1 Structure of Atom —Classical Mechanics

CHAPTER

KEY CONCEPTS AND EQUATIONS

FUNDAMENTAL PARTICLES

An atom consists of the two parts :

(a) Extremely small but dense central part called the nucleus.

(b) Extra nuclear part.

The size of nucleus is of the order 10^{-15} m and it contains positively charged **protons** and neutral particles called **neutrons.** These particles are collectively called **nucleons.**

The charge and mass of these fundamental particles are given in the Table 1.1.

TABLE 1.1. CHARGE AND MASS OF FUNDAMENTAL PARTICLES.

Name of the particle	Charge in coulomb	Unit Charge	Mass (Kg)	Name of the Discoverer
Electron	-1.60×10^{-19}	-1	9.10×10^{-31} kg	J.J. Thomson
Proton	$+1.60 \times 10^{-19}$	$+1$	1.672×10^{-27} kg	E. Goldstein
Neutron	0	0	1.674×10^{-27} kg	Chadwick

ATOMIC NUMBER AND MASS NUMBER

Atomic number is the number of protons present in the nucleus of the atom. It is also equal to the number of electrons in the neutral atom. It is denoted by Z.

Mass Number is the number of nucleons present in the nucleus of an atom. It is denoted by A. An atom is represented by ${}^{A}_{z}X$ or ${}_{Z}X^{A}$. The difference between the mass number and atomic number gives the number of neutrons present in the atom, *i.e.*

$$\text{Number of neutrons} = A - Z$$

PLANCK'S QUANTUM THEORY

The quantum theory proposed by Max Planck in 1900 may be stated as :

(a) Atom absorbs or emits radiations in small units called quanta or photons of energy, *i.e.*

$$E = h\upsilon$$

where υ is the frequency of emitted radiations and h the Planck's constant. Its value is 6.62×10^{-27} erg sec or 6.62×10^{-34} J sec.

(b) An atom can emit or absorb energy equal to $nh\upsilon$ where n is a whole number.

PHOTOELECTRIC EFFECT

When electromagnetic radiations of sufficient energy are allowed to fall on metal surface such as caesium, sodium, etc., electrons are emitted. This phenomenon is called **photoelectric effect.** The frequency which provides enough energy just to release the electron from the metal surface is called **threshold energy,** υ_o. For frequency less than υ_o no electron will be emitted and for higher frequencies $\upsilon > \upsilon_o$, a part of the energy goes to knock the electron and remaining for imparting kinetic energy to the photoelectron emitted. Thus,

$$h\,\upsilon = h\,\upsilon_o + \tfrac{1}{2} m\, v^2$$

where $h\upsilon$ is the energy of the photon of incident light, $h\upsilon_o$ is the minimum energy for an electron to escape from the metal and $\tfrac{1}{2} mv^2$ is the kinetic energy of the photoelectron. A graph between the kinetic energy of photoelectrons against the frequency of incident light is shown in Fig.1.1.

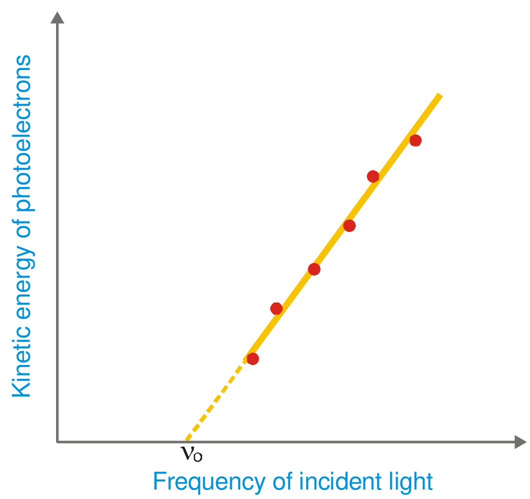


Figure 1.1
Kinetic energy of photoelectrons plotted against frequency of incident light.

EQUATIONS DERIVED FROM BOHR'S THEORY

To calculate the radius (r) and energy (E) of permissible orbits for one electron species, Bohr derived following equations :

Angular momentum, $$m\,v\,r = \frac{n\,h}{2\,\pi} \qquad \text{.... (i)}$$

Radius of an orbit, $$r_n = \frac{n^2\,h^2}{4\,\pi^2\,m\,Z\,e^2} \qquad \text{.... (ii)}$$

Energy of an electron in nth orbit

$$E_n = -\frac{2\,\pi^2 Z\,e^2}{n\,h} \qquad \text{.... (iii)}$$

Velocity of an electron in nth orbit

$$v = \frac{2\,\pi\,Z\,e^2}{n\,h} \qquad \text{.... (iv)}$$

In CGS units, m is the mass of an electron in grams, e its charge in esu, v its velocity in cm sec^{-1}, r the radius of the orbit in cm, h the Planck's constant, E the energy in ergs, Z the atomic number and n is the number of an integer having value 1,2,3 ...... representing the first, second, third, ... orbits respectively.

For first orbit of H atom (or He^+ or Li^{2+} species) we have

$$r_1 = \frac{h^2}{4\pi m Z e^2} \qquad E_1 = -\frac{2\pi^2 Z^2 e^4 m}{h^2} \quad \text{and} \quad v_1 = \frac{2\pi e^2}{h}$$

and for nth orbit we have

$$r_n = n^2 \times r_1 \qquad E_n = -\frac{E_1}{n^2} \quad \text{and} \quad v_n = \frac{v_1}{n}$$

The difference between two energy states n_2 (higher) to n_1 (lower) is given by

$$\Delta E = E_f - E_i$$

where E_f is the electronic energy in final state and E_i is the electronic energy in the initial state. Using Equation (iii) it can be shown that

$$\Delta E = -\frac{2\pi^2 Z^2 e^4 m}{h^2}\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$$

or $$\Delta E = \frac{2\pi^2 Z^2 e^4 m}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$\frac{hc}{\lambda} = \frac{2\pi^2 Z^2 e^4 m}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$\frac{1}{\lambda} = \frac{2\pi^2 Z^2 e^4 m}{c h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

where λ is the wavelength

or $$\text{wave number } \overline{\upsilon} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$R = \frac{2\pi^2 e^4 m}{c\,h^3} \qquad (\because Z = 1 \text{ for H atom})$$

and R is called Rydberg constant. Its value is 109677 cm^{-1} or 1.09677×10^{-7} m.

ADDITIONAL SOLVED PROBLEMS

SOLVED PROBLEM 1. Calculate the velocity of the electron in the first Bohr orbit ($h = 6.625 \times 10^{-27}$ erg sec ; $r = 0.529$ Å ; $m = 9.109 \times 10^{-28}$ g).

SOLUTION :

Formula used

$$m v r = \frac{n h}{2\pi}$$

or $$v = \frac{n h}{2\pi m r}$$

Quantities given

$n = 1$ $h = 6.625 \times 10^{-27}$ erg sec $r = 0.529$ Å $= 0.529 \times 10^{-8}$ cm

Substitution of values

We have $$v = \frac{1 \times 6.625 \times 10^{-27} \text{ erg sec}}{2 \times 3.14 \times (9.109 \times 10^{-28} \text{ g}) \times (0.529 \times 10^{-8} \text{ cm})}$$
$$= 0.2189 \times 10^{9} \text{ cm sec}^{-1}$$
$$= \mathbf{2.189 \times 10^{8} \text{ cm sec}^{-1}}$$

SOLVED PROBLEM 2. Calculate the energy of transition involving $n_1 = 6$ to $n_2 = 3$ in a hydrogen atom, given that Rydberg constant $R = 109737.32$ cm^{-1} and $h = 6.63 \times 10^{-34}$ J sec.

SOLUTION :

Formula used

$$\Delta E = \frac{h\,c}{\lambda}$$

or $$= h\,c\,R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \qquad \left(\because \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]\right)$$

Quantities given

$h = 6.63 \times 10^{-34}$ J sec $n_1 = 6$ $n_2 = 3$

$c = 3 \times 10^{10}$ cm sec^{-1} $R = 109737.32$ cm^{-1}

Substitution of values

$$\Delta E = (6.63 \times 10^{-34} \text{ J sec}) \times (3 \times 10^{10} \text{ cm sec}^{-1}) \times (109737.32 \text{ cm}^{-1}) \left[\frac{1}{6^2} \quad \frac{1}{3^2}\right]$$
$$= 2182675.295 \times 10^{-24}\left[\frac{1}{36} - \frac{1}{9}\right] \text{J}$$
$$= 2182675.295 \times 10^{-24} \,(-0.0833) \text{ J}$$
$$= -181886.9 \times 10^{-24} \text{ J}$$
$$= \mathbf{-1.819 \times 10^{-19} \text{ J}}$$

SOLVED PROBLEM 3. Calculate the wavelength associated with an electron moving with a velocity of 1×10^{8} cm sec^{-1}. Mass of an electron $= 9.1 \times 10^{-28}$ g.

SOLUTION :

Formula used $$\lambda = \frac{h}{m\,v}$$

Quantities given

$m = 9.1 \times 10^{-28}$ g $v = 1 \times 10^{8}$ cm sec^{-1} $h = 6.625 \times 10^{-27}$ erg sec

Substitution of values

We have $$\lambda = \frac{6.625 \times 10^{-27} \text{ erg sec}}{(9.1 \times 10^{-28} \text{ g}) \times (1 \times 10^{-8} \text{ cm sec}^{-1})}$$
$$= 0.728 \times 10^{-9} \text{ cm}$$
$$= \mathbf{7.28 \times 10^{-10} \text{ cm}}$$

SOLVED PROBLEM 4. Calculate the radius of third orbit of hydrogen atom ($h = 6.625 \times 10^{-27}$ erg sec; $m = 9.1091 \times 10^{-28}$ g ; e = 4.8×10^{-10} esu).

