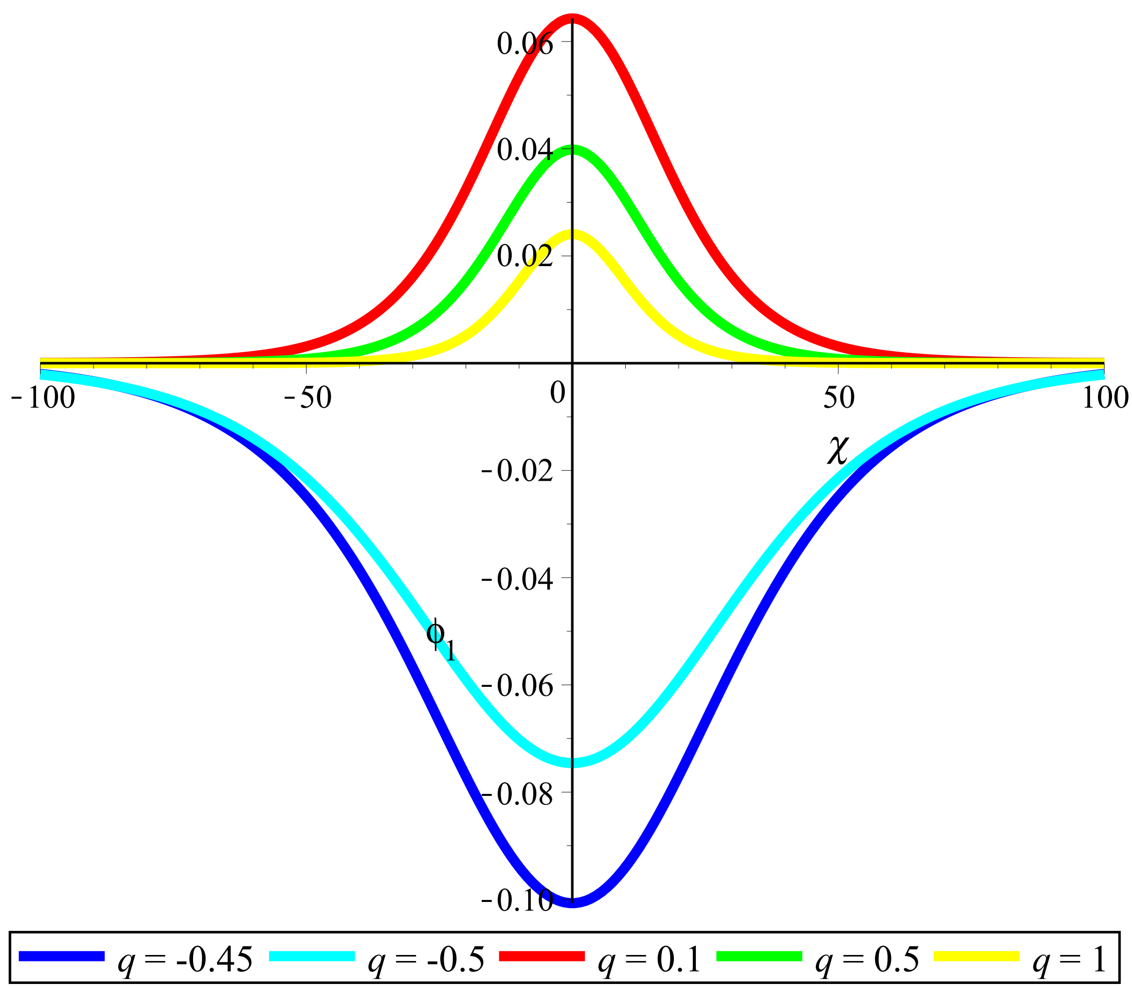

Figure 3.1: The influence of q on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.1$, $\sigma = 0.05$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 45^0$, $\beta = 0.1$ and $U_0 = 0.0075$.

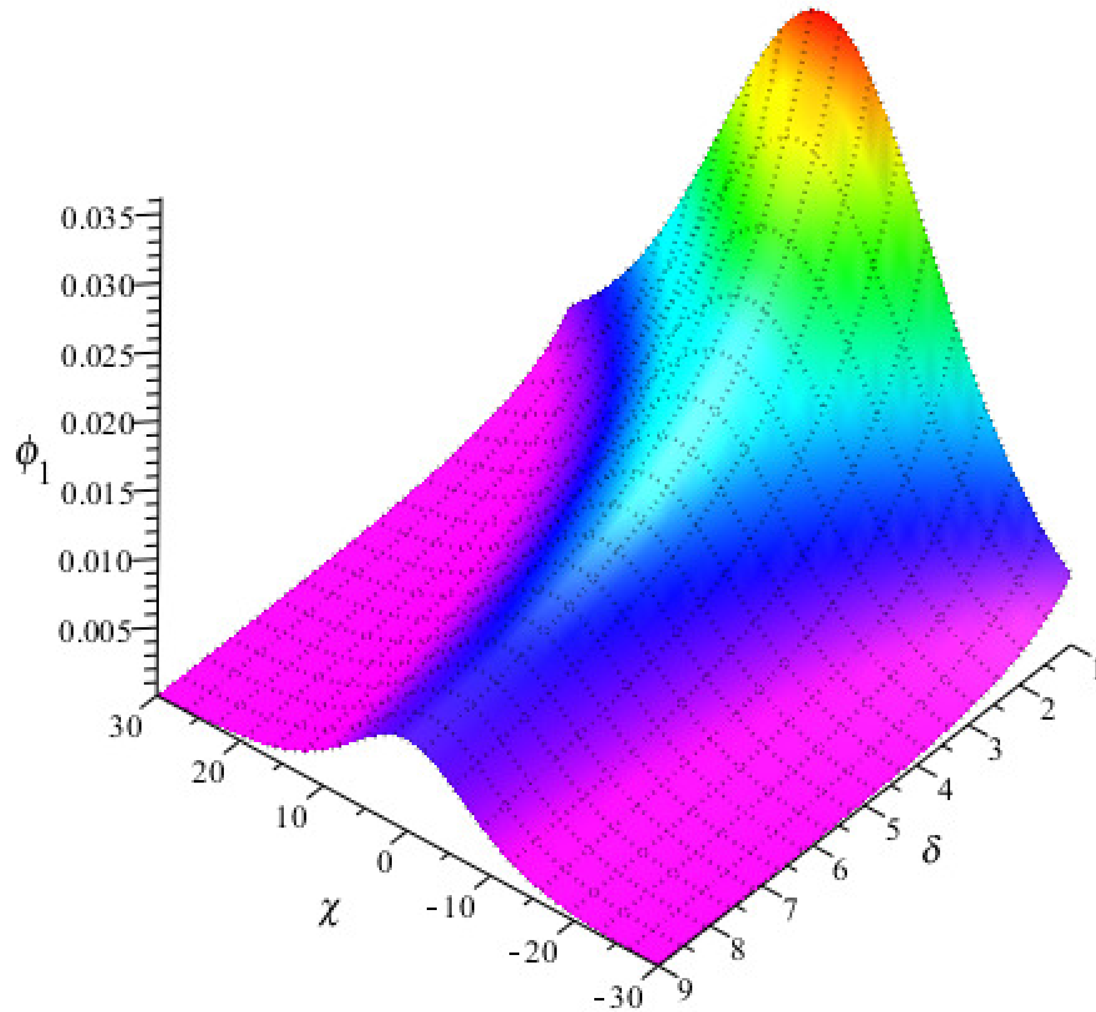

Figure 3.2: The influence of δ on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.2$, $q = 0.1$, $\sigma = 0.1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 30^0$, $\beta = 0.5$ and $U_0 = 0.0075$.

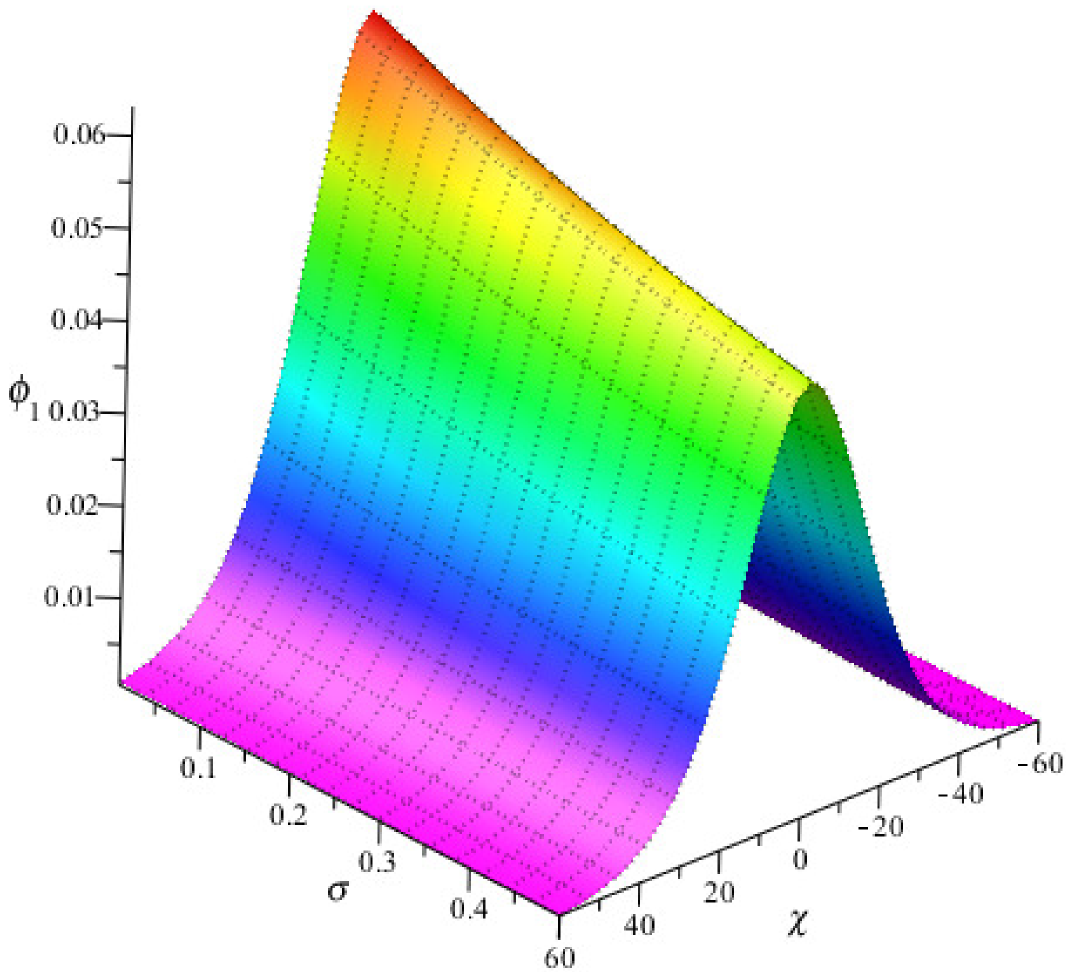

Figure 3.3: The influence of σ on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.1$, $q = 0.1$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 30^0$, $\beta = 0.5$ and $U_0 = 0.0075$.

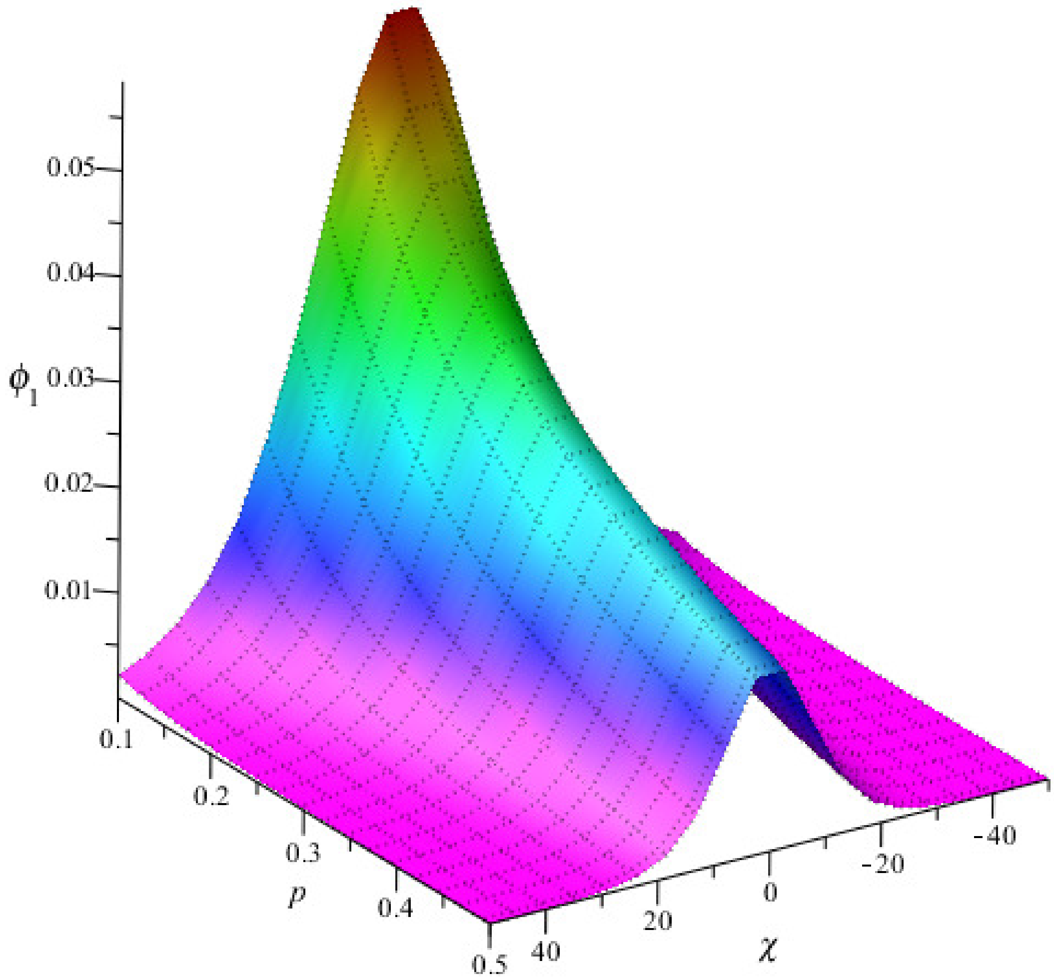

Figure 3.4: The influence of p on the IASs in e-p-i relativistic rotating magnetized plasmas with $q = 0.1$, $\sigma = 0.1$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 30^0$, $\beta = 0.5$ and $U_0 = 0.0075$.

