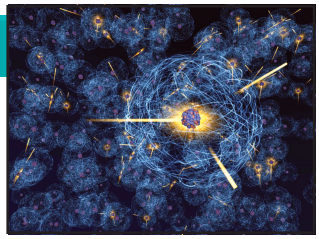
4 Nuclear Chemistry

CHAPTER



KEY CONCEPTS AND EQUATIONS

RADIOACTIVITY AND TYPES OF RADIATIONS

The disintegration or decay of unstable atoms accompanied by emission of radiation is called **Radioactivity**. The substances having this property are called **Radioactive substances**. The radioactive emissions are of three types - α, β, or γ radiations as shown in Fig. 4.1.

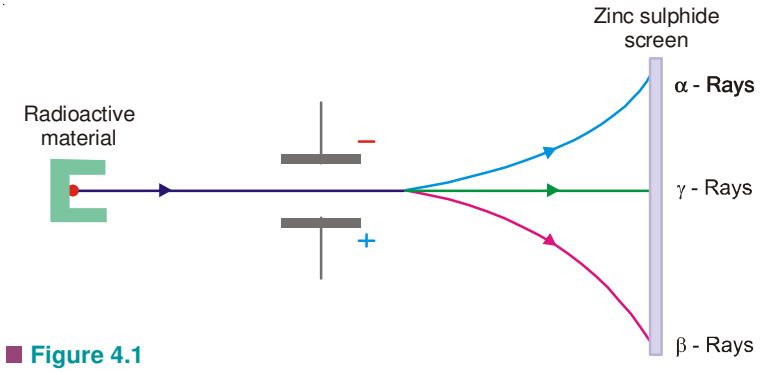


Figure 4.1
Detection of α, β and γ-Rays.

The charge and mass of α, β and γ – particles are given in Table 4.1

TABLE 4.1 CHARGE AND MASS OF α, β AND γ–EMISSIONS

Particle	Charge	Mass	Representation
α	2	4	$_2He^4$ or $_2\alpha^4$
β	–1	0	$_{-1}e^0$ or $_{-1}\beta^0$
γ	0	0	$_0\gamma^0$

RATE OF RADIOACTIVE DECAY (No. of disintegrations per unit time)

The rate of decay is characteristic of an isotope and depends only on the number of atoms present. It is given by

$$-\frac{dN}{dt} = \lambda N$$

where λ is disintegration constant and N is number of atoms present.

On integration we have

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N} \quad \ldots.(i)$$

where N^0 is the number of atoms at $t = 0$

Radioactive disintegration follows the first order kinetics and the equation

$$\lambda = \frac{2.303}{t} \log \frac{a}{a-x} \quad \ldots.(ii)$$

is also applicable.

HALF LIFE PERIOD

The half life period of a radioactive isotope is the time required for one half of the isotope to decay. It is represented by $t_{½}$ or $t_{0.5}$

$$\text{When} \quad t = t_{½} \qquad N = \frac{N^0}{2}$$

Putting in equation (i) we have

$$t_{½} = \frac{2.303}{\lambda} \log 2 = \frac{0.693}{\lambda}$$

It is independent of initial concentration of radioactive substance.

AVERAGE LIFE

The average life of a radioactive substance is the reciprocal of disintegration constant *i.e.*,

$$\lambda' = \frac{1}{\lambda} = \frac{t_{½}}{0.693} = 1.44 \times t_{½}$$

UNITS OF RADIOACTIVITY

The unit of radioactivity is disintegration per second (dps). It is also expressed in curie or rutherford or becquerel. These are related to dps as follows :

$$1 \text{ curie} = 3.7 \times 10^{10} \text{ dps}$$
$$1 \text{ rutherford} = 1 \times 10^{6} \text{ dps}$$
$$1 \text{ becquerel} = 1 \text{ dps}$$

RADIOACTIVE EQUILIBRIUM

Let a radioactive substance A decay to give another radioactive substance B which decays to form the substance C. If λ_A and λ_B are their decay constants then we have

$$A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C$$

$$\frac{N_A}{N_B} = \frac{\lambda_B}{\lambda_A} = \frac{\left(t_{½}\right)_B}{\left(t_{½}\right)_A}$$

GROUP DISPLACEMENT LAW

In an α emission the parent element is displaced to a group **two** places to left and in β emission it will be displaced to a group **one** place to the right in the periodic table as illustrated in Fig.4.2. This is called Group Displacement Law. It was first stated by Fajans and Soddy (1913) and is often named as Fajans-Soddy Group Displacement law.