SOLUTION :

Formula used

$$r = \frac{n^2 h^2}{4 \pi^2 m e^2}$$

Quantities given

$n = 3$ $\qquad$ $h = 6.625 \times 10^{-27}$ erg sec

$m = 9.1091 \times 10^{-28}$ g $\qquad$ $e = 4.8 \times 10^{-10}$ esu

Substitution of values

We have

$$r = \frac{3^2 \times (6.625 \times 10^{-27} \text{ erg sec})^2}{4 \times (3.14)^2 \times (9.1091 \times 10^{-28} \text{ g}) \times (4.8 \times 10^{-10} \text{ esu})^2}$$

$$= 0.04772 \times 10^{-6} \text{ cm}$$

$$= \mathbf{4.772 \times 10^{-8} \text{ cm}}$$

SOLVED PROBLEM 5. Calculate the wavelength of first line in Lyman series of hydrogen spectrum. (R, Rydberg constant = 109677 cm^{-1}).

SOLUTION :

Formula used $\qquad \frac{1}{\lambda} = R Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$

Quantities given

R = 109677 cm^{-1} $\qquad$ Z = 1 (for hydrogen atom)

$n_1 = 1$ $\qquad$ $n_2 = 2$ (for 1st line in Lyman series)

Substitution of values

$$\frac{1}{\lambda} = 109677 \text{ cm}^{-1} \times \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$$

$$= 109677 \text{ cm}^{-1} \times 0.75$$

$$= 82257.75 \text{ cm}^{-1}$$

or

$$\lambda = \frac{1}{82257.75 \text{ cm}^{-1}}$$

$$= 1.215 \times 10^{-5} \text{ cm}$$

$$= 1.215 \times 10^{-5} \times 10^{8} \text{ Å}$$

$$= \mathbf{1215 \text{ Å}}$$

SOLVED PROBLEM 6. In the photoelectric effect experiment irradiation of a metal with light of frequency 5×10^{20} sec^{-1} yields electrons with maximum kinetic energy 6.63×10^{-14} J. Calculate υ_0, threshold frequency of the metal.

SOLUTION :

Formula used

$$h \upsilon = h \upsilon_0 + \tfrac{1}{2} m v^2$$

or

$$\text{K.E., } \tfrac{1}{2} m v^2 = h \upsilon - h \upsilon_0$$

$$= h (\upsilon - \upsilon_0)$$

Quantities given

$KE = 6.63 \times 10^{-14}$ J $\qquad \upsilon = 5 \times 10^{20}$ sec^{-1} $\qquad h = 6.63 \times 10^{-34}$ J sec

Substitution of values

$$6.63 \times 10^{-14} \text{ J} = 6.63 \times 10^{-34} \text{ J sec} \times (5 \times 10^{20} \text{ sec}^{-1} - \upsilon_0)$$

or
$$5 \times 10^{20} \text{ sec}^{-1} - \upsilon_0 = \frac{6.63 \times 10^{-14} \text{ J}}{6.63 \times 10^{-34} \text{ J sec}}$$
$$= 1 \times 10^{20} \text{sec}^{-1}$$
$$\upsilon_0 = 5 \times 10^{20} \text{ sec}^{-1} - 1 \times 10^{20} \text{ sec}^{-1}$$
$$= \mathbf{4 \times 10^{20} \text{ sec}^{-1}}$$

SOLVED PROBLEM 7. The kinetic energy of a subatomic particle is 4.55×10^{-25} J. Calculate the frequency of the particle wave. Planck's constant $h = 6.62 \times 10^{-34}$ kg m^2 sec^{-1}.

SOLUTION :

Formula used

$$Frequency,\ \upsilon = \frac{m v^2}{h}$$

Quantities given

$KE = \frac{1}{2} mv^2 = 4.55 \times 10^{-25}$ $\qquad$ or $\qquad m v^2 = 2 \times 4.55 \times 10^{-25}$ J

$h = 6.62 \times 10^{-34}$ J sec

Substitution of values

We have
$$\upsilon = \frac{9.10 \times 10^{-25} \text{ J}}{6.62 \times 10^{-34} \text{ J sec}}$$
$$= \mathbf{1.37 \times 10^{-9} \text{ sec}^{-1}}$$

SOLVED PROBLEM 8. What is the frequency of emission spectrum when the electron in hydrogen atom falls from 5th to 2nd orbit ?

SOLUTION :

Formula used

$$\Delta E = h\, c\, R \left[\frac{1}{n_1^{\,2}} - \frac{1}{n_2^{\,2}} \right]$$

Quantities given

$n_1 = 5$ $\qquad n_2 = 2$ $\qquad h = 6.625 \times 10^{-34}$ J sec

$c = 3 \times 10^8$ m sec^{-1} $\qquad R = 1.09677 \times 10^7$ m^{-1}

Substitution of values

$$\Delta E = (6.625 \times 10^{-34} \text{ J sec}) \times (3 \times 10^8 \text{ m sec}^{-1}) \times (1.09677 \times 10^7 \text{ m}^{-1}) \times \left[\frac{1}{5^2} - \frac{1}{2^2} \right]$$
$$= 21.798 \times 10^{-34+8+7} \times (0.04 - 0.25) \text{ J}$$
$$= -4.5776 \times 10^{-19} \text{ J}$$

Now
$$\text{Frequency},\ \upsilon = \frac{\Delta E}{h}$$
$$= -\frac{4.5776 \times 10^{-19} \text{ J}}{6.625 \times 10^{-34} \text{ J sec}}$$
$$= -0.6909 \times 10^{15} \text{ sec}^{-1}$$
$$= \mathbf{-6.909 \times 10^{14} \text{ sec}^{-1}}$$

The negative sign simply indicates the loss of energy.

SOLVED PROBLEM 9. Calculate the energy difference, frequency and wavelength of light emitted when the electron in a hydrogen atom undergoes transition from 4th energy level to 2nd energy level (Rydberg constant = 1.09677×10^7 m^{-1}).

SOLUTION :

Formula used

$$\Delta E = h\,c\,R \left[\frac{1}{n_1^{\,2}} - \frac{1}{n_2^{\,2}}\right]$$

Quantities given

$n_1 = 2$ $\quad n_2 = 4$ $\quad R = 1.09677 \times 10^7 \text{ m}^{-1}$ $\quad c = 3 \times 10^8 \text{ m sec}^{-1}$

Substitution of values

$$\Delta E = \frac{(6.625 \times 10^{-34} \text{ J sec}^{-1}) \times (3 \times 10^8 \text{ m sec}^{-1}) \times (1.09677 \times 10^7 \text{ m}^{-1})}{\left(\frac{1}{2^2} - \frac{1}{4^2}\right)}$$

$$= 21.798 \times 10^{-34+8+7} \times 0.1875 \text{ J}$$

$$= \mathbf{4.087 \times 10^{-19} \text{ J}}$$

Also frequency, $\upsilon = \dfrac{\Delta E}{h} = \dfrac{4.087 \times 10^{-19} \text{ J}}{6.625 \times 10^{-34} \text{ J sec}}$

$$= 0.6169 \times 10^{15} \text{ sec}^{-1}$$

$$= \mathbf{6.169 \times 10^{14} \text{ sec}^{-1}}$$

and wavelength, $\lambda = \dfrac{c}{\upsilon} = \dfrac{3 \times 10^8 \text{ m sec}^{-1}}{6.619 \times 10^{14} \text{ m}}$

$$= 0.4532 \times 10^{-6} \text{ m}$$

$$= \mathbf{4.532 \times 10^{-7} \text{ m}}$$

SOLVED PROBLEM 10. If the energy difference between the ground state of an atom and its excited state is 4.4×10^{-19} J, what is the wavelength of the photon required to produce this transition ?

SOLUTION :

Formula used

$$\Delta E = \frac{h\,c}{\lambda}$$

or

$$\lambda = \frac{h\,c}{\Delta E}$$

Quantities given

$\Delta E = 4.4 \times 10^{-19}$ J $\quad h = 6.625 \times 10^{-34}$ J sec $\quad c = 3 \times 10^8$ m sec^{-1}

Substitution of values

$$\lambda = \frac{(6.625 \times 10^{-34} \text{ J sec}) \times (3 \times 10^8 \text{ m sec}^{-1})}{(4.4 \times 10^{-19} \text{ J})}$$

$$= 4.517 \times 10^{-34+8+19} \text{ m}$$

$$= \mathbf{4.517 \times 10^{-7} \text{ m}}$$

SOLVED PROBLEM 11. The electron in a hydrogen atom revolves in the second orbit. Calculate (i) the energy of electron in this orbit (ii) radius of the second orbit and (iii) frequency and wavelength of the spectral line emitted when this electron jumps to the ground state.