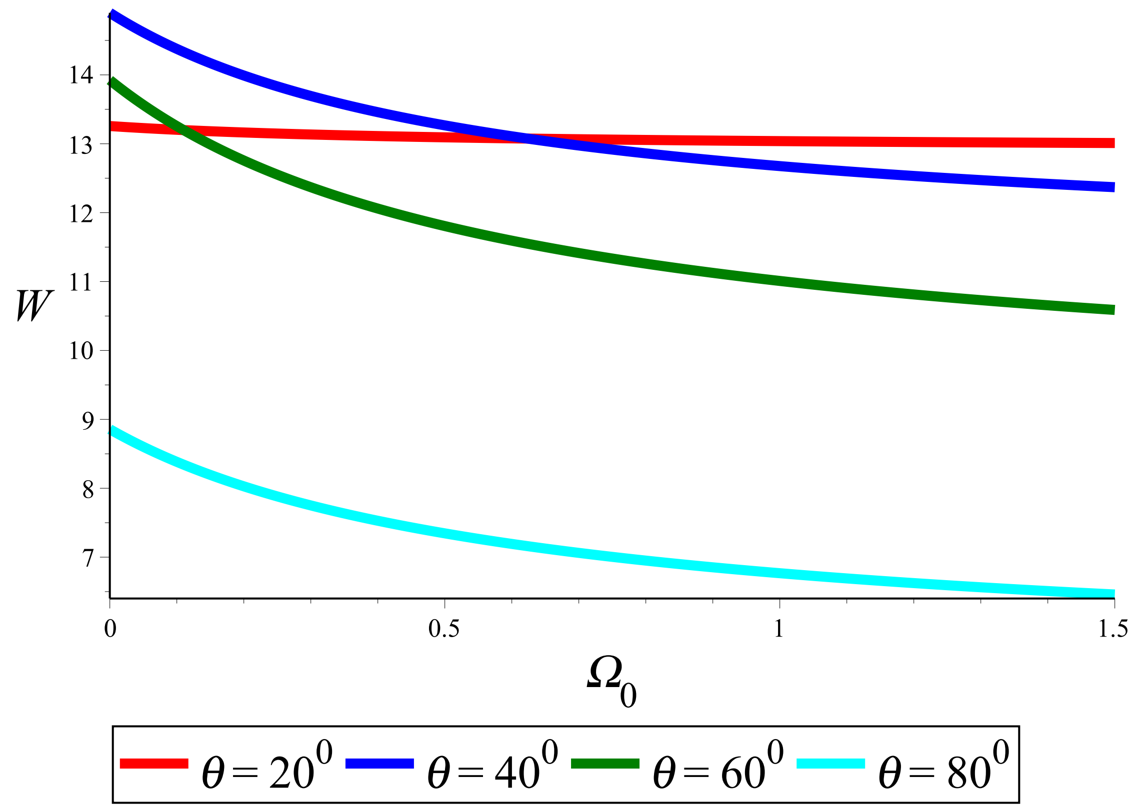

Figure 3.5: The variation of IASs width with regards to Ω_0 and θ for the e-p-i relativistic rotating magnetized plasmas with $p = 0.2$, $q = 0.5$, $\sigma = 0.1$, $\delta = 1$, $\omega_{ci} = 1$, $\beta = 0.5$ and $U_0 = 0.0075$.

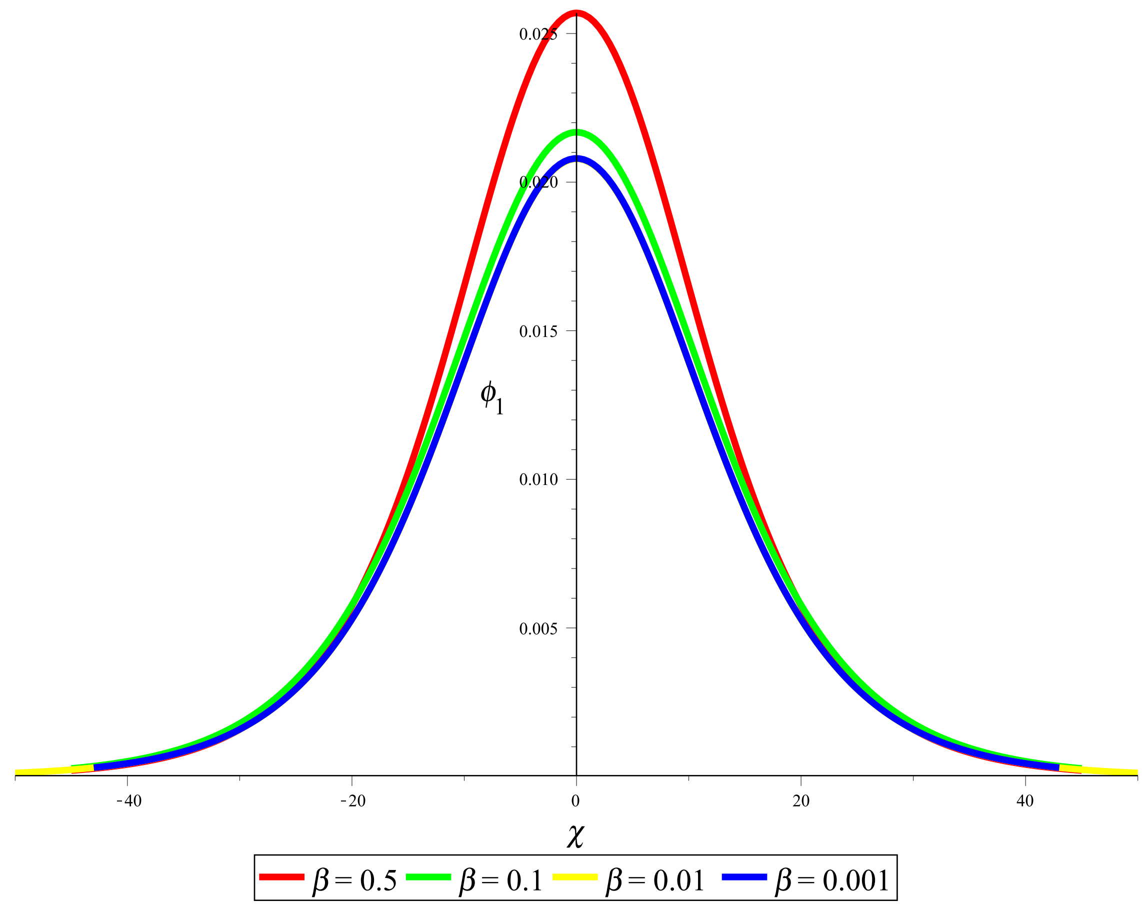

Figure 3.6: The influence of β on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.2$, $q = 0.5$, $\sigma = 0.1$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 30^0$, and $U_0 = 0.0075$.

3.5 RESULTS AND DISCUSSIONS

In this section, the effects of plasma parameters on the small but finite amplitude nonlinear propagation characteristics of IASs have been discussed via the soliton solution of KdVE. In the presented analysis, the parametric values of the parameters are assumed based on the Refs. [27], which are relevant to some astrophysical and space environment [116–119, 122, 123]. Figures 3.1, 3.2, 3.3 and 3.4 display the effect of q,δ, σ and p on the nonlinear propagation of IASs in the relativistic plasma by considering the RLF up to 11 terms and the remaining parameter constant. It is found from these figures that the amplitude and width of IASs are decreasing with the increase of non-exensivity, electron to positron temperature ratio's, ion to electron temperature ratio's and positron to electron density ratio's. Figure 3.1 also indicates that the considered plasma environment supports both of compressive and rarefactive IASs in the presence of super-thermality index, whereas the considered plasma supports only the compressive IASs in the presence of sub-thermality index for electrons and positrons. Figure 3.5 displays the variation of width of the IASs with regards to the magnitude of rotational frequency and obliqueness. It is found from Figure 3.5 that the obliqueness significantly modified the width of IASs in which the width is monotonically increasing upto 45^0 and then decreasing. Whereas, the widths of IASs are decreasing with the increasing values of the magnitude of rotational frequency. Finally, the effect of relativistic streaming index (β) on the nonlinear propagation of IASs is displayed in Figure 3.6 by considering the remaining parameters constant. It is found from this Figure 3.6 that the variation of IASs are very slightly changes with increase of relativistic streaming index up to less than 0.1, like weakly relativistic plasma [105–113]. But, the peak amplitudes of nonlinear propagation of IASs are increasing with the increase of relativistic streaming index up to greater than 0.1 due to the consideration of RLF up to 11 terms. It is provided that one needs to consider not only the RLF up to 11 terms but also more higher order terms of RLF for improving of the nonlinear propagation of IASs in the relativistic plasmas, but the beyond the scope of this investigation. It is also observed from the physical perspective that the deriving force is notably contributed to produce the

IASs with the decrease (increase) of ion temperature (RLF), as a result, the energy of soliton increases with the decrease (increase) of ion temperature (RLF). Whereas the restoring force contributes significantly with the increase of positron temperature. Thus, the investigations made in this article may be very useful in understanding the nature of obliquely propagating IASs dynamics of nonextensive relativistic plasmas not only in laser-plasma interaction, quark-gluon environment, dark-matter hole, solar atmosphere, etc but also laboratory verification.

3.6 CONCLUSIONS

In this work, we have investigated the obliquely propagation characteristics of IASs of the relativistic three component magnetized e-p-i plasmas having ion fluids and q-distributed electrons as well as positrons. The KdVE has been derived by implementing the reductive perturbation method. The effects of obliqueness and plasma parameters on the propagation characteristic of IASs described by KdVE have investigated by taking the RLF up to 11 terms. It is found that the proposed relativistic plasma environment is supported both of compressive and rarefactive IASs in the presence of superthermality. The plasma parameters are significantly modified the amplitudes and widths of IASs by their increasing numeric values. The relativistic streaming factor is remarkably modified the nonlinear propagation of IASs in which the energy of solitons slightly gains (considerably gains) with the increase of relativistic streaming index up to less than 0.1 (greater than 0.1). Thus, the outcomes described in this work may be helpful in understanding the high-energy protons motion existed in the Van Allen radiation belts [124], and pulsar magnetospheres [33], the rotating flows of magnetized plasma in cosmic environment and solar atmosphere [120], etc. It is still now required to do research work on the propagation characteristics of IASs around the critical and super-critical values of any specific plasma parameter by considering the magnetized relativistic plasma environment, which will be reported in our future investigations.

Chapter 4: SOLITON PROPAGATION NEAR CRITICAL VALUES IN MAGNETIZED PLASMA SYSTEM

4.1 INTRODUCTION

This chapter extends the previous one, illustrating methods to overcome challenges in studying soliton propagation around critical values within the considered plasmas. It is found from equation (3.36) that $\phi_1 \to \infty$ when the nonlinear coefficient $A \to 0$, signifying that the KdV equation is incapable of accurately describing the solitary wave phenomena under such circumstances. The nonlinear coefficients A of the KdV equation as in equation (3.27) can nullify for certain critical values of the physical parameters. By considering $A = 0$, one can precisely ascertain the critical values of any specific parameter. To mitigate this limitation, one may derive the KdV-type equation by taking higher-order nonlinearities. In the plasmas, the modified form of KdV equation provides a more precise description of the small amplitude nonlinear waves. The modified form of KdV equation is a nonlinear PDE that appears in various contexts, such as fluid dynamics, nonlinear optics, and plasma physics. Like the KdV equation, the modified form of KdV equation, so called the mKdV equation supports soliton solutions, which are stable, localized waves that can travel long distances without changing its shaped. It is noted that no research work has been done previously to study the IAS propagation in the proposed plasmas that presented in section 3.2 by deriving mKdV equation to best of our knowledge. Thus, this chapter explores the following:

- The derivation of mKdV equation with the existence of critical values by taking higher-order correction of the reductive technique from the considered plasma environment.
- The small but finite amplitude of the IASs are investigated around the critical values with the influences of related plasma parameters.

- The effect of the obliqueness and the magnitude of the rotational frequency on the width of the IASs are investigated.
- The effect of the relativistic streaming factor by taking upto 20 terms are investigated.

4.2 FORMATION OF MKDV EQUATION

To study the acoustic wave phenomena around the critical values, we consider the stretching coordinates by taking the higher order correction of the reductive perturbative method as

$$\xi = \varepsilon(lx + my - v_p t), \tau = \varepsilon^3 t. \tag{4.1}$$

From equation (4.1), the operators can be defined as

$$\left.\begin{aligned} \frac{\partial \tau}{\partial t} &= \varepsilon^3, \\ \frac{\partial \xi}{\partial t} &= -\varepsilon v_p, \\ \frac{\partial \xi}{\partial x} &= \varepsilon l, \\ \frac{\partial \xi}{\partial y} &= \varepsilon m, \\ \frac{\partial \xi}{\partial z} &= 0. \end{aligned}\right\} \tag{4.2}$$

Hence, equations (3.1)-(3.4) and (3.6) are then converts to

$$\varepsilon^3 \frac{\partial n_i}{\partial \tau} - \varepsilon v_p \frac{\partial n_i}{\partial \xi} + \varepsilon l \frac{\partial}{\partial \xi}(n_i u_i) + \varepsilon m \frac{\partial}{\partial \xi}(n_i v_i) = 0, \tag{4.3}$$

$$\varepsilon^3 \frac{\partial}{\partial \tau}(\gamma u_i) - \varepsilon v_p \frac{\partial}{\partial \tau}(\gamma u_i) + \varepsilon l u_i \frac{\partial}{\partial \xi}(\gamma u_i) + \varepsilon m v_i \frac{\partial}{\partial \xi}(\gamma u_i) + \varepsilon l \frac{\partial \phi}{\partial \xi} + \varepsilon l \frac{\sigma}{n_i} \frac{\partial n_i}{\partial \xi} = 0, \tag{4.4}$$

$$\varepsilon^3 \frac{\partial v_i}{\partial \tau} - \varepsilon v_p \frac{\partial v_i}{\partial \tau} + \varepsilon l u_i \frac{\partial v_i}{\partial \xi} + \varepsilon m v_i \frac{\partial v_i}{\partial \xi} + \varepsilon m \frac{\partial \phi}{\partial \xi} + \varepsilon m \frac{\sigma}{n_i} \frac{\partial n_i}{\partial \xi} - \Omega_c w_i = 0, \tag{4.5}$$

$$\varepsilon^3 \frac{\partial w_i}{\partial \tau} - \varepsilon v_p \frac{\partial w_i}{\partial \tau} + \varepsilon l u_i \frac{\partial w_i}{\partial \xi} + \varepsilon m v_i \frac{\partial w_i}{\partial \xi} + \varepsilon l \frac{\partial \phi}{\partial \xi} + \Omega_c v_i = 0, \tag{4.6}$$

$$\varepsilon^2 \left(l^2 + m^2\right) \left(\frac{\partial^2 \phi}{\partial \xi^2}\right) = 1 + k_1 \phi + k_2 \phi^2 + k_3 \phi^3 + k_4 \phi^4 + \ldots - n_i. \tag{4.7}$$

Equations (4.3)-(4.7) are then converted in terms of various power of ε by using equation (4.1) and equation (3.14). From the lowest order of ε, we obtain the same equation as in equation (3.15)-(3.16) and (3.19). The obtained linear phase velocity is also in same form as in equation (3.21). From equation (4.5) and (4.6), we obtain

$$\left.\begin{aligned} -\Omega_c w_1 = 0, \\ \Omega_c v_1 = 0. \end{aligned}\right\} \tag{4.8}$$