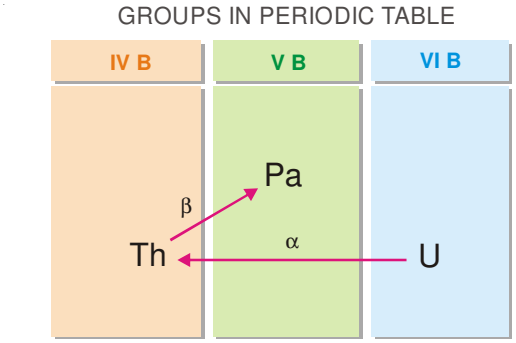


Figure 4.2
Illustration of Group Displacement Law.

NUCLEAR REACTIONS

A nuclear reaction is one which proceeds with a change in the composition of the nucleus so as to produce an atom of a new element. The conversion of one element to another by a nuclear change is called transmutation.

NUCLEAR FISSION REACTIONS

In these reactions an atomic nucleus is broken or fissioned into two or more fragments. For example, $_{7}N^{14}$ when struck by an α - particle first forms an intermediate unstable compound nucleus, $_{9}F^{18}$, which at once cleaves to form stable $_{8}O^{17}$ and $_{1}H^{1}$.

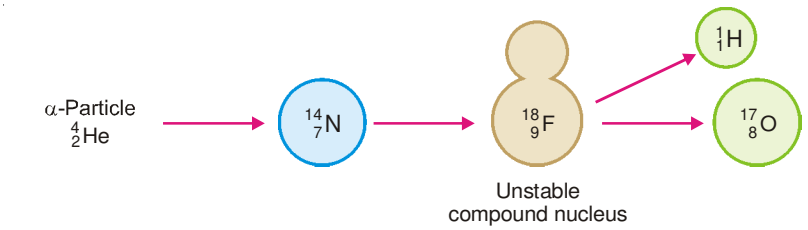


Figure 4.3
Mechanism of a Nuclear fission reaction.

NUCLEAR FUSION REACTIONS

These reactions take place by combination or fusion of two small nuclei into a larger nucleus. For example two hydrogen nuclei, $_{1}H^{1}$, fuse to produce a deuterium nucleus, H^{2} as shown in Fig. 4.4.

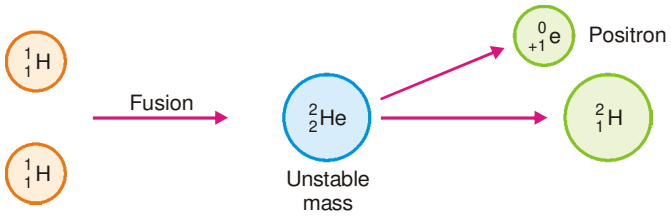


Figure 4.4
Mechanism of a Nuclear fission reaction.

MASS-ENERGY EQUIVALENCE

Mass and energy are equivalent and are related to each other by the equation.

$$E = m c^2$$

where c is the speed of light, m sec^{-1}.

This equation is known as Einstein equation.

MASS DEFECT AND BINDING ENERGY

The difference between the experimental and calculated mass of the nucleus is called the mass defect.

Mass defect = Experimental mass of the nucleus – (mass of protons + electrons + neutrons).

It is equal to the total nucleon mass minus the nuclear mass. This loss of mass is converted into energy which stabilizes the nucleus. This energy is known as binding energy. It is also equal to the energy needed to break a nucleus into its individual protons and neutrons. Einstein equation is used to calculate the binding energy. With mass defect of 1 amu 931.5 MeV of energy is produced *i.e.*

$$1 \text{ amu} = 931.5 \text{ MeV}$$

To compare the stabilities of various nuclei, the binding energy per nucleon is compared.