SOLUTION : (i) To calculate the energy of electron in 2nd orbit

Formula used

$$E_n = \frac{2\,\pi^2 m\, e^4}{n^2\, h^2}$$

Quantities given

$n = 2$ $\quad$ $e = 4.80 \times 10^{-10}$ esu $\quad$ $h = 6.625 \times 10^{-27}$ erg sec $\quad$ m = 9.1×10^{-28} g

Substitution of values

$$E_2 = -\frac{2 \times (3.14)^2 \times (4.8 \times 10^{-10}\,\text{esu})^4 \times (9.1 \times 10^{-28}\,\text{g})}{2^2 \times (6.625 \times 10^{-27}\,\text{erg sec})^2}$$

$$= -\ 3594.59 \times 10^{-14}\ \text{erg}$$

$$= -\ 3.59459 \times 10^{-14}\ \text{erg}$$

$$= \mathbf{-3.5946 \times 10^{-11}\ erg}$$

(ii) To calculate the radius of 2nd orbit

Formula used

$$r_2 = \frac{n^2\, h^2}{4\,\pi^2 m\, e^2}$$

Quantities given

$n = 2$ $\quad$ $h = 6.625 \times 10^{-27}$ erg sec $\quad$ $m = 9.1 \times 10^{-28}$ g $\quad$ $e = 4.8 \times 10^{-10}$ esu

Substitution of values

$$r_2 = \frac{2^2 \times (6.625 \times 10^{-27}\,\text{erg sec})^2}{4 \times (3.14)^2 \times (9.1 \times 10^{-28}\,\text{g}) \times (4.8 \times 10^{-10}\,\text{esu})^2}$$

$$= 0.0212 \times 10^{-54+28+20}\ \text{cm}$$

$$= 0.0212 \times 10^{-6}\ \text{cm}$$

$$= 2.12 \times 10^{-8}\ \text{cm}$$

$$= \mathbf{2.12\ Å}$$

(iii) To calculate the frequency and wavelength of the line emitted

Formula used

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^{\,2}} - \frac{1}{n_2^{\,2}}\right]$$

Quantities given

$n_1 = 1$ $\quad$ $n_2 = 2$ $\quad$ $R = 109677\ \text{cm}^{-1}$

Substitution of values

$$\frac{1}{\lambda} = (109677\ \text{cm}^{-1}) \times \left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$= 109677 \times 0.75\ \text{cm}^{-1}$$

$$= 82257.75\ \text{cm}^{-1}$$

or

$$\lambda = \frac{1}{82257.57\ \text{cm}^{-1}} = \mathbf{1.2157 \times 10^{-5}\ cm}$$

and frequency, $\upsilon = \dfrac{c}{\lambda} = \dfrac{3 \times 10^{10}\,\text{cm}}{1.2157 \times 10^{-5}\,\text{cm}}$

$$= \mathbf{2.4677 \times 10^{15}\ Hz}$$

SOLVED PROBLEM 12. According to Bohr's theory, the electronic energy of the hydrogen atom in nth Bohr's orbit is given by $E_n = -\dfrac{21.76 \times 10^{-19} Z^2}{n^2}$ J. Calculate the longest wavelength of light that will be needed to remove an electrons from 3rd Bohr orbit of He^+ ion.

SOLUTION :

Formula used

$$E_n = -\frac{21.76 \times 10^{-19}\ Z^2}{n^2}$$

Quantities given

$Z = 2$ $\qquad n = 3$

Substitution of values

$$E_n = -\frac{21.76 \times 10^{-19} \times 4}{9}$$

$$= \mathbf{-9.671 \times 10^{-19}\ J}$$

Thus the energy required to remove an electron from 3rd Bohr's orbit of He^+ ion is 9.671×10^{-19} J.

SOLVED PROBLEM 13. What transition in the hydrogen spectrum would have the same wavelength as the Balmer transition $n = 4$ to $n = 2$ of He^+ spectrum ?

SOLUTION :

Formula used

$$\frac{1}{\lambda} = R Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Quantities given (for helium atom)

$Z = 2$ $\qquad n_1 = 2$ $\qquad n_2 = 4$

Substitution of values

$$\frac{1}{\lambda_1} = R \times 4 \times \left[\frac{1}{4} - \frac{1}{16}\right]$$

$$= \frac{4\,R \times 3}{16}$$

Quantities given (for hydrogen atom)

$Z = 1$

Substitution of values

$$\frac{1}{\lambda_2} = R \times 1 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

since $\lambda_1 = \lambda_2$

We have $\dfrac{4\,R \times 3}{16} = R \times \left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

or
$$\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = \frac{3}{4}$$

This is possible if $n_1 = 1$ and $n_2 = 2$

Thus the transition $n_2 = 2$ to $n_1 = 1$ (Lyman series) in hydrogen atom has the same wavelength as the Balmer series transition $n_2 = 4$ to $n_1 = 2$ of He^+ spectrum.

SOLVED PROBLEM 14. Find out the number of waves made by a Bohr electron in one complete revolution in its 3rd orbit.

SOLUTION :

Formula used

$$m\,v\,r = \frac{n\,h}{2\,\pi}$$

Quantities given

$$n = 3$$

Substitution of values

$$m\,v\,r = \frac{3\,h}{2\,\pi}$$

or
$$2\,\pi\,r = \frac{3\,h}{m\,v} \qquad \left[\because\ \lambda = \frac{h}{m\,v}\right]$$
$$= 3\,\lambda$$

Hence three times the wavelength of electron is equal to the circumference of 3rd orbit. So the electron makes three waves around 3rd orbit.

SOLVED PROBLEM 15. Iodine molecule dissociates after absorbing light of 4500Å. If one quantum radiation is absorbed by each molecule, calculate the kinetic energy of iodine atom. (Bond energy of $I_2 = 240 \times 10^3$ kJ mol^{-1}).

SOLUTION :

Formula used

$$\Delta E = \frac{h\,c}{\lambda}$$

Quantities given

$h = 6.625 \times 10^{-34}$ J sec $\qquad c = 3 \times 10^{-8}$ m sec^{-1} $\qquad \lambda = 4500$ Å $= 4500 \times 10^{-10}$ m

Substitution of values

$$\Delta E = \frac{(6.625 \times 10^{-34}\text{ J sec}) \times (3 \times 10^{8}\text{ m sec}^{-1})}{(4500 \times 10^{-10}\text{ m})}$$
$$= 4.416 \times 10^{-19}\text{ J}$$

Now Bond energy of I_2 per molecule
$$= \frac{240 \times 10^3}{\text{Avogadro's No.}}\text{ J/molecule}$$
$$= \frac{240 \times 10^3}{6.023 \times 10^{23}}\text{ J/molecule}$$
$$= 3.984 \times 10^{-19}\text{ J/molecule}$$

Kinetic energy of iodine atoms
$$= \text{Energy absorbed} - \text{Bond energy}$$
$$= 4.416 \times 10^{-19}\text{ J} - 3.984 \times 10^{-19}\text{ J}$$
$$= 0.432 \times 10^{-19}\text{ J}$$

$\therefore$ Kinetic energy per iodine atom
$$= \frac{0.432 \times 10^{19}}{2}\text{ J}$$
$$= \mathbf{2.16 \times 10^{-20}\text{ J}}$$

SOLVED PROBLEM 16. Calculate the wave number for the shortest wavelength transition in the Balmer series of hydrogen atom.

SOLUTION :

Formula used

$$\bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Quantities given

$n_1 = 2$ $n_2 = \infty$ $R = 109677\ \text{cm}^{-1}$

Substitution of values

$$\bar{\upsilon} = 109677\ \text{cm}^{-1}\left[\frac{1}{2^2} - \frac{1}{\infty}\right]$$

$$= 109677\left[\frac{1}{4} - 0\right]\text{cm}^{-1}$$

$$= \mathbf{27419.25\ cm^{-1}}$$

SOLVED PROBLEM 17. Calculate the energy emitted when electrons of 1.0 g atom of hydrogen undergo transition giving the spectral line of lowest energy in the visible region of its atomic spectrum. ($R = 1.1 \times 10^7\ \text{m}^{-1}$).

SOLUTION :

Formula used

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Quantities given

$R = 1.10 \times 10^7\ \text{m}^{-1}$ For visible region we have Balmer series and

For Balmer series $n_1 = 2$ $n_2 = 3$ (lowest energy)

Substitution of values

$$\frac{1}{\lambda} = 1.1 \times 10^7\text{m}^{-1} \times \left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= 1.1 \times 10^7\ \text{m}^{-1} \times \frac{5}{36}$$

$$= 0.15277 \times 10^7\ \text{m}^{-1}$$

or
$$\lambda = \frac{1}{0.15277 \times 10^7\ \text{m}^{-1}}$$

$$= 6.546 \times 10^{-7}\ \text{m}$$

To calculate energy emitted

Formula used

$$\Delta E = \frac{h\,c}{\lambda}$$

Quantities given

$\lambda = 6.545 \times 10^{-7}$ m $h = 6.625 \times 10^{-34}$ J sec $c = 3 \times 10^{-8}$ m sec^{-1}

Substitution of values

$$\Delta E = \frac{(6.625 \times 10^{-34}\ \text{J sec}) \times (3 \times 10^8\ \text{m sec}^{-1})}{6.545 \times 10^{-7}\ \text{m}}$$

$$= 3.037 \times 10^{-34+8+7}\ J$$
$$= 3.037 \times 10^{-19}\ J$$

Now energy emitted from 1 g atom of hydrogen

$$= 3.037 \times 10^{-19}\ J \times \text{Avogadro's No.}$$
$$= 3.037 \times 10^{-19} \times 6.02 \times 10^{23}\ J$$
$$= \mathbf{182.82\ kJ}$$

SOLVED PROBLEM 18. Calculate the wavelength of radiation emitted, producing a line in Lyman series, when an electron falls from fourth stationary state in hydrogen atom. ($R = 1.1 \times 10^7\ m^{-1}$)

SOLUTION :

Formula used

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Quantities given

$n_2 = 4$ $\qquad$ $n_1 = 1$ (Lyman series) $\qquad$ $R = 1.1 \times 10^7\ m^{-1}$

Substitution of values

$$\frac{1}{\lambda} = (1.1 \times 10^7\ m^{-1}) \times \left[\frac{1}{1^2} - \frac{1}{4^2}\right]$$
$$= 1.1 \times 10^7\ m^{-1} \times \left[1 - \frac{1}{16}\right]$$
$$= 1.1 \times 10^7 \times \frac{15}{16}\ m^{-1}$$
$$= 1.0313 \times 10^7\ m^{-1}$$
$$= 1.0313 \times 10^7\ m^{-1}$$

or

$$\lambda = \frac{1}{1.0313 \times 10^7\ m^{-1}}$$
$$= 0.9696 \times 10^{-7}$$
$$= \mathbf{9.696 \times 10^{-8}\ m}$$

ADDITIONAL PRACTICE PROBLEMS

1. Calculate the velocity of the electron in the first Bohr's orbit ($h = 6.625 \times 10^{-27}$ erg sec; $r = 0.529$ Å ; $m = 9.109 \times 10^{-28}$ g).
Answer. 2.189×10^8 cm sec^{-1}