The solution of the first order equations of ε are determined as

$$\left.\begin{aligned} n_1 &= \zeta_1 u_1, \\ u_1 &= \zeta_2 \phi_1, \\ w_1 &= 0, \\ v_1 &= 0. \end{aligned}\right\} \tag{4.9}$$

From the next order of ε, we determine the following PDEs:

$$(lu_0 - v_p)\frac{\partial n_2}{\partial \xi} + l\frac{\partial u_2}{\partial \xi} + l\frac{\partial}{\partial \xi}n_1 u_1 + m\frac{\partial v_1}{\partial \xi} = 0, \tag{4.10}$$

$$(lu_0 - v_p)\gamma_1\frac{\partial u_2}{\partial \xi} + l\sigma\frac{\partial n_2}{\partial \xi} + l\frac{\partial \phi_2}{\partial \xi} + (lu_0\gamma_2 + l\gamma_1 - v_p\gamma_2)u_1\frac{\partial u_1}{\partial \xi} - l\sigma n_1\frac{\partial n_1}{\partial \xi} = 0, \tag{4.11}$$

$$-\Omega_c w_2 = 0, \tag{4.12}$$

$$\Omega_c v_2 = 0, \tag{4.13}$$

$$-k_2\phi_1^2 - k_1\phi_2 + n_2 = 0. \tag{4.14}$$

The solutions of these equations are also obtained as

$$\left.\begin{aligned} n_2 &= \zeta_1^2\zeta_2^2\phi_1^2+\zeta_1 u_2, \\ u_2 &= \zeta_4\phi_1^2+\zeta_2\phi_2, \\ w_2 &= 0, \\ v_2 &= 0. \end{aligned}\right\} \tag{4.15}$$

where $\zeta_4=\left\{\frac{\zeta_2^3}{2}\left(\sigma\zeta_1^2-\frac{\gamma_2}{\zeta_1}+\gamma_1\right)\right\}$. By implying equation (4.15) into equation (4.14), we obtain

$$\left(\zeta_1^2\zeta_2^2+\zeta_1\zeta_4-k_2\right)\phi_1^2+(\zeta_1\zeta_2-k_1)\phi_2=0, \tag{4.16}$$

which gives the critical point $C_f=\left(\zeta_1^2\zeta_2^2+\zeta_1\zeta_4-k_2\right)=0$. Finally, the next order of ε yields

$$(lu_0-v_p)\frac{\partial n_3}{\partial\xi}+l\frac{\partial u_3}{\partial\xi}+m\frac{\partial v_2}{\partial\xi}+l\frac{\partial}{\partial\xi}(n_1u_2)+l\frac{\partial}{\partial\xi}(n_2u_1)+m\frac{\partial}{\partial\xi}(n_1v_1)+\frac{\partial n_1}{\partial\tau}=0, \tag{4.17}$$

$$\begin{aligned} &(lu_0-v_p)\gamma_1\tfrac{\partial u_3}{\partial\xi}+l\sigma\tfrac{\partial n_3}{\partial\xi}+l\tfrac{\partial\phi_3}{\partial\xi}+(lu_0\gamma_2+l\gamma_1-v_p\gamma_2)\tfrac{\partial}{\partial\xi}(u_1u_2) \\ &-l\sigma\tfrac{\partial}{\partial\xi}(n_1n_2)+l\sigma n_1^2\tfrac{\partial n_1}{\partial\xi}+l\gamma_2u_1^2\tfrac{\partial u_1}{\partial\xi}+m\gamma_1v_1\tfrac{\partial u_1}{\partial\xi}+\gamma_1\tfrac{\partial u_1}{\partial\tau}=0, \end{aligned} \tag{4.18}$$

$$-\Omega_c w_3=0, \tag{4.19}$$

$$\Omega_c v_3=0, \tag{4.20}$$

$$(l^2+m^2)\frac{\partial^2\phi_1}{\partial\xi^2}-k_3\phi_1^3-2k_2\phi_1\phi_2-k_1\phi_3+n_3=0. \tag{4.21}$$

By eliminating the third-order quantities, we obtain the following nonlinear mKdV equation which describes obliquely propagating ion-acoustic waves in relativistic, magnetized and collisionless e-p-i plasmas:

$$\frac{\partial\phi_1}{\partial\tau}+A'\phi_1^2\frac{\partial\phi_1}{\partial\xi}+B'\frac{\partial^3\phi_1}{\partial\xi^3}=0, \tag{4.22}$$

where

$$A' = \frac{l}{2}\left\{3\zeta_1\zeta_2^2 - \frac{3k_3}{\zeta_1\zeta_2^2\gamma_1} + 9\zeta_4 - \frac{3\zeta_4\gamma_2}{\zeta_1\gamma_1} - \frac{\left(2\zeta_1\zeta_2^2 + 3\zeta_4\right)\zeta_1^2\sigma}{\gamma_1} + \frac{\zeta_2^2\gamma_2}{\gamma_1}\right\},$$

$$B' = \frac{l}{2\zeta_1\zeta_2^2\gamma_1}.$$

4.3 SOLUTION OF MKDV EQUATION

To determine the soliton solution of the mKdV equation as in equation (4.22), one can be considered the following reference frame:

$$\chi = \xi - U_0\tau, \tag{4.23}$$

where U_0 stands for the constant reference speed. Using the transformation (4.23) in the modified KdV equation (4.22), one can obtain

$$-U_0\frac{d\phi_1}{d\chi} + A'\phi_1^2\frac{d\phi_1}{d\chi} + B'\frac{d^3\phi_1}{d\chi^3} = 0. \tag{4.24}$$

Now integrating equation (4.24) with respect to χ using boundary conditions, $\phi_1 \to 0, \frac{d\phi_1}{d\chi} \to 0, ...$ as $\chi \to \pm\infty$, one can obtain

$$-U_0\phi_1 + \frac{A'}{3}\phi_1^3 + B'\frac{d^2\phi_1}{d\chi^2} = 0, \tag{4.25}$$

$$\text{or, } \frac{d^2\phi_1}{d\chi^2} = \frac{U_0}{B'}\phi_1 - \frac{A'}{3B'}\phi_1^3. \tag{4.26}$$

Equation (4.26) can be represent in planar dynamical system as

$$\begin{cases} \frac{d\phi_1}{d\chi} = z', \\ \frac{dz'}{d\chi} = \frac{U_0}{B'}\phi_1 - \frac{A'}{3B'}\phi_1^3. \end{cases} \tag{4.27}$$

The dynamical system (4.27) is a wamiltonian system with Hamiltonian function

$$H(\phi_1, z^{'}) = \frac{(z^{'})^2}{2} - \frac{U_0}{2B^{'}}\phi_1^2 + \frac{A^{'}}{12B^{'}}\phi_1^4. \tag{4.28}$$

For any homoclinic orbit of the dynamical system (4.28) at $(0,0)$, one can have $H(\phi_1, z^{'}) = 0$, which gives

$$\frac{(z^{'})^2}{2} - \frac{U_0}{2B^{'}}\phi_1^2 + \frac{A^{'}}{12B^{'}}\phi_1^4 = 0,$$

$$\text{or, } z^{'} = \pm\sqrt{\frac{U_0}{B^{'}}}\phi_1\sqrt{1 - \frac{A^{'}}{6U_0}\phi_1^2},$$

$$\text{or, } \frac{d\phi_1}{d\chi} = \pm\sqrt{\frac{U_0}{B^{'}}}\phi_1\sqrt{1 - \frac{A^{'}}{6U_0}\phi_1^2},$$

$$\text{or, } \frac{d\phi_1}{\phi_1\sqrt{1 - \frac{A^{'}}{6U_0}\phi_1^2}} = \pm\sqrt{\frac{U_0}{B^{'}}}d\chi. \tag{4.29}$$

Let $\frac{A^{'}}{6U_0}\phi_1^2 = (f^{'})^2$. By applying this into equation (4.29) and then integrating, we obtain

$$\int \frac{df^{'}}{f^{'}\sqrt{1-(f^{'})^2}} = \pm\int\sqrt{\frac{U_0}{B^{'}}}d\chi,$$

$$\text{or, } sech^{-1}f^{'} = \pm\sqrt{\frac{U_0}{B^{'}}}\chi,$$

$$\text{or, } f^{'} = sech\left(\pm\sqrt{\frac{U_0}{B^{'}}}\chi\right). \tag{4.30}$$

Using $f^{'}$ in equation (4.30), one can obtain

$$\phi_1 = \phi_m sech\left\{\frac{\chi}{W^{'}}\right\}. \tag{4.31}$$

Equation (4.31) represents the solitary wave solution of the modified KdV equation (4.22), where $\phi_m = \sqrt{\frac{6U_0}{A^{'}}}$ and $W^{'} = \sqrt{\frac{B^{'}}{U_0}}$ are the amplitude and width of the soliton, respectively.

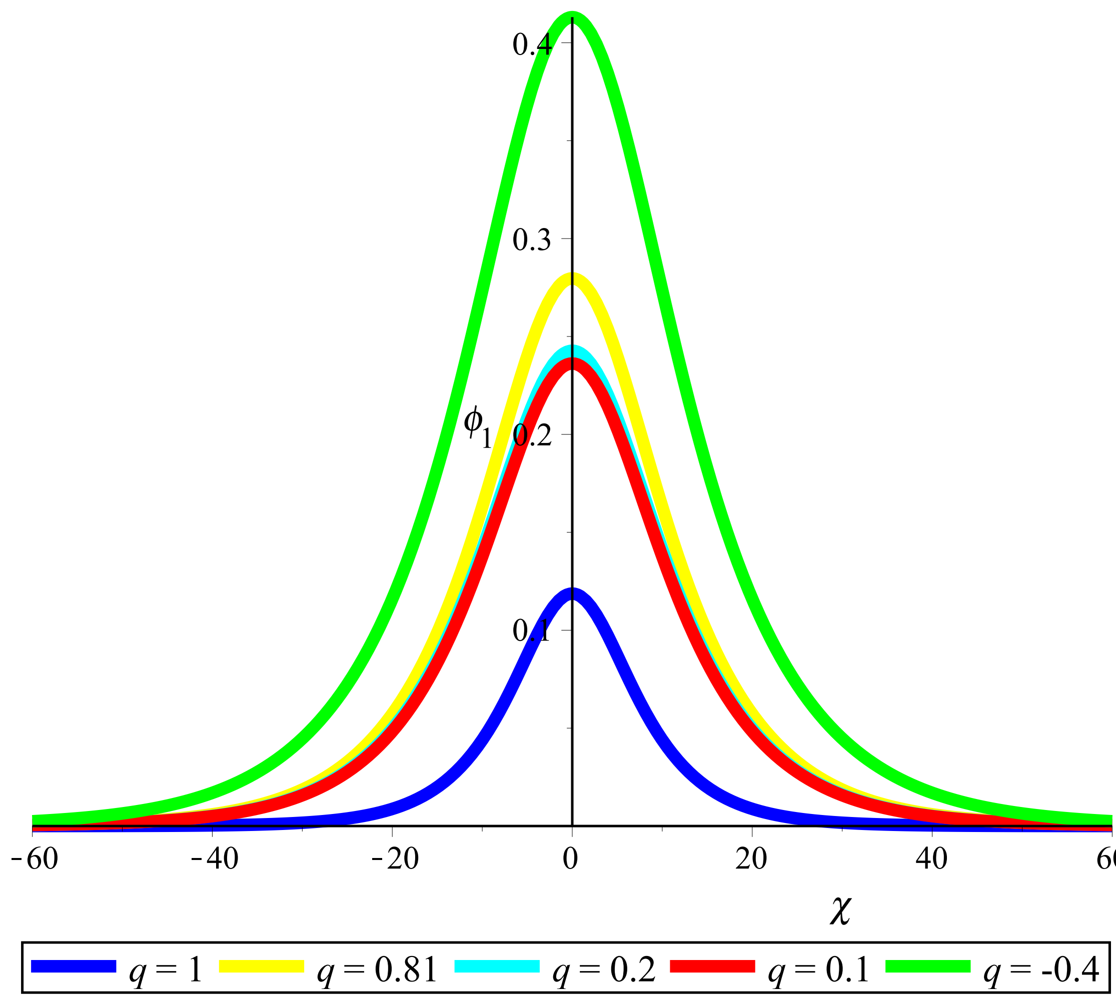

Figure 4.1: The influence of q $(q_c < q)$ on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.2$, $\alpha = 0.1$, $\sigma = 0.1$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 30^0$, $\beta = 0.5$ and $U_0 = 0.0075$.

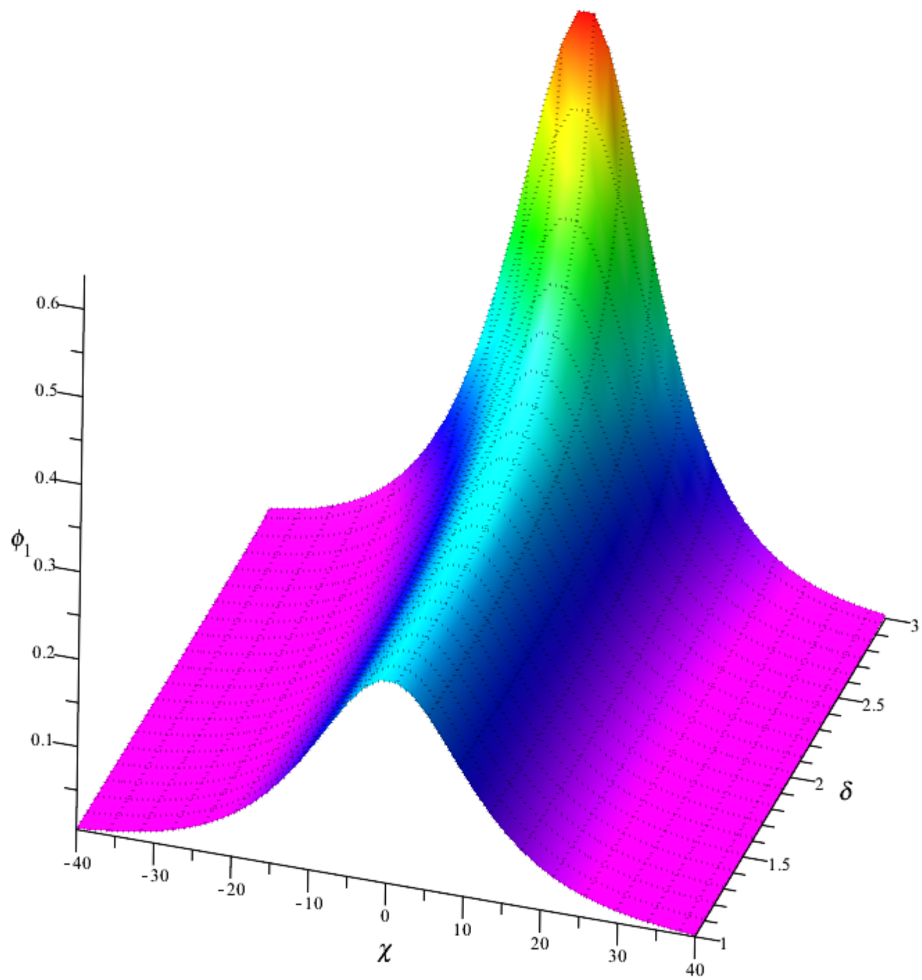

Figure 4.2: The influence of δ on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.1$, $\alpha = 0.01$, $q = 0.1(q_c < q)$, $\sigma = 0.1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 30^0$, $\beta = 0.5$ and $U_0 = 0.0075$.