2. Calculate the energy of transition involving $n_1 = 6$ to $n_2 = 3$ in a hydrogen atom given that Rydberg constant, $R = 109737.32\ cm^{-1}$ and $h = 6.63 \times 10^{-34}$ J sec.
Answer. 1.818×10^{-19} J

3. Calculate the radius of the third orbit of hydrogen atom ($h = 6.625 \times 10^{-27}$ erg sec ; $m = 9.1091 \times 10^{-28}$ g; $e = 4.8 \times 10^{-10}$ esu).
Answer. 4.763×10^{-8}

4. Calculate the wavelength of the first line in Balmer series of hydrogen spectrum. (R = 109677 cm^{-1})
Answer. 1215 Å

5. What is frequency of a red light source having wavelength of 700 nm (nanometer) ? Calculate the energy of a photon of light.
Answer. 4.28 × 10^{14} sec^{-1} ; 2.78 × 10^{-19} J

6. What is the frequency of emission spectra when the electron in a hydrogen atom falls from n_6 orbit to n_2 orbit ?
Answer. 7.30 × 10^{14} sec^{-1}

7. A light beam has a frequency of 1.0 × 10^{12} Hertz. What is the wavelength of the beam in meters ? The velocity of light is 3.0 × 10^{8} m sec^{-1}.
Answer. 3.0 × 10^{4} m sec^{-1}

8. Sound waves travel about 203 m sec^{-1} in air. What is the wavelength of the sound produced by a tuning fork of 512 vibrations per second ?
Answer. 0.396 m

9. Calculate the loss of energy in Joules per mole of photons, when atoms release radiations at a frequency of 10^{12} sec^{-1}.
Answer. 4 × 10^{2} J mol^{-1}

10. An electron in an atom drops to a lower energy level with the release of 5.0 × 10^{14} sec^{-1} radiations. What is the loss of energy in J mol^{-1} ?
Answer. 2.0 × 10^{5} J mol^{-1}

11. Visible light is in the range of 4.0 × 10^{14} Hz to 7.5 × 10^{14} Hz. Calculate the range of visible radiation in meters.
Answer. 4 × 10^{-7} m to 7.5 × 10^{-7} m

12. In hydrogen atom the energy of the electron in first Bohr's orbit is – 1312 × 10^{5} J mol^{-1}. What is the energy required for the excitation of second Bohr's orbit ?
Answer. 9.84 × 10^{5} J mol^{-1}

13. The kinetic energy of an electron is 4.55 × 10^{-25} J. Calculate its wavelength (h = 6.6 × 10^{-34} kg m^{2} sec^{-1}; m = 9.1 × 10^{-31} kg)
Answer. 7.2 × 10^{-7} m

14. The reaction $I_2 \xrightarrow{\text{energy}} 2\,I$ was brought about by light radiation. It was found that 151 kJ mol^{-1} of light was required to dissociate the molecular iodine. Assuming that one quantum of light energy was required to dissociate one molecule of iodine, calculate the energy in Joules in one quantum of light and the wavelength of the light radiations in meters.
Answer. 2.51 × 10^{-19} J ; 7.92 × 10^{-7} m

15. Using Bohr theory, calculate the radius and velocity of the electron in tenth orbit of hydrogen atom.
Answer. 0.53 × 10^{-6} cm ; 21.9 × 10^{7} cm sec^{-1}

16. Calculate the frequency and energy associated with photons of radiations having a wavelength of 6000 Å. Planck's constant = 6.625 × 10^{-27} erg sec.
Answer. 5 × 10^{14} cps ; 3.312 × 10^{-27} erg

17. Calculate the range of frequencies of visible light (3800 Å to 7600 Å)
Answer. 3.948 × 10^{14} cps to 7.894 × 10^{14} cps

18. Calculate the wave number of lines having frequency of 4.5 × 10^{16} cps.
Answer. 1.5 × 10^{8} m^{-1}

19. Calculate the energy in kcal mol^{-1} of photons of an electromagnetic radiations of wavelength 4000 Å.
Answer. 71.47 kcal mol^{-1}

20. Electromagnetic radiation of wavelength 242 nm is just sufficient to ionize the sodium atom. Calculate the ionization energy of sodium in kJ mol^{-1} ; h = 6.6256 × 10^{-34} Joule sec.
Answer. 494.5 kJ mol^{-1}

21. In a hydrogen atom, an electron jumps from 3rd orbit to first orbit. Find out the frequency and wavelength of the spectral line.

Answer. 1025.6 Å

22. Calculate the wavelength and energy of radiations emitted for the electronic transition from infinity (∝) to stationary state of the hydrogen atom. (R = 1.09678 × 10^7 m^{-1} ; h = 6.625 × 10^{-34} Joule sec and c = 2.9979 × 10^8 m sec^{-1})

Answer. 9.11 × 10^{-6} m ; 217.9 × 10^{-23} kJ

23. Calculate the wavelength in Å of the photon that is emitted when an electron in Bohr orbit n = 2 returns to the orbit n = 1 in the hydrogen atom. The ionization potential in the ground state of hydrogen atom is 2.17 × 10^{-11} erg per atom.

Answer. 1220 Å

24. What transition in the hydrogen spectrum would have the same wavelength as the Balmer transition n = 4 to n = 2 of He^+ transition ?

Answer. n = 2 to n = 1

25. A line at 434 nm in Balmer series of spectrum corresponds to a transition of an electron from the nth to 2nd Bohr orbit. What is the value of n ?

Answer. n = 5

26. The electron energy in hydrogen atom is given by $E = -21.7 \times 10^{-12}/n^2$ ergs. Calculate the energy required to remove an electron completely from the n = 2 orbit. What is the longest wavelength (in cm) of light that can be used to cause this transition ?

Answer. – 5.42 × 10^{-12} erg ; 3.67 × 10^{-5} cm

27. The energy transition in hydrogen atom occurs from n = 3 to n = 2 energy level. (R = 1.097 × 10^7 m^{-1}). (i) Calculate the wavelength of the emitted electron (ii) Will this electron be visible ? (iii) Which spectrum series does this photon belong to ?

Answer. 6564 Å ; Yes ; Balmer series

28. Calculate the energy emitted when electrons of 1.0 g of hydrogen undergo transition giving the spectral line of lowest energy in the visible region of its atomic spectrum (R = 1.1 × 10^7 m^{-1}; c = 3 × 10^8 m sec^{-1}; h = 6.62 × 10^{-34} J sec)

Answer. 182.5 kJ

29. The energy of the electron in the second and third Bohr orbits of the hydrogen atom is – 5.42 × 10^{-12} erg and – 2.41 × 10^{-12} erg respectively. Calculate the wavelength of the emitted radiation when the electron drops from third to second orbit.

Answer. 6600 Å

30. A doubly ionized lithium atom is hydrogen like with an atomic number 3. (i) Find the wavelength of the radiation required to excite the electron in Li from the first to third Bohr orbit (Ionization energy of hydrogen atom is 13.6 eV) (ii) How many spectral lines are observed in the emission spectrum of the above excited system ?

Answer. (i) 113.7Å (ii) 3

31. An electron beam can undergo diffraction by crystals. Through what potential should a beam of electrons be accelerated so that its wavelength becomes equal to 1.54 Å.

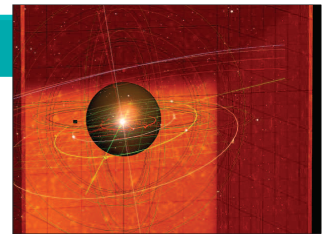
Answer. 8.0459]

32. With what velocity should an α - particle travel towards the nucleus of a copper atom so as to arrive at a distance of 10^{-13} meter from the nucleus of the copper atom ?

Answer. 6.318 × 10^6 m s^{-1}

CHAPTER 2

Structure of the Atom —Wave Mechanics Approach



KEY CONCEPTS AND EQUATIONS

WAVE MECHANICS

According to "Wave Mechanical Theory' light exhibits both a wave and a particle nature under suitable conditions. This theory was further extended to matter *i.e.* electrons, protons and atoms. The new quantum mechanics which takes into account the particulate and wave nature of matter is termed as wave mechanics.

DE BROGLIE EQUATION

de Broglie in 1924 proposed that matter (atoms, neutrons, protons, electrons, etc.) has dual character *i.e.* it is associated with the properties of waves and their wavelengths λ is given by the relation

$$\lambda = \frac{h}{mv} = \frac{h}{momentum}$$

where m is the mass, v, the velocity of the particle and h the Planck's constant. de Broglie equation has no significance for large objects.

HEISENBERG'S UNCERTAINTY PRINCIPLE

It is impossible to measure simultaneously both the position and velocity (or momentum) of a microscopic particle with absolute accuracy or certainty. Mathematically,

$$\Delta x \times \Delta p \geq \frac{h}{4\pi}$$

or

$$\Delta x \times m\,\Delta v \geq \frac{h}{4\pi}$$

where Δx is uncertainty in position and Δp the uncertainty in momentum or Δv is the uncertainty in velocity and m the mass of the particle. It also has no significance for large objects.

SCHRÖDINGER'S WAVE EQUATION

Schrödinger derived an equation based upon the idea of the electron as 'standing wave' around the nucleus. This equation, known as Schrödinger's wave equation, is

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} + \frac{8\pi^2 m}{h^2}(E - PE)\,\psi = 0$$

where ψ is a mathematical function representing the amplitude of the wave, m the mass of the particle, v, the velocity, E the total energy of the particle, $P.E.$, the potential energy and x,y,z are three space coordinates.

It may be noted that square of the wave function, ψ^2, represents the probability of finding the electron in different regions.

QUANTUM NUMBERS

According to modern theory of electronic structure an electron in an atom is specified in terms of four quantum numbers which describe the probable location of the electron. The values of quantum numbers govern its energy, orientation in space and its possible interaction with other electrons.

PRINCIPAL QUANTUM NUMBER

It denotes the principal shell to which the electron belongs. It also represents the average size of electron cloud. The distance of the electron from the nucleus in hydrogen atom is given by

$$r_n = 0.529 \times n^2 \text{ Å}$$

where n is the principal quantum number. It can have integral values 1,2,3,4....... The maximum number of electrons in the principal shell is equal to $2n^2$.

AZIMUTHAL QUANTUM NUMBER

It describes the angular momentum of the electron and defines the shape of the orbital occupied by the electron. It is denoted by 'l' and it can have all integral values from 0 to n–1. The total number of sublevels in each principal level is numerically equal to the principal quantum number of that level. For example,

$$\begin{array}{lllll}
n = 1 & l = 0 & & & \\
n = 2 & l = 0, & l = 1 & & \\
n = 3 & l = 0, & l = 1, & l = 2 & \\
n = 4 & l = 0, & l = 1, & l = 2, & l = 3
\end{array}$$

The shape of the orbital is related to the azimuthal quantum number in the following manner :

$$\begin{array}{cccc}
l = 0 & l = 1 & l = 2 & l = 3 \\
s\text{–subshell} & p\text{–subshell} & d\text{–subshell} & f\text{–subshell}
\end{array}$$

MAGNETIC QUANTUM NUMBER

It accounts for the splitting of the spectral line in a magnetic field (Zeeman effect). It gives the orientation of the electron cloud and is denoted by m. For each value of l, m can have all integral values between $-l$ to $+l$ through zero. For example,

$$\begin{array}{llllllll}
l = 0 & m = 0 & & & & & & \\
l = 1 & m = -1, & m = 0, & m = +1 & & & & \\
l = 2 & m = -2, & m = -1, & m = 0, & m = +1, & m = +2 & & \\
l = 3 & m = -3, & m = -2, & m = -1, & m = 0, & m = +1, & m = +2, & m = +3
\end{array}$$

SPIN QUANTUM NUMBER

It accounts for the spin of the electron about its own axis. It is denoted by s and it can have either +½ or –½ value depending upon whether the electron spin is clockwise or anticlockwise. The summary of the four quantum numbers is given in Table 2.1.

TABLE 2.1 THE QUANTUM NUMBERS AND THEIR PERMITTED VALUES.

Quantum Number	Designation	Permitted Values
Principal Quantum Number	n	1,2,3,4,..............
Azimuthal Quantum Number	l	0,1,2,3,..............
Magnetic Quantum Number	m	$-l$,.....,0,.......$+l$
Spin Quantum Number	s	+1/2 or –1/2

PAULI'S EXCLUSION PRINCIPLE

No two electrons in an atom can have the same set of all the four identical quantum numbers. This means if two electrons have the same values for n, l and m, they must have different values of s. It helps us to find out the number of electrons that can be accommodated in an orbital or sublevel or principal energy level. The total number of electrons in first four energy shells are given in Table 2.2.

TABLE 2.2. MAXIMUM NUMBER OF ELECTRONS IN FIRST FOUR ENERGY LEVELS.

Principal Quantum Number	Number of electrons in				Total number of electrons
	s–subshell	p–subshell	d–subshell	f–subshell	
1	2	–	–	–	2
2	2	6	–	–	8
3	2	6	10	–	18
4	2	6	10	14	32

($n + l$) RULE

The energy of the electrons increases as the sum $(n+l)$ increases, i.e. lower the value of $(n+l)$ for an electron in an atom lower is its energy.

For two electrons with same $(n+l)$ value, the one with lower value of n has lower energy.

AUFBAU PRINCIPLE

In the ground state of an atom, the electron enters the orbital of lowest energy first and subsequent electrons are added in the order of increasing energies. Fig. 2.1 shows the energy level scheme of filling up of electrons in various orbitals.

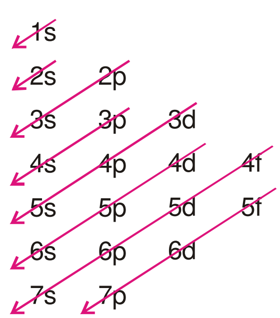


Figure 2.1
Aufbau order of filling up of electrons in various orbitals.

The increasing order of energy of various orbitals is as follows :

$$1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s \ldots\ldots\ldots\ldots\ldots$$

HUND'S RULE OF MAXIMUM MULTIPLICITY

Electrons are distributed among the orbitals of a subshell in such a way as to give maximum number of unpaired electrons with the same direction of spin.

ADDITIONAL SOLVED PROBLEMS

SOLVED PROBLEM 1. A neutral atom has 2K, 8L and 3M electrons. Predict from this (i) its atomic number; (ii) total number of *s*-electrons; (iii) total number of *p*-electrons and (iv) total number of *d*-electrons.

SOLUTION :

(i) *Atomic number*

Atomic number = No. of Protons = No. of electrons
= 2 + 8 + 3
= 13

(ii) *Total number of s-electrons*

The electronic configuration of the atom with atomic number 13 is

$$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^1$$

Total no. of *s*-electrons = 2 + 2 + 2 $[1s^2, 2s^2, 3s^2]$
= **6**

(iii) *Total no. of p-electrons*

Total no. of *p*-electrons = 6 + 1 $[2p^6, 3p^1]$
= **7**

(iv) *Total no. of d-electrons*

Total no. of *d*-electrons = **0** [$\because$ $3d$ is vacant]

SOLVED PROBLEM 2. The velocity of a ball bowled by Kapil Dev is 25 m sec^{-1}. Calculate the wavelength of the matter wave associated with the ball (mass of the cricket ball is 158.5 g; $h = 6.625 \times 10^{-27}$ erg sec).

SOLUTION :

Formula used

$$\lambda = \frac{h}{m\,v}$$

Quantities given

$m = 158.5$ g $\quad v = 25$ m sec^{-1} = 2500 cm sec^{-1} $\quad h = 6.625 \times 10^{-27}$ erg sec

Substitution of values

$$\lambda = \frac{6.625 \times 10^{-27}\ \text{erg sec}}{158.5\ \text{g} \times 2500\ \text{cm sec}^{-1}}$$

$$= 0.0000167 \times 10^{-27}\ \text{cm}$$

$$= \mathbf{1.67 \times 10^{-32}\ cm}$$

SOLVED PROBLEM 3. Give the orbital configuration and number of unpaired electrons in the ground state for the following atoms :

(i) Nitrogen (At. No. 7)
(ii) Chlorine (At. No. 17)

(iii) Manganese (At. No. 25)

(iv) Cadmium (At. No. 48)

SOLUTION : The electronic configurations and number of unpaired electrons are as follows :

			No. of unpaired electrons
(i) Nitrogen (At. No.7)	=	$1s^2\ 2s^2\ 2p_x^1\ 2p_y^1\ 2p_z^1$	3
(ii) Chlorine (At. No. 17)	=	$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p_x^2\ 3p_y^2\ 3p_z^1$	1
(iii) Manganese (At. No. 25)	=	$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^5$	5
(iv) Cadmium (At. No. 48)	=	$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4s^2\ 4p^6\ 5s^2\ 4d^{10}$	0

SOLVED PROBLEM 4. A particle having wavelength of 6.6×10^{-4} cm is moving with a velocity of 10^6 cm sec^{-1}. Find the mass of the particle. (Planck's constant = 6.62×10^{-27} erg sec)

SOLUTION :

Formula used

$$\lambda = \frac{h}{m\,v}$$

or
$$m = \frac{h}{v\,\lambda}$$

Quantities given

$h = 6.62 \times 10^{-27}$ erg sec $\qquad v = 10^6$ cm sec^{-1} $\qquad \lambda = 6.6 \times 10^{-4}$ cm

Substitution of values

$$m = \frac{6.62 \times 10^{-27} \text{ erg sec}}{6.6 \times 10^{-4} \text{ cm} \times 10^{6} \text{ cm sec}^{-1}}$$

$$= \mathbf{1.003 \times 10^{-29}\ g}$$

SOLVED PROBLEM 5. What is the wavelength associated with a particle of mass 0.1 g moving with a speed of 1×10^5 cm sec^{-1}? ($h = 6.6 \times 10^{-27}$ erg sec)

SOLUTION :

Formula used

$$\lambda = \frac{h}{m\,v}$$

Quantities given

$h = 6.6 \times 10^{-27}$ erg sec $\qquad m = 0.1$ g $\qquad v = 1 \times 10^5$ cm sec^{-1}

Substitution of values

$$\lambda = \frac{6.6 \times 10^{-27} \text{ erg sec}}{(0.1 \text{ g}) \times (1 \times 10^{5} \text{ cm sec}^{-1})}$$

$$= 66 \times 10^{-32} \text{ g}$$

$$= \mathbf{6.6 \times 10^{-31}\ g}$$

SOLVED PROBLEM 6. Calculate the de Broglie wavelength for a ball of 200 g moving with a velocity 3×10^{10} cm sec^{-1} and an electron moving with the same velocity. (Mass of electron = 9.109 $\times 10^{-28}$ g; $h = 6.624 \times 10^{-27}$ erg sec).

SOLUTION :

Formula used

$$\lambda = \frac{h}{m\,v}$$

Quantities given

(i) For the ball

$m = 200$ g $\qquad v = 3 \times 10^{10}$ cm sec^{-1} $\qquad h = 6.624 \times 10^{-27}$ erg sec

Substitution of values

$$\lambda = \frac{6.624 \times 10^{-27} \text{ erg sec}}{200 \text{ g} \times 3 \times 10^{10} \text{ cm sec}^{-1}}$$

$$= \mathbf{1.04 \times 10^{-39} \text{ cm}}$$

(ii) For the electron

$m = 9.109 \times 10^{-28}$ g $\qquad v = 3 \times 10^{10}$ cm sec^{-1} $\qquad h = 6.624 \times 10^{-27}$ erg sec

Substitution of values

$$\lambda = \frac{6.624 \times 10^{-27} \text{ erg sec}}{9.109 \times 10^{-28} \text{ g} \times 3 \times 10^{10} \text{ cm sec}^{-1}}$$

$$= 0.24239 \times 10^{-9} \text{ cm}$$

$$= \mathbf{2.4239 \times 10^{-10} \text{ cm}}$$

SOLVED PROBLEM 7. If an electron is accelerated by 100 volts, calculate the de Broglie wavelength associated with it. Also calculate the velocity acquired by the electron. (Mass of electron = 9.1×10^{-28} g; $h = 6.62 \times 10^{-27}$ erg sec).