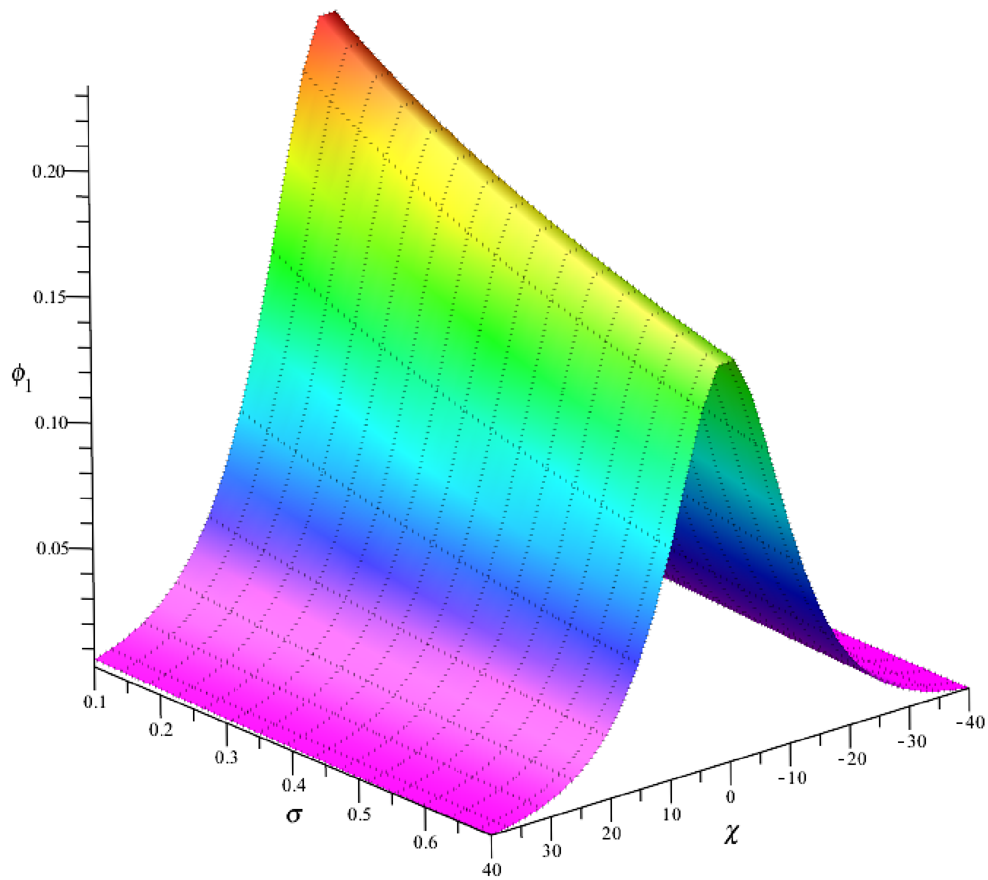

Figure 4.3: The influence of σ on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.1$, $\alpha = 0.01$, $q = 0.1(q_c < q)$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 30^0$, $\beta = 0.5$ and $U_0 = 0.0075$.

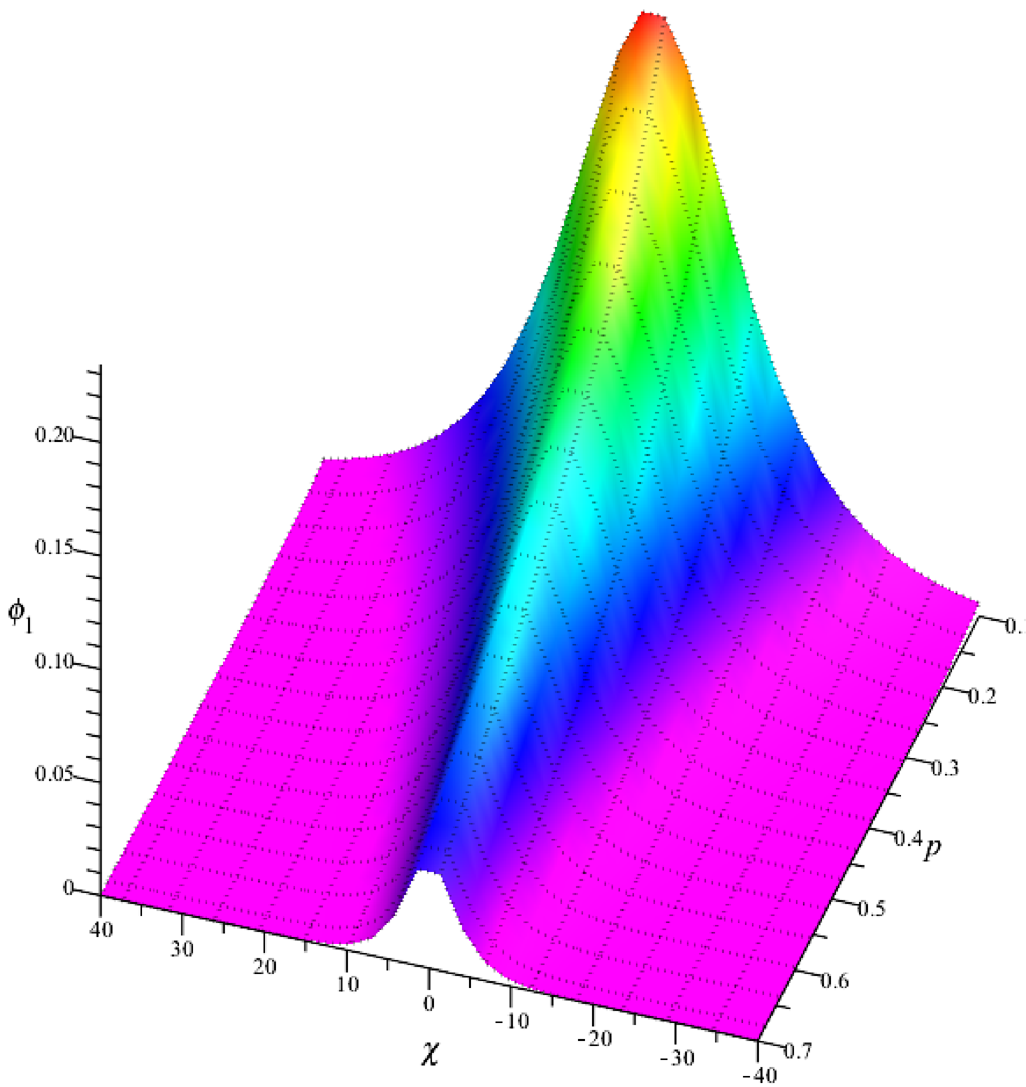

Figure 4.4: The influence of p on the IASs in e-p-i relativistic rotating magnetized plasmas with $\alpha = 0.01$, $q = 0.1(q_c < q)$, $\sigma = 0.1$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 30^0$, $\beta = 0.5$ and $U_0 = 0.0075$.

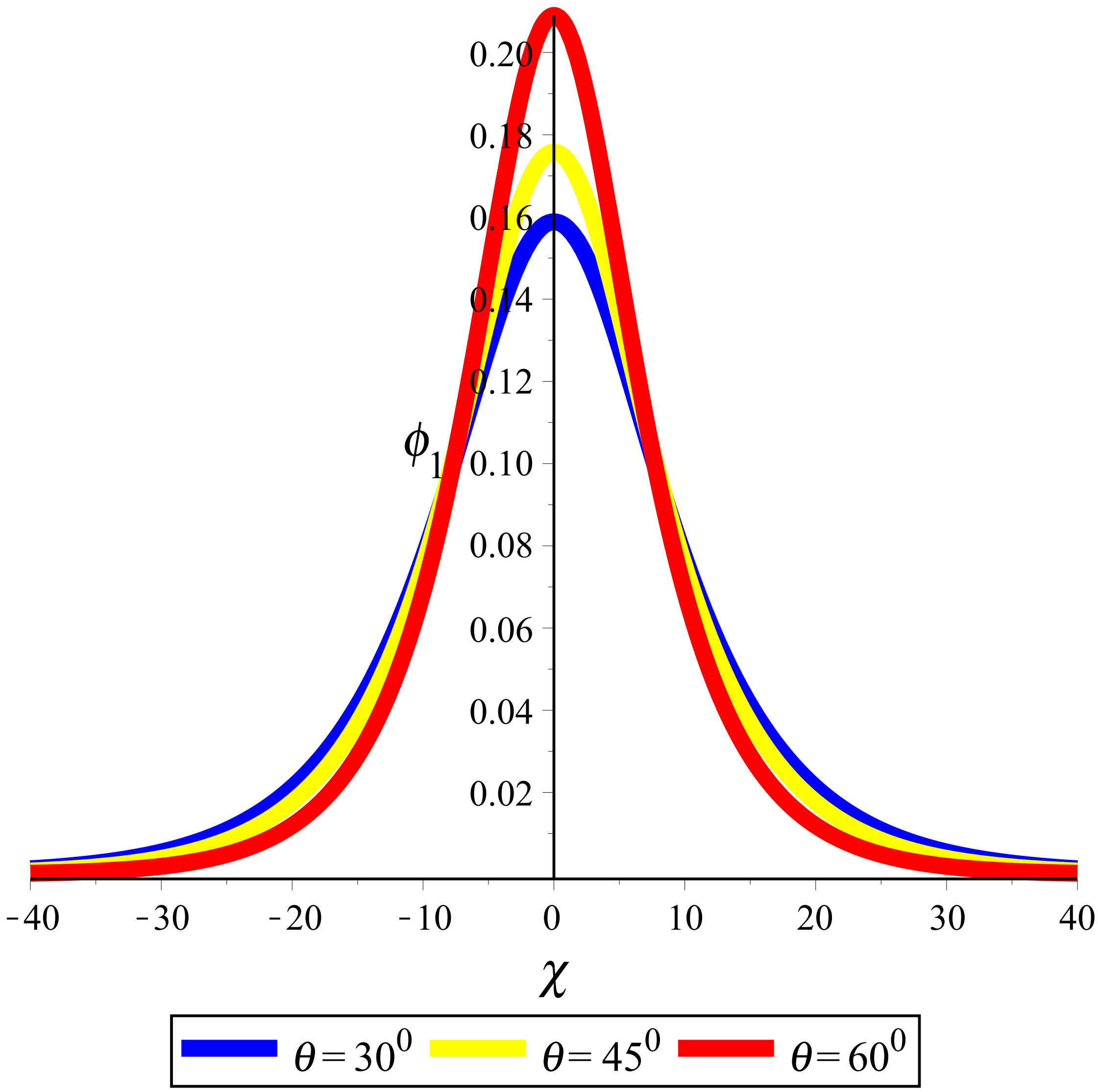

Figure 4.5: The influence of θ on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.2$, $\alpha = 0.01$, $q = 0.1 (q_c < q)$, $\sigma = 0.1$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\beta = 0.5$ and $U_0 = 0.0075$.

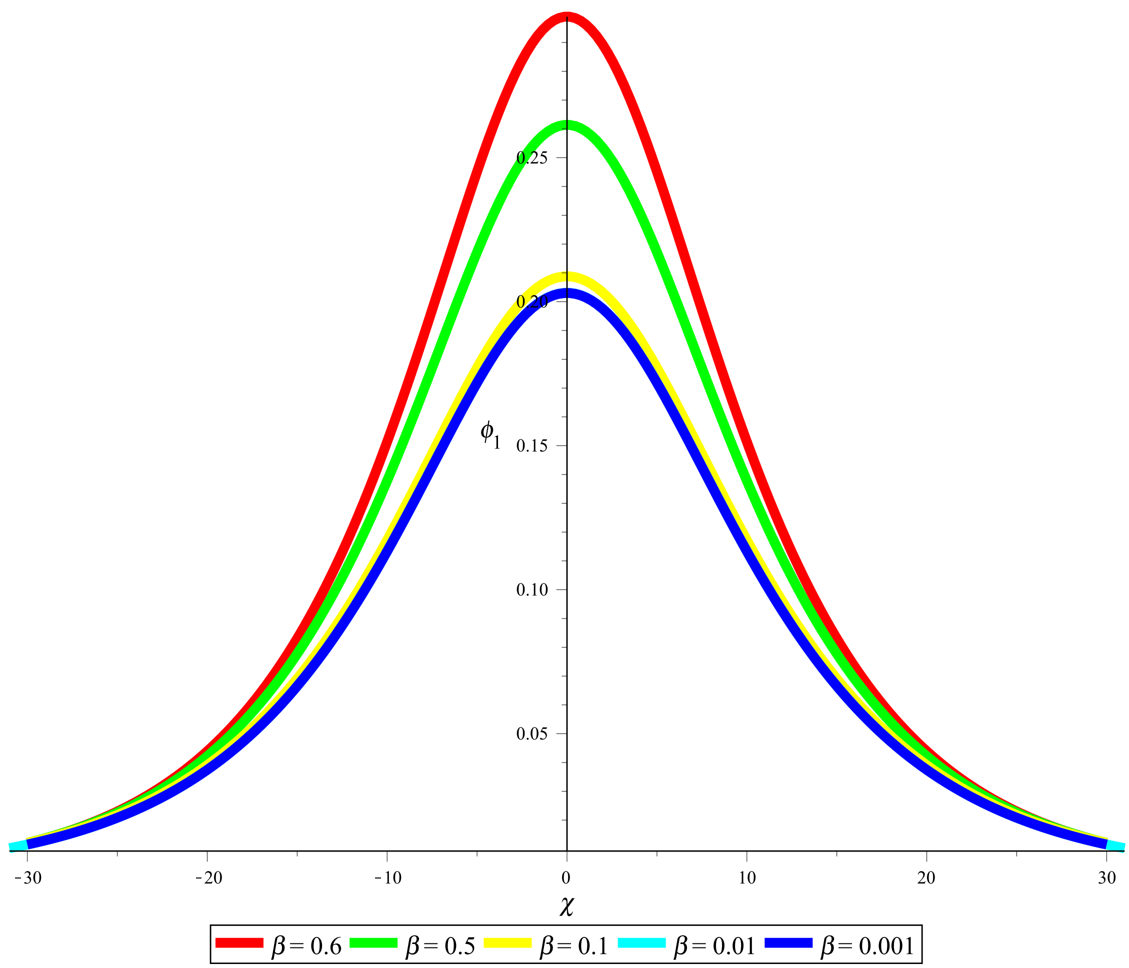

Figure 4.6: The influence of β on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.2$, $\alpha = 0.1$, $q = 0.1(q_c < q)$, $\sigma = 0.1$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 45^0$ and $U_0 = 0.0075$.

4.4 RESULTS AND DISCUSSIONS

In this section, the propagation characteristics of the small but finite amplitude nonlinear IASs have been discussed for the effects of plasma parameters by analyzing the soliton solution of the mKdV equation. In the presented study, the values of the parameters are assumed based on ref [27], which are relevant for astrophysical and space environments.

To study the soliton propagation, one needs to determine the critical values of any one parameter. With the parametric values of the parameters held constant at $p = 0.2, \sigma = 0.1, \delta = 1$ and $\alpha = 0$, the critical value is determined to be $q = q_c = -0.49$. Figures 4.1, 4.2, 4.3, and 4.4 displays the effect of q, δ, σ, and p on the nonlinear propagation of IASs around the critical values in the relativistic plasma by taking the RLF up to 20 terms and the remaining parameter constants. It is found the investigated e-p-i plasma provides finite-amplitude solitary structures, with characteristics such as polarity, amplitude, and width being significantly dependent on the plasma parameters. Figure 4.1 shows the amplitude and width of the IASs decreases (increases) within the range $-0.41 < q < 0.08$ ($0.08 < q < 0.21$), and there are no IASs between the range $0.22 < q < 0.8$ and then again decreases with the increase of q. It is also found for the considered plasmas that the effect of the nonextensivity parameter q supports compressive IASs in the presence of the super-thermality index as well as the sub-thermality index, for electrons and positrons. It is further noted that for $q > q_c$, hump-shaped IASs are present. Analysis of figures 4.2, 4.3, and 4.4 shows that the amplitude and width of the IASs are decreasing (increasing) with the increase of the ion to electron temperature ratio's and positron to electron density ratio's (electron to positron temperature ratio's). In Figure 4.5 the obliqueness significantly modifies the width and amplitude of the IASs, in which the amplitude increases with the increasing value of obliqueness while the width is decreasing. However, it is also found that the magnitude of the rotational frequency has no impact on the width of the IASs.

Finally, Figure 4.6 displays the effect of relativistic streaming index (β) on the nonlinear propagation of IASs by considering the remaining parameters constant. The variation of IASs are very slightly changing with the relativistic streaming index up to less than 0.1, like weakly relativistic plasma [105–113]. But the peak amplitudes of nonlinear propagation of IASs are increasing with the increase of relativistic streaming index up to greater than 0.1 due to the consideration of RLF up to 20 terms. It is recommended that, for advancing the nonlinear propagation of IASs in relativistic plasmas, one should consider not only the RLF up to 20 terms but also additional higher-order terms of the RLF, although this is beyond the scope of this investigation. From a physical perspective, it is evident that the driving force remarkably influences the generation of IASs with a decrease in ion temperature (or an increase in RLF), resulting in an increase in the soliton's energy. The restoring force, on the other hand, becomes notably significant with an increase in positron temperature. Thus, the investigations presented in this article may be highly valuable for understanding the dynamics of obliquely propagating IASs in $(\alpha,q)-$distributed relativistic plasmas not only in contexts such as laser-plasma interactions, quark-gluon environments, dark-matter anomalies, and the solar atmosphere but also for laboratory verification.

4.5 CONCLUSIONS

This chapter has been investigated the oblique propagation characteristics of IASs for the relativistic three-component magnetized e-p-i plasmas having ion fluids and (α,q)-distributed electrons as well as positrons. The mKdV equation has been derived by using the reductive perturbation method. By using the solution of this equation, the effects of obliqueness and plasma parameters on the propagation characteristic of IASs incorporating the RLF up to 20 terms have investigated. It is found that the proposed relativistic plasma environment has supported the compressive IASs in the presence of superthermality. Also this analysis helps in understanding how varying q influences the formation and characteristics of solitons in a relativistic plasma environment in the presence of population parameter α. Therefore, the plasma parameters significantly modified the amplitudes and widths of IASs by their increasing numeric values. The relativistic streaming factor is remarkably modified the nonlinear propagation of IASs in which the energy of solitons slightly gains (considerably gains) with the increase of relativistic streaming index up to less than 0.1 (greater than 0.1) around the critical values for nonextensivity. Consequently, the findings presented in this chapter may be contributed to understanding the nature of wave phenomena in some astrophysical and space environments, such as high-energy proton motion in the Van Allen radiation belts [124], pulsar magnetospheres [33], magnetized plasma rotating flows in cosmic environments, and solar atmospheres [120].

Chapter 5: SOLITON PROPAGATION IN MAGNETIZED RELATIVISTIC PLASMAS INVOLVING QUARTIC NONLINEARITY

5.1 INTRODUCTION

This chapter extends the previous chapters 3 and 4, illustrating methods to overcome challenges in studying the propagation IASs in a relativistic magnetized plasma around super-critical values. It is found from equations (3.36) and (4.31) that $\phi_1 \to \infty$ when the nonlinear coefficients $A \to 0$ and $A' \to 0$ of the KdV and mKdV equations, respectively at the same time. This indicates that both KdV and mKdV equations fails to accurately describe the solitary wave phenomena under the above conditions. The nonlinear coefficients A and A' of the KdV and mKdV equations as in equations (3.27) and (4.22) can become zero at the same time for specific values of the physical parameters. One can precisely determine the supercritical values of any particular parameter by setting $A = 0$ and $A' = 0$ at the same time with the consideration of remaining parameters constant. To address this limitation, the higher-order nonlinearities can be considered, leading to the derivation of the KdV equation with quartic nonlinearity. Thus, the main contribution of this chapter is to derive KdV equation involving more higer-order nonlinearity with the existence of supercritical values from our considered plasma system. Some other contributions are as follows:

- The small but finite amplitude of the IASs around the super-critical values with consideration of the effects of related plasma parameters.
- The impact of obliqueness and the magnitude of rotational frequency on the IASs width around the super-critical values.
- The influence of the relativistic streaming factor, analyzing up to 20 terms of the RLF on the IASs around the super-critical values.

5.2 FORMATION OF KDV EQUATION WITH QUARTIC NONLINEARITY

To study the acoustic wave phenomena around the super-critical values, we consider the stretching coordinates by taking the higher order correction of the reductive perturbative method as

$$\xi = \varepsilon^{\frac{3}{2}}(lx + my - v_p t), \tau = \varepsilon^{\frac{9}{2}} t. \tag{5.1}$$

From equation (5.1), the operators can be defined as

$$\left.\begin{array}{c} \frac{\partial \tau}{\partial t} = \varepsilon^{9/2}, \\ \frac{\partial \xi}{\partial t} = -\varepsilon^{3/2} v_p, \\ \frac{\partial \xi}{\partial x} = \varepsilon^{3/2} l, \\ \frac{\partial \xi}{\partial y} = \varepsilon^{3/2} m, \\ \frac{\partial \xi}{\partial z} = 0. \end{array}\right\} \tag{5.2}$$

Hence, equation (3.1)-(3.4) and (3.6) are then converts to

$$\varepsilon^{9/2} \frac{\partial n_i}{\partial \tau} - \varepsilon^{3/2} v_p \frac{\partial n_i}{\partial \xi} + \varepsilon^{3/2} l \frac{\partial}{\partial \xi}(n_i u_i) + \varepsilon^{3/2} m \frac{\partial}{\partial \xi}(n_i v_i) = 0, \tag{5.3}$$

$$\begin{array}{c} \varepsilon^{9/2} \frac{\partial}{\partial \tau}(\gamma u_i) - \varepsilon^{3/2} v_p \frac{\partial}{\partial \tau}(\gamma u_i) + \varepsilon^{3/2} l u_i \frac{\partial}{\partial \xi}(\gamma u_i) + \varepsilon^{3/2} m v_i \frac{\partial}{\partial \xi}(\gamma u_i) + \\ \varepsilon^{3/2} l \frac{\partial \phi}{\partial \xi} + \varepsilon^{3/2} l \frac{\sigma}{n_i} \frac{\partial n_i}{\partial \xi} = 0, \end{array} \tag{5.4}$$

$$\varepsilon^{9/2} \frac{\partial v_i}{\partial \tau} - \varepsilon^{3/2} v_p \frac{\partial v_i}{\partial \tau} + \varepsilon^{3/2} l u_i \frac{\partial v_i}{\partial \xi} + \varepsilon^{3/2} m v_i \frac{\partial v_i}{\partial \xi} + \varepsilon^{3/2} m \frac{\partial \phi}{\partial \xi} + \varepsilon^{3/2} m \frac{\sigma}{n_i} \frac{\partial n_i}{\partial \xi} - \Omega_c w_i = 0, \tag{5.5}$$

$$\varepsilon^{9/2} \frac{\partial w_i}{\partial \tau} - \varepsilon^{3/2} v_p \frac{\partial w_i}{\partial \tau} + \varepsilon^{3/2} l u_i \frac{\partial w_i}{\partial \xi} + \varepsilon^{3/2} m v_i \frac{\partial w_i}{\partial \xi} + \varepsilon^{3/2} l \frac{\partial \phi}{\partial \xi} + \Omega_c v_i = 0, \tag{5.6}$$

$$\varepsilon^3 \left(l^2 + m^2\right) \left(\frac{\partial^2 \phi}{\partial \xi^2}\right) = 1 + k_1 \phi + k_2 \phi^2 + k_3 \phi^3 + k_4 \phi^4 + \ldots - n_i. \tag{5.7}$$

Using equation (5.1) and the quantities from equation (3.14) in to equations (5.3)-(5.7), one can convert the above equation in terms of the power of ε. From the lowest order of ε, we obtain the same equation as in equations (3.15)-(3.16),(3.19) and (4.8). The solution of these equations are same as in equation (4.9) and the obtained linear phase velocity is also in same form as in equation (3.21). From the next order of ε, we obtain the same equations as in equations (4.10)-(4.11) and (4.14) that yields

$$m\sigma\frac{\partial n_1}{\partial \xi}+m\frac{\partial \phi_2}{\partial \xi}-\Omega_c w_2=0, \tag{5.8}$$

$$(lu_0-v_p)\frac{\partial w_1}{\partial \xi}+\Omega_c v_2=0. \tag{5.9}$$

The solution of these equations are

$$\left.\begin{array}{r}n_2=\zeta_1^2\zeta_2^2\phi_1^2+\zeta_1 u_2,\\ u_2=\zeta_4\phi_1^2+\zeta_2\phi_2,\\ w_2=\frac{m(\zeta_1\zeta_2\sigma+1)}{\Omega_c}\frac{\partial \phi_1}{\partial \xi},\\ v_2=0.\end{array}\right\} \tag{5.10}$$

Using equation (5.10) in equation (4.14), the similar mathematical equation as in equation (4.16) is obtained, which allow us to determine the critical point for any one parameter with the remaining parameters constant by setting the nonlinear coefficient of KdV equation is equal to zero. Now, the next order of ε yields

$$(lu_0-v_p)\frac{\partial n_3}{\partial \xi}+l\frac{\partial u_3}{\partial \xi}+m\frac{\partial v_2}{\partial \xi}+l\frac{\partial}{\partial \xi}(n_1u_2)+l\frac{\partial}{\partial \xi}(n_2u_1)+m\frac{\partial}{\partial \xi}(n_1v_1)=0, \tag{5.11}$$

$$\begin{array}{c}(lu_0-v_p)\gamma_1\frac{\partial u_3}{\partial \xi}+l\sigma\frac{\partial n_3}{\partial \xi}+l\frac{\partial \phi_3}{\partial \xi}+(lu_0\gamma_2+l\gamma_1-v_p\gamma_2)\frac{\partial}{\partial \xi}(u_1u_2)\\ -l\sigma\frac{\partial}{\partial \xi}(n_1n_2)+l\sigma n_1^2\frac{\partial n_1}{\partial \xi}+l\gamma_2u_1^2\frac{\partial u_1}{\partial \xi}+m\gamma_1v_1\frac{\partial u_1}{\partial \xi}=0,\end{array} \tag{5.12}$$

$$(lu_0-v_p)\frac{\partial v_1}{\partial \xi}-m\sigma n_1\frac{\partial n_1}{\partial \xi}+m\sigma\frac{\partial n_2}{\partial \xi}+m\frac{\partial \phi_2}{\partial \xi}-\Omega_c w_3=0, \tag{5.13}$$

$$(lu_0-v_p)\frac{\partial w_2}{\partial \xi}+lu_1\frac{\partial w_1}{\partial \xi}+\Omega_c v_3=0, \tag{5.14}$$

$$-k_3\phi_1^3-2k_2\phi_1\phi_2-k_1\phi_3+n_3=0. \tag{5.15}$$

It is found that one can only solve the above equations by setting $A = 0$ and $A' = 0$ at the same time, which allows us to determine the supercritical values of any one parameter along with the constant parametric values of the remaining parameters. As a result, the solution of the above equations are determined as

$$\left.\begin{array}{c} n_3 = \left(\zeta_1^3\zeta_2^3 + 2\zeta_1^2\zeta_2\zeta_4\right)\phi_1^3 + 2\zeta_1^2\zeta_2^2\phi_1\phi_2 + \zeta_1 u_3, \\ u_3 = \zeta_5\phi_1^3 + 2\zeta_4\phi_1\Phi_2 + \zeta_2\phi_3, \\ w_3 = \frac{m\zeta_1\sigma\left(\zeta_1\zeta_2^2+2\zeta_4\right)}{\Omega_c}\phi_1\frac{\partial\phi_1}{\partial\xi} + \frac{m(\zeta_1\zeta_2\sigma+1)}{\Omega_c}\frac{\partial\phi_2}{\partial\xi}, \\ v_3 = \zeta_6\frac{\partial^2\phi_1}{\partial\xi^2}, \end{array}\right\} \tag{5.16}$$

where $\zeta_5 = \frac{\zeta_2^2}{3}\left(\zeta_1^3\zeta_2^2\sigma + 3\zeta_1^2\zeta_4\sigma + \zeta_2^2\gamma_2 - \frac{3\zeta_4\gamma_2}{\zeta_1} + 3\zeta_4\gamma_1\right)$ and $\zeta_6 = \frac{l}{\Omega_c^2}\left(m\sigma\zeta_2 + \frac{m}{\zeta_1}\right)$. Finally, the following PDEs are obtained from the next order of ε:

$$\begin{array}{c} (lu_0 - v_p)\frac{\partial n_4}{\partial\xi} + l\frac{\partial}{\partial\xi}(n_1u_3) + l\frac{\partial}{\partial\xi}(u_1n_3) + l\frac{\partial}{\partial\xi}(u_2u_2) + m\frac{\partial}{\partial\xi}(n_1v_2) + m\frac{\partial}{\partial\xi}(n_2v_1) \\ +l\frac{\partial u_4}{\partial\xi} + m\frac{\partial v_3}{\partial\xi} + \frac{\partial n_1}{\partial\tau} = 0, \end{array} \tag{5.17}$$

$$\begin{array}{c} (lu_0 - v_p)\gamma_1\frac{\partial u_4}{\partial\xi} + l\sigma\frac{\partial n_4}{\partial\xi} + l\frac{\partial\phi_4}{\partial\xi} + (lu_0\gamma_2 + l\gamma_1 - v_p\gamma_2)\left\{\frac{\partial}{\partial\xi}(u_1u_3) + u_2\frac{\partial u_2}{\partial\xi}\right\} \\ -l\sigma\left\{\frac{\partial}{\partial\xi}(n_1n_3) + n_2\frac{\partial n_2}{\partial\xi}\right\} + l\sigma\left\{n_1^2\frac{\partial n_2}{\partial\xi} - n_1^3\frac{\partial n_1}{\partial\xi}\right\} + l\gamma_2u_1^2\frac{\partial u_2}{\partial\xi} + 2l\gamma_2u_1u_2\frac{\partial u_1}{\partial\xi} \\ +2l\sigma n_1n_2\frac{\partial n_1}{\partial\xi} + m\gamma_1v_2\frac{\partial u_1}{\partial\xi} + m\gamma_1v_1\frac{\partial u_2}{\partial\xi} + m\gamma_2v_1u_1\frac{\partial u_1}{\partial\xi} + \gamma_1\frac{\partial u_1}{\partial\tau} = 0, \end{array} \tag{5.18}$$

$$(lu_0 - v_p)\frac{\partial v_2}{\partial\xi} + lu_1\frac{\partial v_1}{\partial\xi} + m\sigma n_1^2\frac{\partial n_1}{\partial\xi} - m\sigma\frac{\partial}{\partial\xi}(n_1n_2) + m\sigma\frac{\partial n_3}{\partial\xi} + m\frac{\partial\phi_3}{\partial\xi} - \Omega_c w_4 = 0, \tag{5.19}$$

$$(lu_0 - v_p)\frac{\partial w_3}{\partial\xi} + lu_1\frac{\partial w_2}{\partial\xi} + (lu_2 + mv_1)\frac{\partial w_1}{\partial\xi} + \Omega_c v_4 = 0, \tag{5.20}$$

$$(l^2 + m^2)\frac{\partial^2\phi_1}{\partial\xi^2} - k_4\phi_1^4 - 3k_3\phi_1^2\phi_2 - 2k_2\phi_1\phi_3 - k_2\phi_2^2 - k_1\phi_4 + n_4 = 0. \tag{5.21}$$