SOLUTION :

(i) To calculate wavelength

Formula used

$$\lambda = \sqrt{\frac{150}{V \text{ volts}}} \text{ Å}$$

Quantities given

$V = 100$ volts

Substitution of values

$$\lambda = \sqrt{\frac{150}{100}} \text{ Å}$$

$$= \sqrt{1.5 \text{ Å}}$$

$$= 1.2247 \text{ Å}$$

$$= \mathbf{1.2247 \times 10^{-8} \text{ cm}}$$

(ii) To calculate the velocity

Formula used

$$\lambda = \frac{h}{m v}$$

or

$$v = \frac{h}{m \lambda}$$

Quantities given

$m = 9.1 \times 10^{-28}$ g $\qquad h = 6.62 \times 10^{-27}$ erg sec $\qquad \lambda = 1.2247 \times 10^{-8}$ cm

Substitution of values

$$v = \frac{6.62 \times 10^{-27} \text{ erg sec}}{(9.1 \times 10^{-28} \text{ g}) \times (1.2247 \times 10^{-8} \text{ cm})}$$

$$= 0.594 \times 10^{9} \text{ cm sec}^{-1}$$

$$= \mathbf{5.94 \times 10^{8} \text{ cm sec}^{-1}}$$

SOLVED PROBLEM 8. Calculate the wavelength of a particle of mass 1.5 g moving with a velocity of 250 m sec^{-1}.

SOLUTION :

Formula used

$$\lambda = \frac{h}{m\,v}$$

Quantities given

$h = 6.625 \times 10^{-27}$ erg sec $\quad m = 1.5$ g $\quad v = 250$ m sec^{-1} = 25000 cm sec^{-1}

Substitution of values

$$\lambda = \frac{6.625 \times 10^{-27} \text{ erg sec}}{1.5 \text{ g} \times 25000 \text{ cm sec}^{-1}}$$

$$= 0.1766 \times 10^{-27-3} \text{ cm}$$

$$= 0.1766 \times 10^{-30} \text{ cm}$$

$$= \mathbf{1.766 \times 10^{-31} \text{ cm}}$$

SOLVED PROBLEM 9. Calculate the wavelength of an electron of mass 9.1×10^{-28} g moving with a velocity of 4.2×10^{6} cm sec^{-1} ($h = 6.62 \times 10^{-27}$ erg sec).

SOLUTION :

Formula used

$$\lambda = \frac{h}{m\,v}$$

Quantities given

$h = 6.62 \times 10^{-27}$ erg sec $\quad m = 9.1 \times 10^{-28}$ g $\quad v = 4.2 \times 10^{6}$ cm sec^{-1}

Substitution of values

$$\lambda = \frac{6.62 \times 10^{-27} \text{ erg sec}}{(9.1 \times 10^{-28} \text{g}) \times (4.2 \times 10^{6} \text{ cm sec}^{-1})}$$

$$= 0.1732 \times 10^{-27+28-6} \text{ cm}$$

$$= 0.1732 \times 10^{-5} \text{ cm}$$

$$= 1.732 \times 10^{-6} \text{ cm}$$

$$= \mathbf{1.732 \times 10^{-8} \text{ m}}$$

SOLVED PROBLEM 10. Calculate the momentum of a particle which has de Broglie's wavelength of 0.10 nm. ($h = 6.6 \times 10^{-34}$ kg m^{2} sec^{-1}).

SOLUTION :

Formula used

$$\lambda = \frac{h}{m\,v}$$

or momentum, $$m\,v = \frac{h}{\lambda}$$

Quantities given

$h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1} $\qquad \lambda = 0.1 \times 10^{9}$ m

Substitution of values

$$\text{Momentum} = \frac{6.6 \times 10^{-34} \text{ kg m}^2 \text{ sec}^{-1}}{0.1 \times 10^{-9} \text{ m}}$$

$$= \mathbf{6.6 \times 10^{-24} \text{ kg m sec}^{-1}}$$

SOLVED PROBLEM 11. An electron is moving with a kinetic energy of 4.55×10^{-25} J. Calculate de Broglie wavelength for it. (mass of the electron = 9.1×10^{-31} kg; $h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1}).

SOLUTION :

(i) To calculate the velocity of the electron

Formula used

$$K.E. = \tfrac{1}{2} m v^2$$

or

$$v^2 = \frac{2 \times K.E.}{m}$$

Quantities given

$K.E. = 4.55 \times 10^{-25}$ J $\qquad m = 9.1 \times 10^{-31}$ kg

Substitution of values

$$v^2 = \frac{2 \times 4.55 \times 10^{-25} \text{ kg m}^2 \text{ sec}^{-2}}{9.1 \times 10^{-31} \text{ kg}}$$

$$= 1 \times 10^{6} \text{ m}^2 \text{ sec}^{-2}$$

or

$$v = \sqrt{1 \times 10^{6} \text{ m}^2 \text{ sec}^{-2}}$$

$$= 1 \times 10^{3} \text{ m sec}^{-1}$$

(ii) To calculate the wavelength of the electron

Formula used

$$\lambda = \frac{h}{m v}$$

Quantities given

$h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1} $\qquad m = 9.1 \times 10^{-31}$ kg $\qquad v = 1 \times 10^{3}$ m sec^{-1}

Substitution of values

$$\lambda = \frac{6.6 \times 10^{-34} \text{ kg m}^2 \text{ sec}^{-1}}{9.1 \times 10^{-31} \text{ kg} \times 1 \times 10^{3} \text{ m sec}^{-1}}$$

$$= 0.7253 \times 10^{-34+31-3} \text{ m}$$

$$= 0.7253 \times 10^{-6} \text{ m}$$

$$= 7.253 \times 10^{-7} \text{ m}$$

$$= \mathbf{725.3 \text{ nm}}$$

SOLVED PROBLEM 12. Calculate the de Broglie's wavelength of an electron travelling with a speed of 20% of light.

SOLUTION :

Formula used

$$\lambda = \frac{h}{m v}$$

Quantities given

$h = 6.625 \times 10^{-34}$ kg m^2 sec^{-1} $\qquad m = 9.1 \times 10^{-31}$ kg

$$v = 20\% \text{ of } c = \frac{20 \times 3 \times 10^8}{100} \text{ m sec}^{-1} = 0.6 \times 10^8 \text{ m sec}^{-1}$$

Substitution of values

$$\lambda = \frac{6.625 \times 10^{-34} \text{ kg m}^2 \text{ sec}^{-1}}{(9.1 \times 10^{-31} \text{ kg}) \times (0.6 \times 10^8 \text{ m sec}^{-1})}$$
$$= 1.2133 \times 10^{-34+31-8} \text{ m}$$
$$= 1.2133 \times 10^{-11} \text{ m}$$
$$= 12.133 \times 10^{-12} \text{ m}$$
$$= \mathbf{12.133 \text{ pm}} \qquad [\because 1 \text{ pm} = 10^{-12} \text{ m}]$$

SOLVED PROBLEM 13. Compare the relative de Broglie wavelength of an atom of hydrogen and an atom of oxygen both moving with the same velocity.

SOLUTION :

Formula used

$$\lambda = \frac{h}{m v}$$

For hydrogen atom $$\lambda_1 = \frac{h}{m_1 v} \qquad \text{.... (i)}$$

For oxygen atom $$\lambda_2 = \frac{h}{m_2 v} \qquad [\because v \text{ is the same}] \text{ ... (ii)}$$

Dividing (i) by (ii), we have

$$\frac{\lambda_1}{\lambda_2} = \frac{h}{m_1 v} \times \frac{m_2 v}{h}$$
$$= \frac{m_2}{m_1}$$

or $$\frac{\lambda_1}{\lambda_2} = \frac{16 m_1}{m_1} \qquad [\because \text{at. mass of O} = 16 \times \text{at. mass of H}]$$

or $$\lambda_1 : \lambda_2 = 16 : 1$$

SOLVED PROBLEM 14. A good baseball pitcher can throw a 130 g baseball at a speed of 92 km hr^{-1}. What is the de Broglie wavelength in meters produced by the moving baseball ?

SOLUTION :

Formula used

$$\lambda = \frac{h}{m v}$$

Quantities given

$h = 6.625 \times 10^{-34}$ kg m^2 sec^{-1} $\qquad$ $m = 130 \text{ g} = 130 \times 10^{-3}$ kg

$v = 92 \text{ km hr}^{-1} = 92 \times \frac{5}{18}$ m sec^{-1} $\qquad$ $[\because 1 \text{ km hr}^{-1} = \frac{5}{18} \text{ m sec}^{-1}]$

Substitution of values

$$\lambda = \frac{6.625 \times 10^{-34} \text{ kg m}^2 \text{ sec}^{-1}}{(130 \times 10^{-3} \text{ kg}) \times (25.55 \text{ m sec}^{-1})}$$
$$= 0.001994 \times 10^{-34+3} \text{ m}$$
$$= 0.001994 \times 10^{-31} \text{ m}$$
$$= \mathbf{1.994 \times 10^{-34} \text{ m}}$$

SOLVED PROBLEM 15. Calculate the product of uncertainty of displacement and velocity of a moving electron having a mass of 9.1×10^{-31} kg ($h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1}).