By eliminating the fourth-order quantities, we obtain the following nonlinear KdV equation with quartic nonlinearity which describes obliquely propagating IASs in relativistic, magnetized and collisionless e-p-i plasmas with the existence of supercritical values:

$$\frac{\partial \phi_1}{\partial \tau} + A'' \phi_1^3 \frac{\partial \phi_1}{\partial \xi} + B'' \frac{\partial^3 \phi_1}{\partial \xi^3} = 0, \tag{5.22}$$

where

$$A'' = \frac{l}{2\gamma_1} \left\{ \left(4\zeta_1^2 \zeta_2^2 \gamma_1 + 12\zeta_1 \zeta_4 \gamma_1 + 4\zeta_4 \gamma_2 + \frac{12\zeta_5\gamma_1}{\zeta_2} + \frac{2\zeta_4^2\gamma_1}{\zeta_2^2} \right) \zeta_2 \right.$$
$$\left. - \left(3\zeta_1^2 \zeta_2^2 + 8\zeta_1 \zeta_4 + \frac{4\zeta_5}{\zeta_2} + \frac{2\zeta_4^2}{\zeta_2^2} \right) \sigma \zeta_1^2 \zeta_2 - \frac{4k_4}{\zeta_1 \zeta_2^2} - \frac{2\zeta_4^2(\gamma_2 - \zeta_1\gamma_1) + 4\zeta_2\zeta_5\gamma_2}{\zeta_1 \zeta_2} \right\},$$

$$B'' = \frac{1}{2\zeta_1 \zeta_2} \left(m\zeta_6 + \frac{l}{\zeta_2 \gamma_1} \right).$$

5.3 SOLUTION OF KDV EQUATION WITH QUARTIC NONLINEARITY

To determine the soliton solution of the quartic KdV equation as in equation (5.22), one can assume in the following reference frame:

$$\chi = \xi - U_0 \tau, \tag{5.23}$$

where U_0 stands for the constant reference speed and Using the transformation (5.23) in the quartic KdV equation (5.22), one can obtain

$$-U_0 \frac{d\phi_1}{d\chi} + A'' \phi_1^3 \frac{d\phi_1}{d\chi} + B'' \frac{d^3 \phi_1}{d\chi^3} = 0. \tag{5.24}$$

Now integrating equation (5.24) with respect to χ using boundary conditions, $\phi_1 \to 0$, $\frac{d\phi_1}{d\chi} \to 0$,... as $\chi \to \pm\infty$, one can obtain

$$-U_0 \phi_1 + \frac{A''}{4} \phi_1^4 + B'' \frac{d^2 \phi_1}{d\chi^2} = 0,$$

$$\text{or, } \frac{d^2 \phi_1}{d\chi^2} = \frac{U_0}{B''} \phi_1 - \frac{A''}{4B''} \phi_1^4. \tag{5.25}$$

Equation (5.25) can be represent in planar dynamical system as

$$\begin{cases} \frac{d\phi_1}{d\chi} = z^{''}, \\ \frac{dz^{''}}{\partial\chi} = \frac{U_0}{B^{''}}\phi_1 - \frac{A^{''}}{4B^{''}}\phi_1^4. \end{cases} \tag{5.26}$$

The dynamical system (5.26) is a Hamiltonian system with Hamiltonian function

$$H(\phi_1, z^{''}) = \frac{(z^{''})^2}{2} - \frac{U_0}{2B^{''}}\phi_1^2 + \frac{A^{''}}{20B^{''}}\phi_1^5. \tag{5.27}$$

For any homoclinic orbit of the dynamical system (5.26) at $(0,0)$, one can have $H(\phi_1, z^{''}) = 0$, which gives

$$\frac{(z^{''})^2}{2} - \frac{U_0}{2B^{''}}\phi_1^2 + \frac{A^{''}}{20B^{''}}\phi_1^5 = 0,$$

$$\text{or, } z^{''} = \pm\sqrt{\frac{U_0}{B^{''}}}\phi_1\sqrt{1 - \frac{A^{''}}{10U_0}\phi_1^3},$$

$$\text{or, } \frac{d\phi_1}{d\chi} = \pm\sqrt{\frac{U_0}{B^{''}}}\phi_1\sqrt{1 - \frac{A^{''}}{10U_0}\phi_1^3},$$

$$\text{or, } \frac{d\phi_1}{\phi_1\sqrt{1 - \frac{A^{''}}{10U_0}\phi_1^3}} = \pm\sqrt{\frac{U_0}{B^{''}}}d\chi. \tag{5.28}$$

Let $\frac{A^{''}}{10U_0}\phi_1^3 = (f^{''})^2$, applying this in equation (5.28) and by integrating we obtain

$$\int \frac{df^{''}}{f^{''}\sqrt{1-(f^{''})^2}} = \pm\int \frac{3}{2}\sqrt{\frac{U_0}{B^{''}}}d\chi,$$

$$\text{or, } sech^{-1}f^{''} = \pm\frac{3}{2}\sqrt{\frac{U_0}{B^{''}}}\chi,$$

$$\text{or, } f^{''} = sech\left(\pm\frac{3}{2}\sqrt{\frac{U_0}{B^{''}}}\chi\right). \tag{5.29}$$

Using f'' in equation (5.29), one can obtain

$$\phi_1 = \phi_q sech^{\frac{2}{3}} \left\{ \frac{\chi}{W''} \right\}. \tag{5.30}$$

Equation (5.30) represents the solitary wave solution of the quartic KdV equation (5.22) where $\phi_q = \left(\frac{10U_0}{A''}\right)^{\frac{1}{3}}$ and $W'' = \sqrt{\frac{4B''}{9U_0}}$ are the amplitude and width of the soliton, respectively.

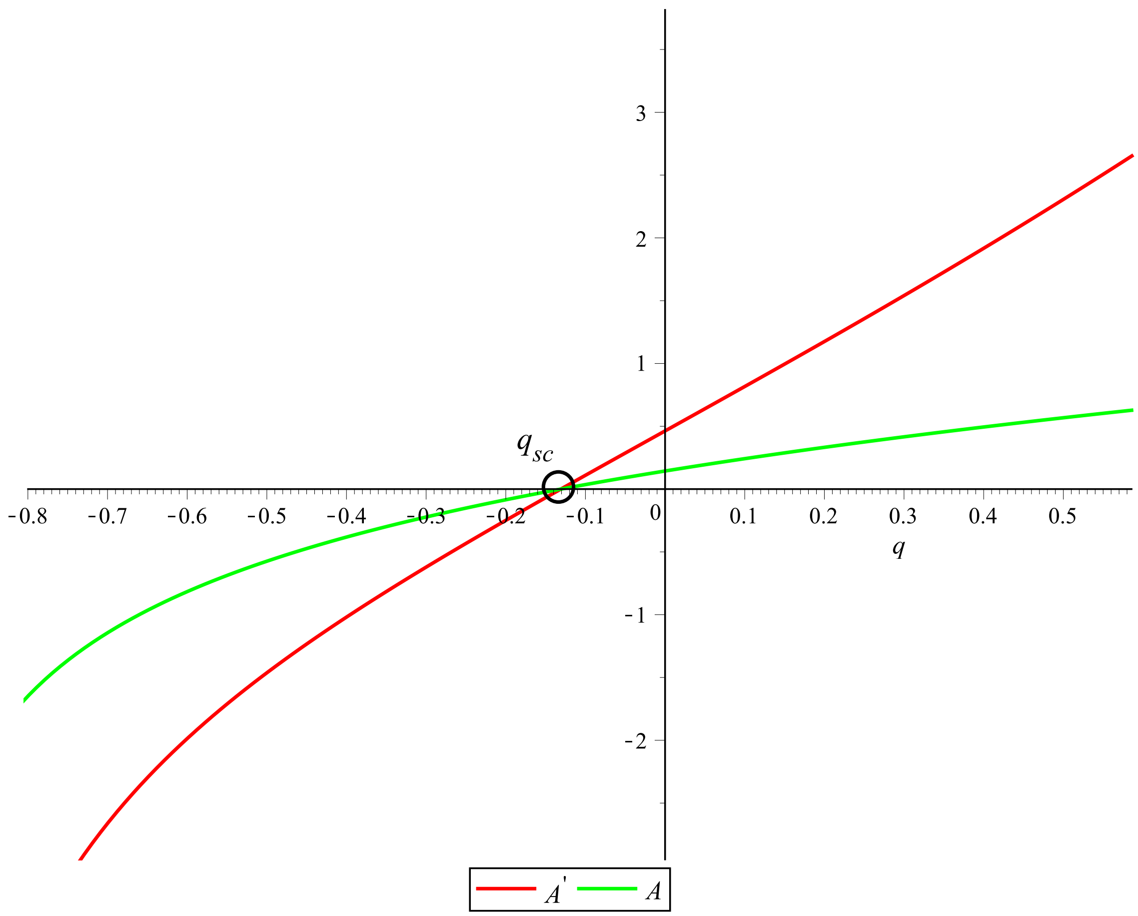

Figure 5.1: The supercritical point q_{sc} of the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.011$, $\alpha = 0$, $\sigma = 0.5$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 45^0$, $\beta = 0.1$ and $U_0 = 0.0075$.

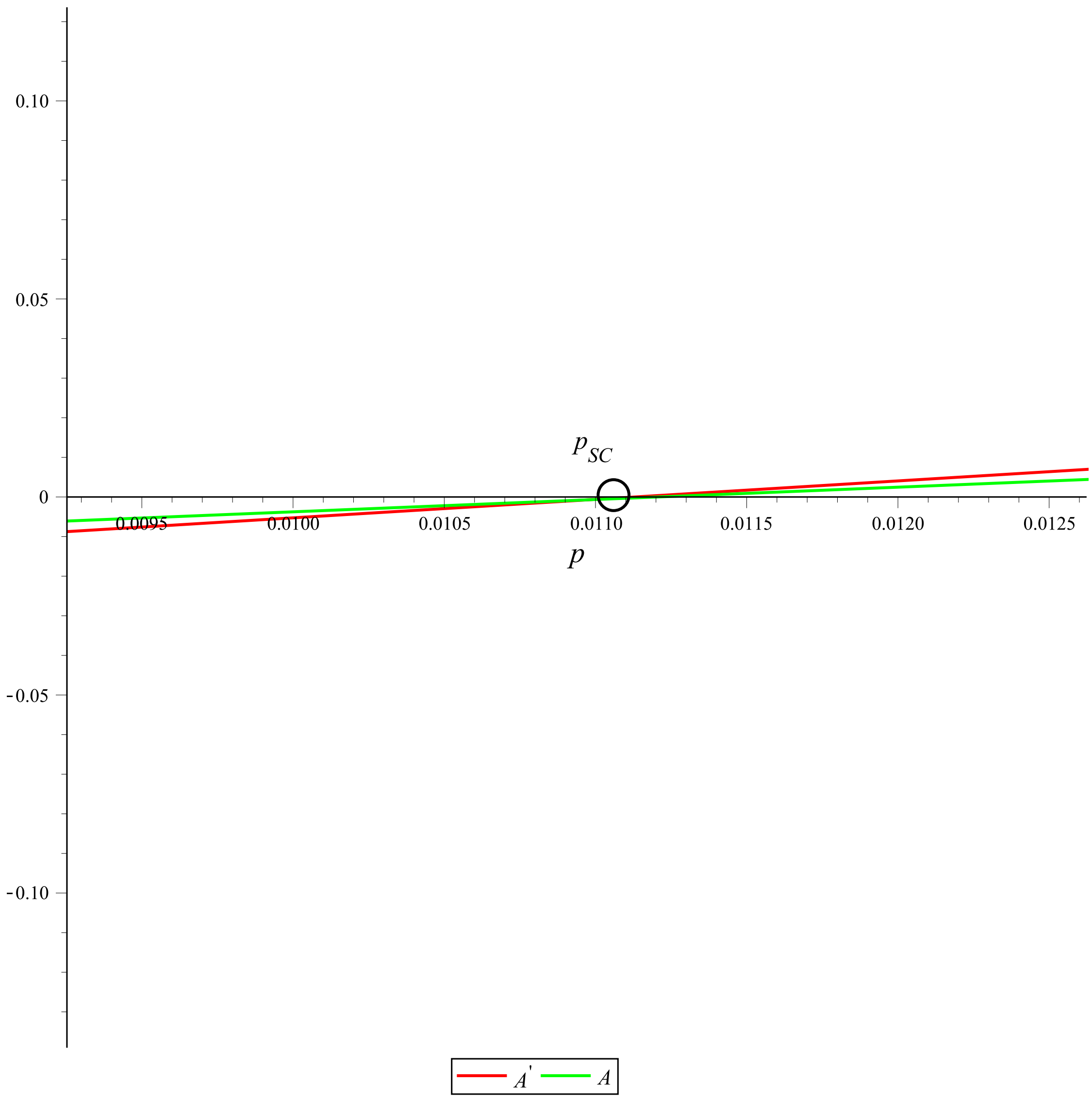

Figure 5.2: The supercritical point p_{sc} of the IASs in e-p-i relativistic rotating magnetized plasmas with $\alpha = 0$, $q = -0.1308$, $\sigma = 0.5$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 45^0$, $\beta = 0.1$ and $U_0 = 0.0075$.

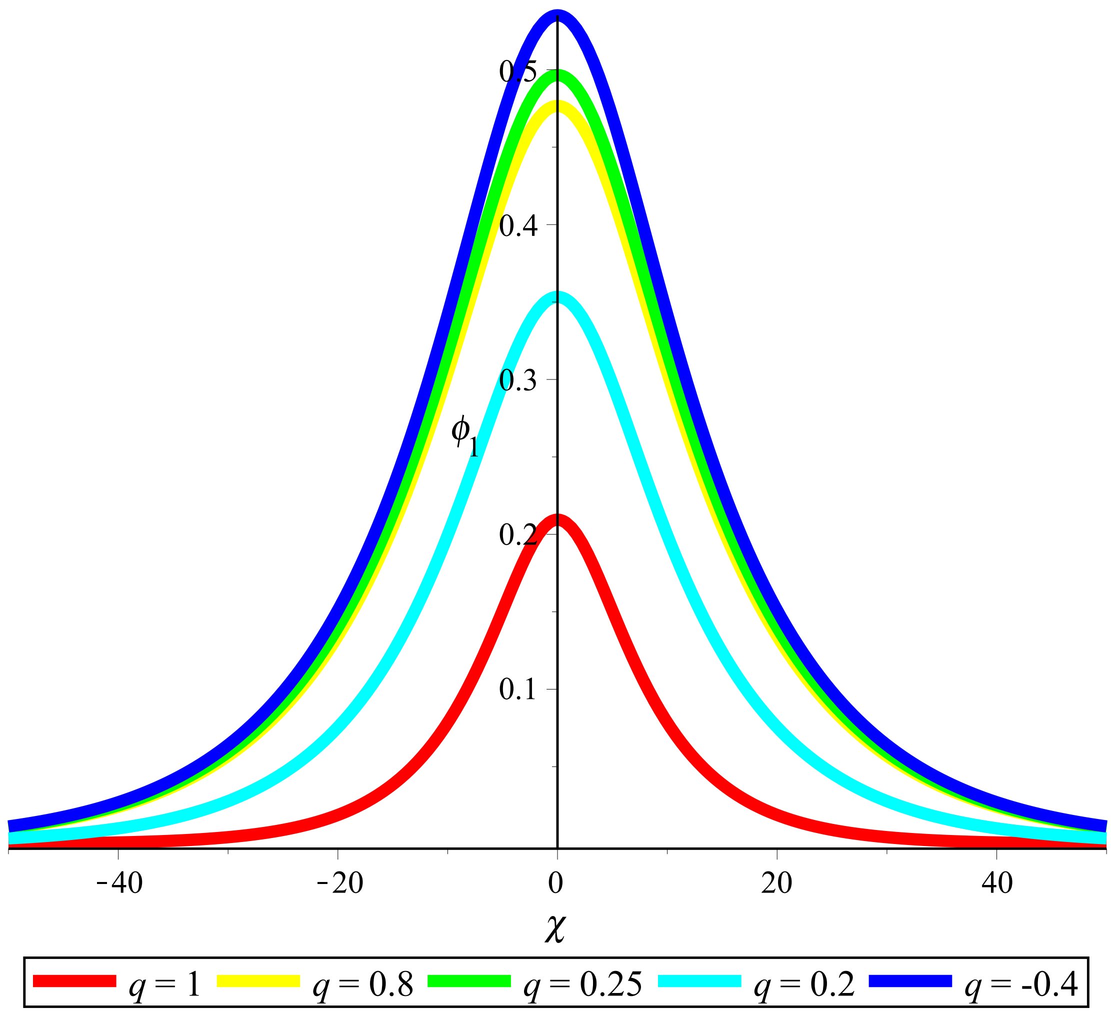

Figure 5.3: The influence of q on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.2$ $(p_{sc} < p)$, $\alpha = 0.1$, $\sigma = 0.1$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 45^0$, $\beta = 0.5$ and $U_0 = 0.0075$.

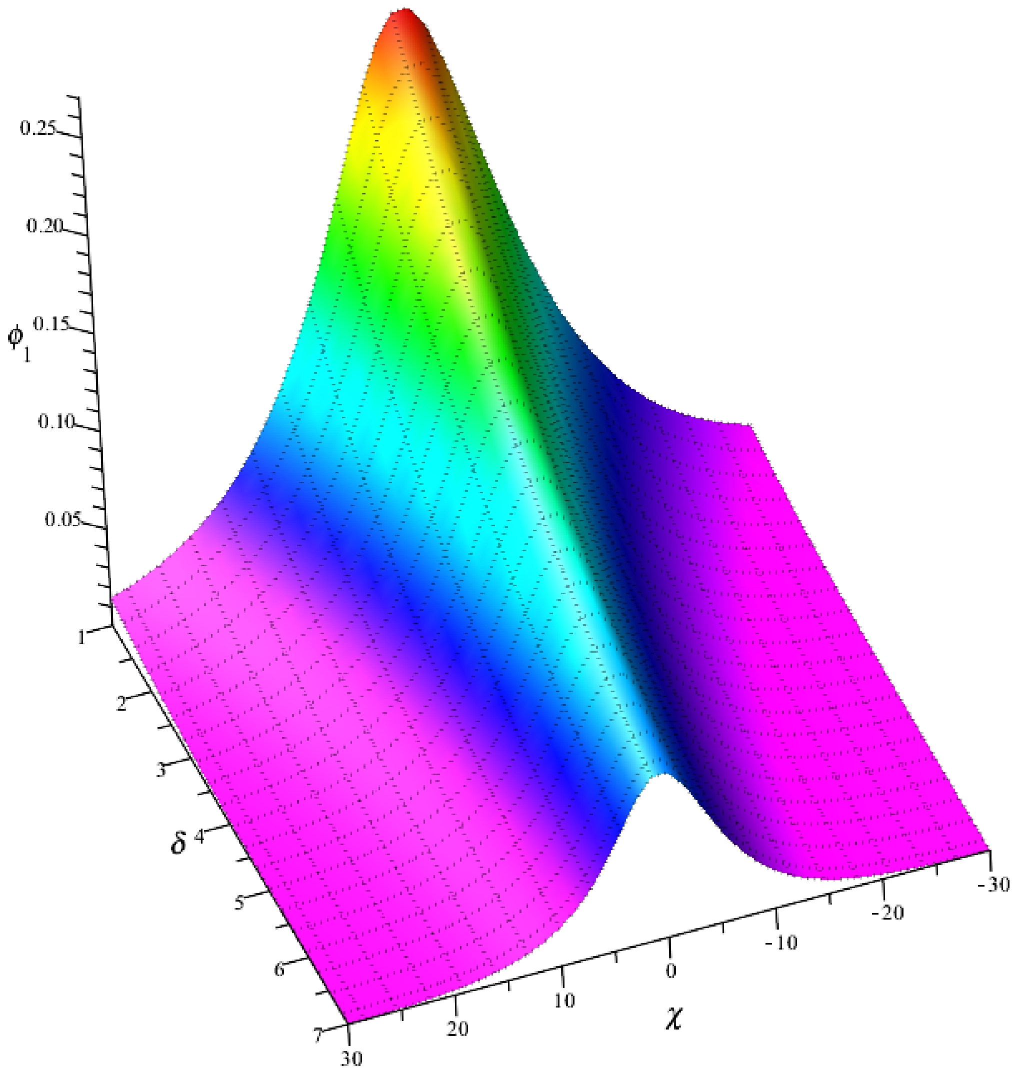

Figure 5.4: The influence of δ on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.2$ $(p_{sc} < p)$, $\alpha = 0.01$, $q = 0.1$ $(q_{sc} < q)$, $\sigma = 0.1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 30^0$, $\beta = 0.5$ and $U_0 = 0.0075$.

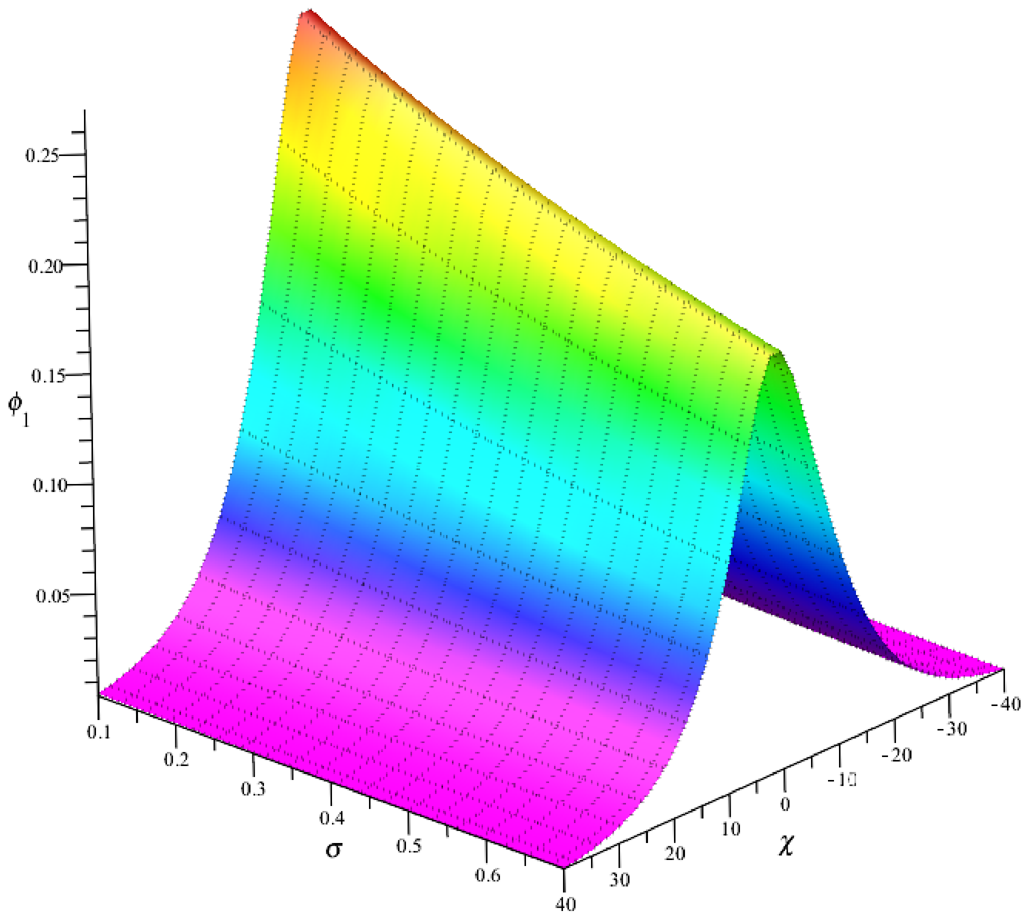

Figure 5.5: The influence of σ on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.2$ $(p_{sc} < p)$, $\alpha = 0.01$, $q = 0.1$ $(q_{sc} < q)$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 30^0$, $\beta = 0.5$ and $U_0 = 0.0075$.

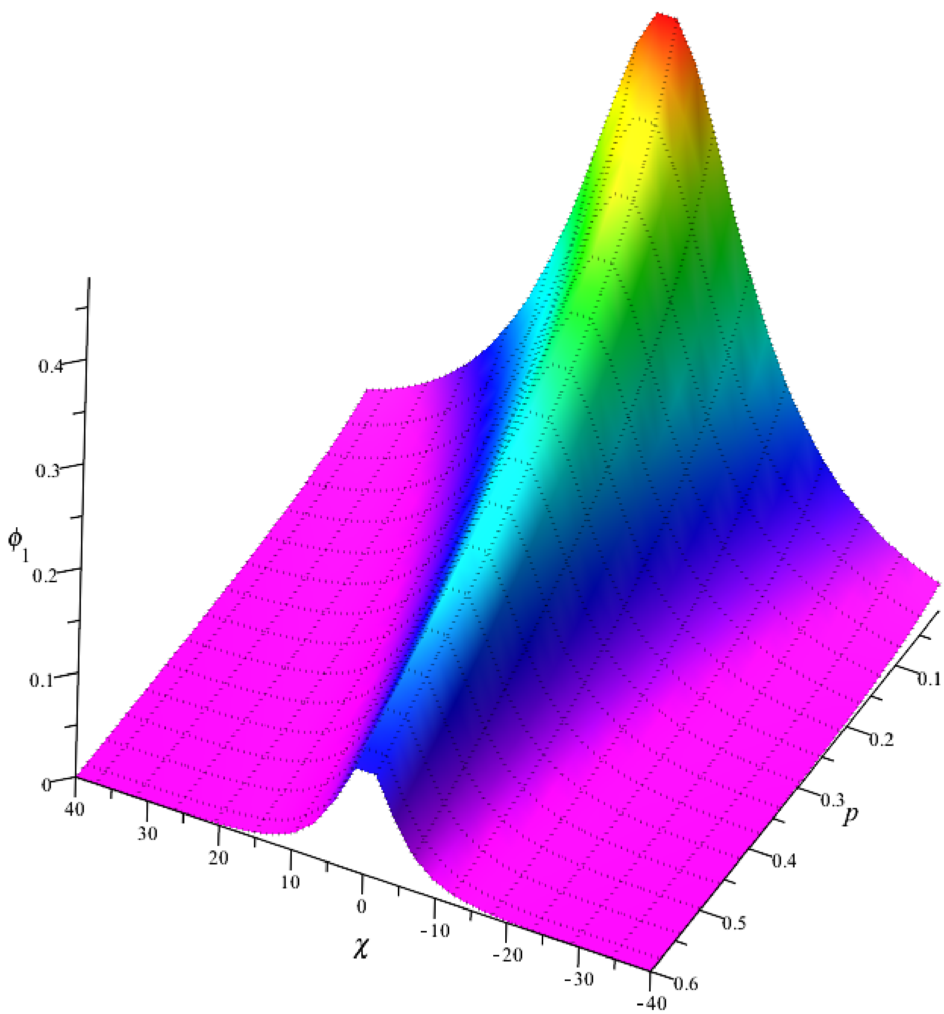

Figure 5.6: The influence of p ($p_{sc} < p$) on the IASs in e-p-i relativistic rotating magnetized plasmas with $\alpha = 0.01$, $q = 0.1$ ($q_{sc} < q$), $\sigma = 0.1$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 30^0$, $\beta = 0.5$ and $U_0 = 0.0075$.