SOLUTION :

Formula used

$$\Delta x \times m\,\Delta v = \frac{h}{4\,\pi}$$

or
$$\Delta x \times \Delta v = \frac{h}{4\,\pi\,m}$$

Quantities given

$h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1} $\qquad m = 9.1 \times 10^{-31}$ kg

Substitution of values

$$\Delta x \times \Delta v = \frac{6.6 \times 10^{-34} \text{ kg m}^2 \text{ sec}^{-1}}{4 \times 9.1 \times 10^{-31} \text{ kg} \times 3.14}$$

$$= 0.05774 \times 10^{-34+31} \text{ m}^2 \text{ sec}^{-1}$$

$$= \mathbf{5.774 \times 10^{-5} \text{ m}^2 \text{ sec}^{-1}}$$

SOLVED PROBLEM 16. Calculate the certainty in position of an electron if uncertainty in velocity is 5.7×10^5 m sec^{-1}. ($h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1} ; mass of electron $= 9.1 \times 10^{-31}$ kg).

SOLUTION :

Formula used

$$\Delta x \times m\,\Delta v = \frac{h}{4\,\pi}$$

or
$$\Delta x = \frac{h}{4\,\pi\,m\,\Delta\,v}$$

Quantities given

$h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1} $\qquad m = 9.1 \times 10^{-31}$ kg $\qquad \Delta v = 5.7 \times 10^5$ m sec^{-1}

Substitution of values

$$\Delta x = \frac{6.6 \times 10^{-34} \text{ kg m}^2 \text{ sec}^{-1}}{4 \times 3.1 \times 9.1 \times 10^{-31} \text{ kg} \times 5.7 \times 10^5 \text{ m sec}^{-1}}$$

$$= 0.0103 \times 10^{-34+31-5} \text{ m}$$

$$= 0.0103 \times 10^{-8} \text{ m}$$

$$= \mathbf{1.03 \times 10^{-10} \text{ m}}$$

SOLVED PROBLEM 17. What are the possible l and m quantum numbers for an electron $n = 3$ quantum level of an atom ?

SOLUTION :

Here $n = 3$

$\therefore$ $\quad l = 0, 1$ and 2 $\qquad [\because\ l = 0, 1 \ldots\ldots n-1]$

and when $\quad l = 0 \qquad m = 0$

$l = 1 \qquad m = -1, 0, +1$

$l = 2 \qquad m = -2, -1, 0, +1, +2$

SOLVED PROBLEM 18. The electronic configuration of an atom in the ground state is $1s^2\,2s^2\,2p^6$. Give the four quantum numbers of the electron in last orbital.

SOLUTION :

The given electron configuration is $1s^2\ 2s^2\ 2p^6$

or $1s^2\ 2s^2\ 2p_x^{\ 2}\ 2p_y^{\ 2}\ 2p_z^{\ 2}$

$\therefore$ The last electron is in $2p_z$ orbital

For $2p_z$ orbital $n = 2$, $l = 1$, $m = 1$ and

$s = +½$ or $-½$

SOLVED PROBLEM 19. Arrange the electrons in the following list in order of increasing energy.

	n	l	m	s
(a)	3	2	–1	+½
(b)	4	0	0	–½
(c)	4	1	1	+½
(d)	2	1	–1	–½

SOLUTION :

$(n + l)$ values in respect of above electrons are

(a) $3 + 2 = 5$

(b) $4 + 0 = 4$

(c) $4 + 1 = 5$

(d) $2 + 1 = 3$

$\therefore$ **Increasing order of energy is (d) < (b) < (a) < (c)**

SOLVED PROBLEM 20. Deduce the possible set of four quantum number when $n = 3$

SOLUTION :

For $n = 3$ $l = 0, 1$ and 2

When	$l = 0$	$m = 0$	$s = +½$ or $-½$
	$l = 1$	$m = -1$	$s = +½$ or $-½$
		$m = 0$	$s = +½$ or $-½$
		$m = +1$	$s = +½$ or $-½$
	$l = 2$	$m = -2$	$s = +½$ or $-½$
		$m = -1$	$s = +½$ or $-½$
		$m = 0$	$s = +½$ or $-½$
		$m = +1$	$s = +½$ or $-½$
		$m = +2$	$s = +½$ or $-½$

SOLVED PROBLEM 21. Arrange the electrons represented by the following sets of quantum numbers in the decreasing order of energy.

(a)	$n = 4$	$l = 0$	$m = 0$	$s = +½$
(b)	$n = 3$	$l = 1$	$m = 0$	$s = –½$
(c)	$n = 3$	$l = 2$	$m = 0$	$s = +½$
(d)	$n = 3$	$l = 0$	$m = 0$	$s = –½$

SOLUTION :

The $(n + l)$ values of above electrons are as follows :

(a)	$n=4$	$l=0$	$m=0$	$s=+½$	$n+l=4$	(4s)
(b)	$n=3$	$l=1$	$m=0$	$s=–½$	$n+l=4$	(3p)
(c)	$n=3$	$l=2$	$m=0$	$s=+½$	$n+l=5$	(3d)
(d)	$n=3$	$l=0$	$m=0$	$s=–½$	$n+l=3$	(3s)

Thus the decreasing order of energy is 3*d* > 4*s* > 3*p* > 3*s*

SOLVED PROBLEM 22. Calculate the wavelength of a 200 kg drum rolling at a speed of 30 km per hour.

SOLUTION :

Formula used

$$\lambda = \frac{h}{m\,v}$$

Quantities given

$h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1} $\quad m = 200$ kg $\quad v = 30$ km hr^{-1} $= \frac{30\times10^3}{60\times60}$ m sec^{-1}

$= 8.33$ m sec^{-1}

Substitution of values

$$\lambda = \frac{6.6\times10^{-34}\ \text{kg m}^2\ \text{sec}^{-1}}{200\ \text{kg}\times 8.33\ \text{m sec}^{-1}}$$

$$= \mathbf{3.96\times10^{-37}\ m}$$

SOLVED PROBLEM 23. Calculate the momentum of a particle which has a de Broglie wavelength of 110Å.

SOLUTION :

Formula used

$$\lambda = \frac{h}{m\,v}$$

or

$$m\,v = \frac{h}{\lambda}$$

Quantities given

$h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1} $\lambda = 110$ Å $= 110 \times 10^{-10}$ m

Substitution of values

$$m\,v = \frac{6.6\times10^{-34}\ \text{kg m}^2\ \text{sec}^{-1}}{110\times10^{-10}\ \text{m}}$$

$$= \mathbf{6\times 10^{-26}\ kg\ m\ sec^{-1}}$$

SOLVED PROBLEM 24. By applying de Broglie's equation, calculate wavelength associated with the motion of the earth, a ball and an electron. Following data is given :

Mass of earth = 6.0×10^{24} kg.

Mass of ball = 0.1 kg

Mass of electron = 9.1×10^{-31} kg

Velocity of orbital motion of the earth = 3×10^{4} m sec^{-1}

Velocity of the ball = 100 m sec^{-1}

Velocity of the electron = 1.0×10^{6} m sec^{-1}

$h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1}

In case of which of these three objects, will the wavelength be measurable ?

SOLUTION :

Formula used

$$\lambda = \frac{h}{m\,v}$$

(i) For earth

Quantities given

$h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1} $m = 6.0 \times 10^{24}$ kg $v = 3 \times 10^{4}$ m sec^{-1}

Substitution of values

$$\lambda = \frac{6.6 \times 10^{-34}\text{ kg m}^2\text{ sec}^{-1}}{6.0 \times 10^{24}\text{ kg} \times 3 \times 10^{4}\text{ m sec}^{-1}}$$

$$= \mathbf{3.66 \times 10^{-63}\ m}$$

(ii) For ball

Quantities given

$h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1} $m = 0.1$ kg $v = 100$ m sec^{-1}

Substitution of values

$$\lambda = \frac{6.6 \times 10^{-34}\text{ kg m}^2\text{ sec}^{-1}}{0.1\text{ kg} \times 100\text{ m sec}^{-1}}$$

$$= \mathbf{6.6 \times 10^{-35}\ m}$$

(ii) For electron

Quantities given

$h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1} $m = 9.1 \times 10^{-31}$ kg $v = 1.0 \times 10^{6}$ m sec^{-1}

Substitution of values

$$\lambda = \frac{6.6 \times 10^{-34}\text{ kg m}^2\text{ sec}^{-1}}{(9.1 \times 10^{-31}\text{ kg}) \times (1.0 \times 10^{6}\text{ m sec}^{-1})}$$

$$= \mathbf{7.25 \times 10^{-10}\ m}$$

Thus the wavelength is measurable in case of electron only.