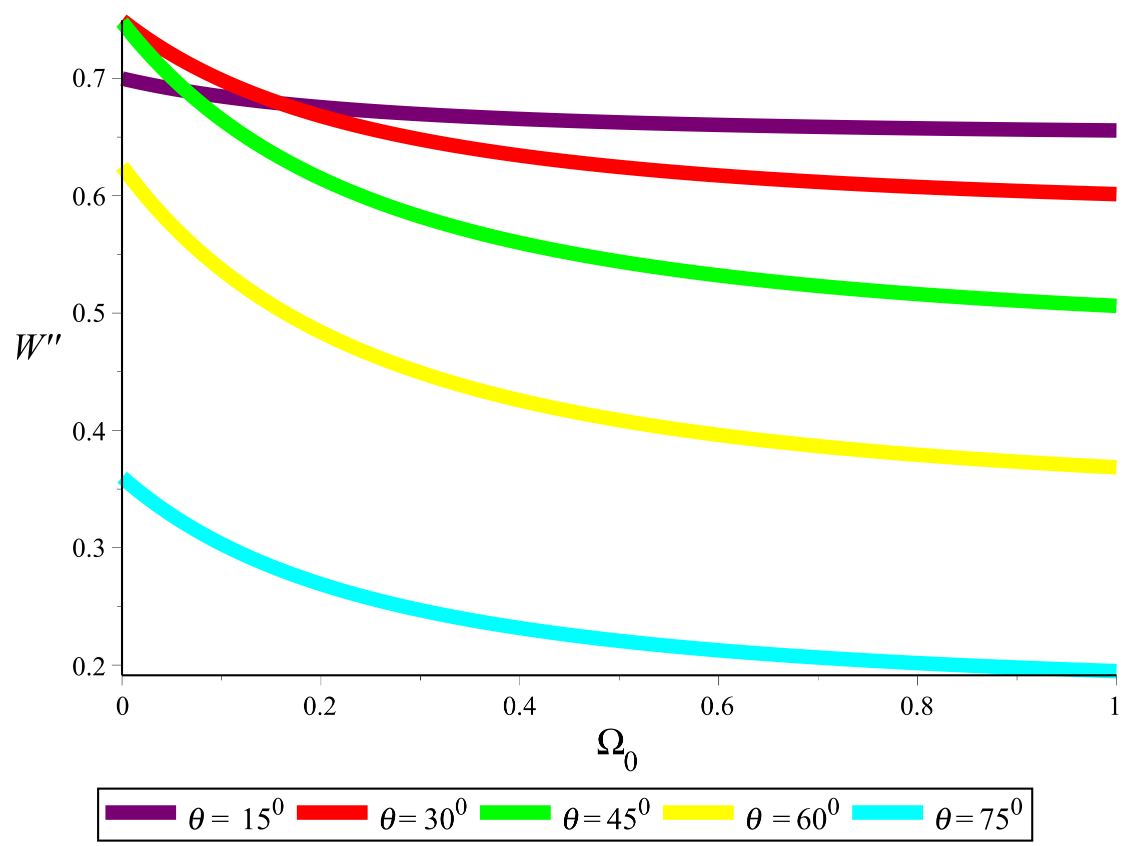

Figure 5.7: The variation of IASs width with regards to Ω_0 and θ for the e-p-i relativistic rotating magnetized plasmas with $p = 0.2$ ($p_{sc} < p$), $\alpha = 0.1$, $q = 0.1$ ($q_{sc} < q$), $\sigma = 0.1$, $\delta = 1$, $\omega_{ci} = 1$, $\beta = 0.5$ and $U_0 = 0.0075$.

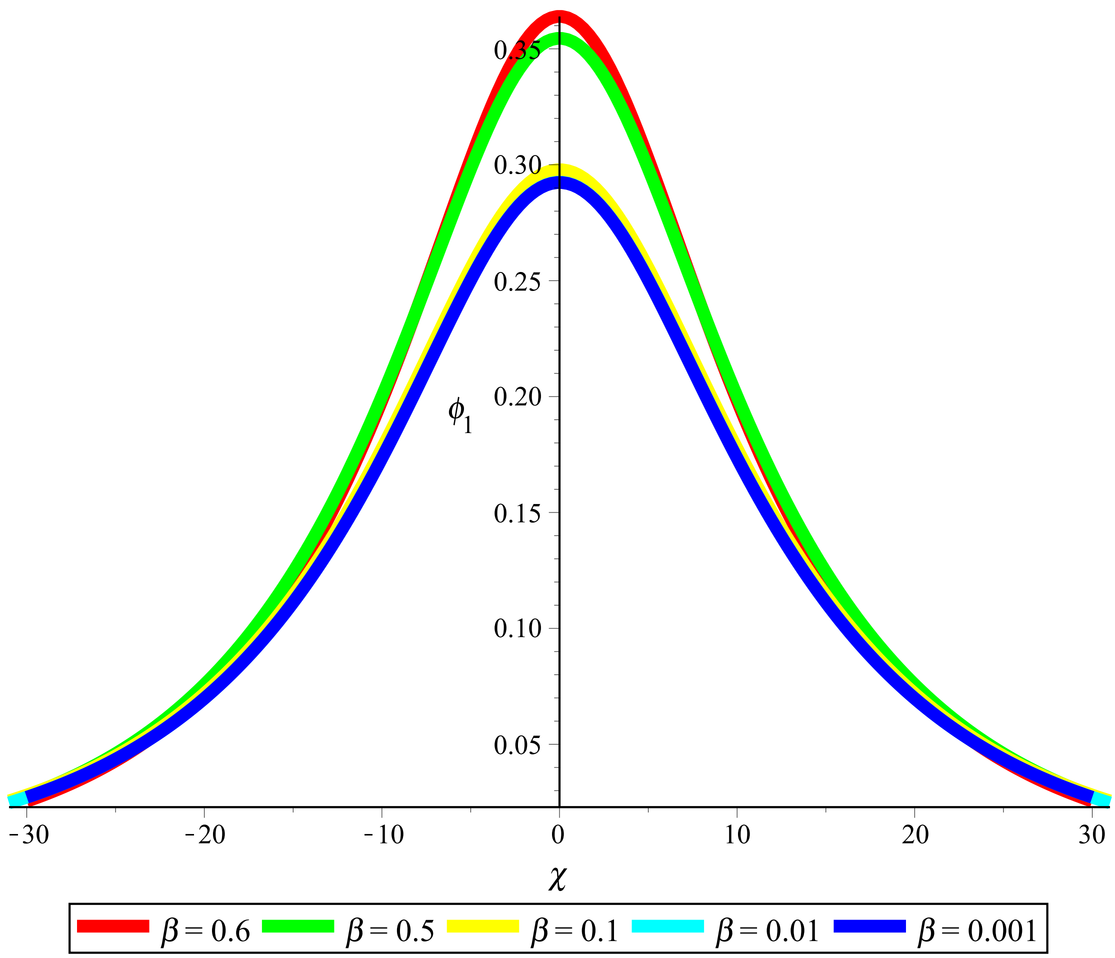

Figure 5.8: The influence of β on the IASs in e-p-i relativistic rotating magnetized plasmas with $p = 0.2$ $(p_{sc} < p)$, $\alpha = 0.1$, $q = 0.1$ $(q_{sc} < q)$, $\sigma = 0.1$, $\delta = 1$, $\Omega_0 = 0.001$, $\omega_{ci} = 1$, $\theta = 45^0$ and $U_0 = 0.0075$.

5.4 RESULTS AND DISCUSSIONS

In this section, the propagation characteristics of the small but finite amplitude nonlinear IASs are discussed, considering the effects of plasma parameters by analyzing the soliton solution of the KdV equation with quartic nonlinearity. In the presented analysis, the values of the parameters are assumed based on reference [27], which are relevant for astrophysical and space environments. It is observed that the KdV equation with quartic nonlinearity are only obtained when the nonlinear coefficients of both KdV and mKdV equations are zero. As a result, one can determine the supercritical values by setting the nonlinear coefficients of these equations are equal to zero. It is found that our plasma environment supports the supercritical values. Thus, our finding based on the above assumptions are given below.

The appearance of super-critical points q_{sc} and p_{sc} for the nonextensivity parameter q and density ratio p, respectively along with the constant values of the remaining parameters are presented in Figures 5.1 and 5.2 by plotting the nonlinear coefficients A and A' of the KdV and mKdV equations. It is found from these figures that both A and A' are becomes zero at the points q_{sc} and p_{sc}, which allows us to study the IASs propagation around q_{sc} and p_{sc}. Based on the appearance of super-critical points, Figures 5.3, 5.4, 5.5, and 5.6 show the effects of q, δ, σ, and p on the nonlinear propagation of IASs in relativistic plasma with the consideration of RLF up to 20 terms and other parameters held constant. The findings indicate that the studied e-p-i plasma exhibits finite-amplitude solitary structures, with their polarity, amplitude, and width being heavily dependent on the plasma parameters. In Figure 5.1, increasing the nonextensivity parameter q leads to a decrease in the amplitude and width of IASs within the range $-0.41 < q < 0.2$, followed by an increase in the range $0.2 < q < 0.27$. There are no IASs found between $0.28 < q < 0.76$. Beyond this interval, further increases in q result in a decrease in the amplitude and width of IASs. It is also observed that the nonextensivity parameter q supports compressive IASs in the presence of both super-thermality and sub-thermality indices for electrons and positrons. The analysis of Figures 5.4, 5.5, and 5.6 is also showed that the amplitude and width of the IASs decrease with the increase in the electron to positron temperature ratio's, ion to electron temperature ratio's, and positron to electron density ratio's. Figure 5.7 displays the variation in the width of the IASs concerning the magnitude of rotational frequency and obliqueness. It is observed that obliqueness significantly affects the width of the IASs, with the width monotonically increasing between 30° and 45°, then decreasing. Conversely, the width of the IASs decreases with increasing values of the magnitude of the rotational frequency. Finally, Figure 5.8 displays the influence of the relativistic streaming index (β) on the nonlinear propagation of IASs, assuming constant values for the other parameters. The variation in IASs are very slightly changing with the relativistic streaming index up to less than 0.1, similar to weakly relativistic plasma [105–113]. However, the peak amplitudes of nonlinear IAS propagation increase with the relativistic streaming index up to 0.5 followed by a very slight increase due to the inclusion of the RLF up to 20 terms. It is recommended that, for advancing the nonlinear propagation of IASs in relativistic plasmas, one should consider not only the RLF up to 20 terms but also additional higher-order terms of the RLF and increasing the value of the relativistic streaming index (β), although this is beyond the scope of this investigation. From a physical perspective, it is evident that the driving force remarkably influences the generation of IASs with a decrease in ion temperature (or an increase in RLF), resulting in an increase in the soliton's energy. The restoring force, on the other hand, becomes notably significant with an increase in positron temperature. Thus, the investigations presented in this article may be highly valuable for understanding the dynamics of obliquely propagating IASs in $(\alpha,q)-$distributed relativistic plasmas not only in contexts such as laser-plasma interactions, quark-gluon environments, dark-matter anomalies, and the solar atmosphere but also for laboratory verification.

5.5 CONCLUSIONS

The investigation of this chapter has been focussed on the oblique propagation characteristics of IASs in the considered relativistic three-component magnetized e-p-i plasmas with the appearance of super-critical values of any one parameter. To archive our goals, the

KdV equation with quartic nonlinearity has been determined for the first time by taking more higher-order correction of the reductive perturbation method. The solution of this equation has also been determined. The analysis made in this chapter have demonstrated how the obliqueness and plasma parameters affect IAS propagation, incorporating the RLF up to 20 terms. It is found that the proposed relativistic plasma environment has supported the compressive IASs in the presence of both super-thermality and sub-thermality around the super-critical values. This analysis would be helpful in understanding how varying q influences the formation and characteristics of solitons in a relativistic plasma environment in the presence of population parameter α. It is also found that the relativistic streaming factor significantly influences the nonlinear propagation of IASs, with soliton energy slightly increasing for relativistic streaming index less than 0.1 and considerably increasing for relativistic streaming index up to 0.5 and followed by a very slight increase. Thus, the presented results may be enhanced our understanding of wave phenomena in many astrophysical and space environments such as high-energy proton motion in the Van Allen radiation belts [124], pulsar magnetospheres [33], magnetized plasma flows in cosmic settings, and solar atmospheres [120], among other, where the plasma environment supports the super-critical values. One may design a new laboratory experiment to verify the theoretical results that presented in this chapter.

Chapter 6: CONCLUDING REMARKS AND FUTURE ASPECTS

This work has made a theoretical approach on the propagation characteristics of IASs in a rotational, magnetized e-p-i plasma consisting of relativistic ion fluid and $(\alpha, q)-$distributed electrons and positrons. By using the reductive perturbation technique, various types of KdV equations has been derived. The effect of plasma parameters on the nonlinear propagation characteristics has been investigated by determine the solutions of these equations. The results found from the previous chapters are summarized below.

Chapters 1 and 2 primarily cover a lucid description of the foundational concepts of plasma, criteria for defining plasma, magnetized and relativistic plasma, theoretical model and soliton formulation, and the methodology used in the study.

In Chapter 3, the proposed theoretical model equations for a rotational, magnetized e-p-i plasma consisting of relativistic plasmas have provided. The KdV equation was derived using the reductive perturbation method. The study investigated the effects of obliqueness and various plasma parameters on the propagation characteristics of IASs by extending the RLF up to the 11th order. It was observed that in the proposed relativistic plasma environment, both compressive and rarefactive IASs are supported in the presence of superthermality. The plasma parameters significantly influence the amplitudes and widths of the IASs as their numerical values increase. Particularly, the relativistic streaming factor plays a notable role in modifying the nonlinear propagation of IASs, resulting in a slight increase (or considerable increase) in soliton energy as the relativistic streaming index increases, either just below 0.1 or above 0.1.

Chapters 4 and 5 extend the findings and discussions presented in Chapter 3. In Chapters 4 and 5, the mKdV equation and KdV equation with quartic nonlinearity were derived by adjusting the stretching coordinates within the framework of reductive perturbation theory. It was observed that the mKdV equation applies to soliton propagation near critical values, whereas the KdV equation with quartic nonlinearity describes soliton propagation around supercritical values. Importantly, these equations were derived for the first time in the context of relativistic magnetized plasmas. Solutions to these equations were determined, enabling the investigation of the nonlinear propagation characteristics of ion-acoustic solitons (IASs) around critical and supercritical values, respectively. It is found that the considered plasma supports only the compressive soliton propagation around critical and supercritical values.

Therefore, the results presented in this thesis would be enhanced our understanding of wave phenomena in numerous astrophysical and space environments, such as high-energy proton dynamics in the Van Allen radiation belts [125], pulsar magnetospheres [27], magnetized plasma flows in cosmic settings, and solar atmospheres [39]. These environments often involve plasma conditions that support not only critical but also supercritical values. It may be beneficial to design new laboratory experiments to validate the theoretical findings presented in this Thesis. Such experiments could provide valuable insights and further confirm the applicability of theoretical models to real-world astrophysical scenarios.

However, there is lot of scope to do further research works by proposing new theoretical model equation in magnetized relativistic plasma under various types of plasma assumption. Addition, one can consider the expansion of RLF not only up to 20 terms but also more than 20 terms. One may also study (i) the dynamical behaviours of acoustic wave propagation by displaying the phase portrait diagrams, (ii) the chaotic motion of acoustic wave phenomena with the influence of external periodic forces, (iii) rouge wave phenomena dy deriving complex nonlinear evolution equations, and so on in the relativistic plasmas. In the presented thesis, we have considered the homogeneous relativistic plasma environments. The inhomogeneous relativistic plasma environments can also be considered to investigate the effect of plasma parameters on the dynamics of acoustic wave phenomena, but beyond the scope of this thesis.

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