ADDITIONAL PRACTICE PROBLEMS

1. Calculate the product of uncertainty of displacement and velocity of a moving electron. ($m = 9.1 \times 10^{-31}$ kg).
Answer. 5.80×10^{-1} m^2 sec^{-1}

2. Calculate the momentum of a particle which has a de Broglie's wavelength of 0.1 nm.
Answer. 6.6×10^{-24} kg m^2 sec^{-1}

3. The kinetic energy of an electron is 4.55×10^{-25} J. Calculate its wavelength ($h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1} ; mass of electron = 9.1×10^{-31} kg).
Answer. 7.25×10^{-1} m

4. The kinetic energy of a subatomic particle is 5.60×10^{-25} J. Calculate the frequency of the particle wave (Planck's Constant $h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1}).
Answer. 1.696×10^{9} sec^{-1}

5. Calculate the wavelength associated with an electron moving with a velocity of 1×10^{8} cm sec^{-1}. (mass of the electron = 9.1×10^{-28} g).
Answer. 7.28×10^{-8} cm

6. Calculate the de Broglie wavelength of an electron moving with a velocity of 6×10^{5} m sec^{-1}.
Answer. 1.456×10^{-8} m

7. Calculate the uncertainty in position of an electron if uncertainty in velocity is 5.7×10^{5} m sec^{-1}. ($h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1} ; mass of electron = 9×10^{-31} kg)
Answer. 10^{-10} m

8. A body moving with a speed of 100 m sec^{-1} has a wavelength of 5×10^{-36} m. Calculate the mass of the body. ($h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1}).
Answer. 1.32 kg

9. Calculate the uncertainty in momentum of an electron if uncertainty in its position is approximately 100 pm. (5.0×10^{-12} m).
Answer. 5.27×10^{-25} kg m sec^{-1}

10. Calculate the uncertainty in the velocity of a bullet of mass 10 g whose position at time t is known with uncertainty equal to 1.0×10^{-5} m.
Answer. 5.628×10^{-28} m sec^{-1}

11. Calculate the uncertainty in the velocity of an electron if the uncertainty in position is 1×10^{-10} m.
Answer. 5.76×10^{5} m sec^{-1}

12. Calculate the uncertainty in the position of a particle when the uncertainty in the momentum is
(a) 1×10^{-7} kg m sec^{-1} and
(b) Zero
Answer. (a) 5.72×10^{-28} m ; (b) $\propto$

13. Calculate the uncertainty in the velocity of a bullet weighing 10 g whose position is known with an accuracy of $\pm$ 0.1 nm.
Answer. 0.527×10^{-27} m sec^{-1}

14. Calculate the uncertainty in the velocity of a wagon of mass 2000 kg whose position is known with an accuracy of $\pm$ 10 m.
Answer. 5.25×10^{-31} m sec^{-1}

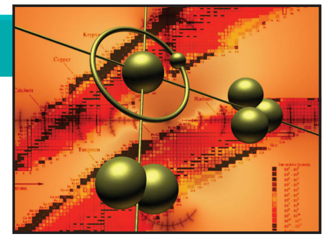
15. Calculate the product of uncertainty of displacement and velocity of a moving electron having a mass of 9.1×10^{-31} kg ($h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1}).
Answer. 5.774×10^{-5} m^2 sec^{-1}

16. A body moving with a speed of 100 m sec^{-1} has a wavelength of 5×10^{-36} m. Calculate the mass of the body ($h = 6.6 \times 10^{-34}$ kg m^2 sec^{-1}).
Answer. 1.32 kg

3

Isotopes, Isobars and Isotones

CHAPTER



KEY CONCEPTS AND EQUATIONS

ISOTOPES

The atoms of an element which have the same atomic number but different atomic masses or mass numbers are called Isotopes. They have the same number of protons (or electrons) and different number of neutrons. They have same position in the periodic table. Isotopes have similar chemical properties as they have the same electronic configuration. However, they differ in respect of physical properties which depend on their atomic masses. Examples of isotopes are

${}_{1}H^{1}$	${}_{1}H^{2}$	${}_{1}H^{3}$
Hydrogen	Deuterium	Tritium
${}_{6}C^{12}$	${}_{6}C^{13}$	${}_{6}C^{14}$
${}_{8}O^{16}$	${}_{8}O^{17}$	${}_{8}O^{18}$
${}_{92}U^{234}$	${}_{92}U^{235}$	${}_{92}U^{238}$

DETERMINING ATOMIC MASSES FROM ISOTOPIC MASSES AND FRACTIONAL ABUNDANCES

Multiply each of the isotopic masses by its fractional abundance and then add. For example to calculate the atomic mass of neon which has following isotopes in percentages as :

${}_{10}Ne^{20}$	90.92
${}_{10}Ne^{21}$	0.26
${}_{10}Ne^{22}$	8.82

Thus we have

$$\begin{aligned} 20 \times 0.9092 &= 18.18 \\ 21 \times 0.0026 &= 0.055 \\ 22 \times 0.0882 &= \underline{\ \ 1.94} \\ &\ \ \ \ \underline{20.175} \end{aligned}$$

$\therefore$ The atomic mass of Neon is 20.175

RATE OF DIFFUSION

The rate of diffusion is inversely proportional to the square root of its atomic mass (Graham's Law) *i.e.*

$$\text{Rate of Diffusion} \propto \sqrt{\frac{1}{\text{Molecular mass}}}$$

Thus when a mixture of two isotopes is allowed to diffuse through a porous partition, **the lighter isotope passes through more rapidly than the heavier one.**

DETERMINING THE NUMBER OF ELECTRONS, PROTONS AND NEUTRONS

The isotopes of an element are characterised by different number of neutrons in the nucleus. The number of neutrons in the nucleus is equal to the difference between mass number (A) and the atomic number (Z) *i.e.*

$$\begin{aligned}\text{Number of neutrons} &= \text{Mass number} - \text{Atomic number}\\ &= A - Z\end{aligned}$$

WHAT ARE ISOBARS ?

The atoms which have the same mass number but different atomic numbers are called Isobars. They have different number of protons (or electrons) and neutrons but **the sum of protons and neutrons is the same.** Examples are :

	$_{18}Ar^{40}$	$_{19}K^{40}$	$_{20}Ca^{40}$
No. of Protons	18	19	20
No. of Neutrons	22	21	20
Sum of Protons and Neutrons	40	40	40

WHAT ARE ISOTONES ?

Atoms which have different atomic numbers and diffrent atomic masses but **the same number of neutrons** are called Isotones. Examples are :

	$_{6}C^{14}$	$_{7}N^{15}$	$_{8}O^{16}$
No. of Protons	6	7	8
No. of Neutrons	8	8	8
Sum of Protons and Neutrons	14	15	16

Isotones have different physical and chemical properties.

COMPARISON OF ISOTOPES, ISOBARS AND ISOTONES

The comparison of Isotopes, Isobars and lsotones is given in the Table 3.1.

TABLE 3.1. COMPARISON OF ISOTOPES, ISOBARS AND ISOTONES

Name	Mass number (A)	Atomic number (Z)	No. of neutrons (A– Z)	Examples
Isotopes	Different	Same	Different	$_{6}C^{12}$, $_{6}C^{13}$, $_{6}C^{14}$
Isobars	Same	Different	Different	$_{18}Ar^{40}$, $_{19}K^{40}$, $_{20}Ca^{40}$
Isotones	Different	Different	Same	$_{14}Si^{30}$, $_{15}P^{31}$, $_{16}S^{32}$

ADDITIONAL SOLVED PROBLEMS

SOLVED PROBLEM 1. Calculate the relative atomic mass of an element which consists of the following isotopes with the indicated relative abundance.

Isotope	Isotopic mass	Natural abundance
1	28	92.0
2	29	5.0
3	30	3.0

SOLUTION :

The atomic mass of an ordinary isotopic mixture is the average of the determined atomic masses of individual isotopes. Thus

$$28 \times 0.92 = 25.76$$
$$29 \times 0.05 = 1.45$$
$$30 \times 0.03 = 0.90$$
$$\text{Total} \quad 28.11$$

Thus the atomic mass is 28.11 amu

SOLVED PROBLEM 2. Chromium, Cr, has following isotopic masses and fractional abundances :

Mass Number	Mass (amu)	Fractional abundance
50	49.9461	0.0435
52	51.9405	0.8379
53	52.9407	0.0950
54	53.9389	0.0236

SOLUTION :

Multiplying each isotope by its fractional abundance, we have

$$49.9461 \text{ amu} \times 0.0435 = 2.17 \text{ amu}$$
$$51.9405 \text{ amu} \times 0.8379 = 43.52 \text{ amu}$$
$$52.9407 \text{ amu} \times 0.0950 = 5.03 \text{ amu}$$
$$53.9389 \text{ amu} \times 0.0236 = 1.27 \text{ amu}$$
$$\text{Total} \quad 51.99 \text{ amu}$$

Thus the atomic mass of chromium is 51.99 amu

SOLVED PROBLEM 3. Calculate the atomic mass of boron, B, from the following data :

Isotope	Atomic mass (amu)	Fractional abundance
${}_5B^{10}$	10.013	0.1978
${}_5B^{11}$	11.009	0.8022

SOLUTION :

Multiplying each isotope by its fractional abundance, we have

$$10.013 \text{ amu} \times 0.1978 = 1.9806 \text{ amu}$$
$$11.009 \text{ amu} \times 0.8022 = 8.8314 \text{ amu}$$
$$\text{Total} \quad 10.8120 \text{ amu}$$

Thus the atomic mass of Boron is 10.8120 amu

SOLVED PROBLEM 3. Naturally occuring boron consists of two isotopes whose atomic masses are 10.01 and 11.01. The atomic mass of naturally occuring boron is 10.81. Calculate the percentage of each isotope in natural boron.

SOLUTION :

Let the percentage of isotope with atomic mass 10.01 be A.

$\therefore$ the percentage of isotope with atomic mass 11.01 = 100 – A

$$\text{Now the atomic mass} = \frac{A \times 10.01 + (100 - A) \times 11.01}{100}$$

$$10.81 = \frac{10.01\,A + 1101 - 11.01A}{100}$$

or $$11.01\,A - 10.01\,A = 1101 - 1081$$

or $$A = 20$$

Hence the percentage of isotope with atomic mass 10.01 = 20

and the percentage of isotope with atomic mass 11.01 = 80

ADDITIONAL PRACTICE PROBLEMS

1. Magnesium has naturally occuring isotopes with the following masses and abundances.

Isotope	Atomic mass (amu)	Fractional abundance
$_{12}MG^{24}$	23.985	0.7870
$_{12}MG^{25}$	24.986	0.1013
$_{12}MG^{26}$	25.983	0.1117

Calculate the atomic mass of magnesium.

Answer. 24.31 amu

2. Silver has two naturally occuring isotopes with atomic masses 106.91 and 108.90 amu. The atomic mass of silver is 107.87 amu. Calculate the fractional abundances for these two isotopes.

Answer. 0.518 ; 0.482

3. Calculate the fractional abundances for the two naturally occuring isotopes of copper. The masses of the isotopes are 62.9298 and 64.9278 amu. The atomic mass of copper is 63.546 amu.

Answer. 0.692 ; 0.308]