BALANCING OF NUCLEAR EQUATIONS

The equations involving the nuclei of the reactants and products are called nuclear equations. The nuclear reactions occur by redistribution of protons and neutrons present in the reactants so as to form the products. Thus the total number of protons and neutrons in the reactants and products is the same. For example in the nuclear equation

$$_{92}U^{238} + {}_{2}He^{4} \longrightarrow ? + {}_{0}n^{1}$$

the atomic number of missing atom is 94 and its mass no. is 238 + 4 – 1 = 241. Hence the atom is $_{94}Pu^{241}$. The balanced equation becomes.

$$_{92}U^{238} + {}_{2}He^{4} \longrightarrow {}_{94}Pu^{241} + {}_{0}n^{1}$$

ADDITIONAL SOLVED PROBLEMS

SOLVED PROBLEM 1. It is found that a sample of radioactive matter is half integrated in 18 hours. How much of it will remain after 42.5 hours ?

SOLUTION :

(i) To calculate disintegration constant

Formula used $\lambda = \dfrac{0.693}{t_{½}}$

Quantities given $t_{½} = 18\,\text{hr}$

Substitution of values $\lambda = \dfrac{0.693}{18\ \text{hr}} = 0.0385\ \text{hr}^{-1}$

(ii) To calculate the amount of sample left after 42.5 hours

Formula used

$$\lambda = \frac{1}{t} \log \frac{a}{a-x}$$

Quantities given

$\lambda = 0.0385\ \text{hr}^{-1}$ $t = 42.5$ hours $a = 100$ $a - x = 100 - x$

Substitution of values

$$0.0385\ \text{hr}^{-1} = \frac{1}{42.5\ \text{hr}} \log \frac{100}{100-x}$$

$$\text{or} \quad 1.63625 = \log \frac{100}{100-x}$$

$$\text{or} \quad \frac{100}{100-x} = \text{Antilog } 1.63625$$

$$= 5.13$$

$$\text{or} \quad 100 = 5.13 \times (100 - x)$$

$$100 = 513 - 5.13x$$

$$\text{or} \quad x = \frac{413}{5.13}$$

$$= 80.51$$

$$\therefore \quad \text{Amount left} = 100 - 80.51$$

$$= \mathbf{19.49\%}$$

SOLVED PROBLEM 2. Radium has atomic mass 226 and a half life of 1600 years. Calculate the number of disintegration produced per second from one gram of radium.

SOLUTION :

(i) To calculate the disintegration constant

Formula used $\lambda = \dfrac{0.693}{t_{½}}$

Quantities given

$t_{½} = 1600$ years $= 1600 \times 365 \times 24 \times 60 \times 60$ sec

$=$

Substitution of values

$\therefore \quad \lambda = \dfrac{0.693}{5.04576 \times 10^{10} \text{ sec}}$

$= 0.1373 \times 10^{-10} \text{ sec}^{-1}$

(ii) To calculate the rate of disintegration

Formula used $-\dfrac{dN}{dt} = \lambda \times N$

Quantities given $\lambda =$

No. of nuclei of Ra^{226} in 1g, $N = \dfrac{\text{Avogadro's No.}}{226}$

$= \dfrac{6.023 \times 10^{23}}{226}$

$= 0.0266 \times 10^{-23}$ atoms

Substitution of values

Rate of disintegration, $-\dfrac{dN}{dt} = 0.1373 \times 10^{-10} \text{ sec}^{-1} \times 0.0266 \times 10^{-23}$ atoms

$= 0.003652 \times 10^{-13}$ atom sec^{-1}

$= \mathbf{3.652 \times 10^{16} \text{ atom sec}^{-1}}$

SOLVED PROBLEM 3. The activity of a radioactive isotope reduces by 25% after 100 minutes. Calculate the decay constant and half life period.

SOLUTION :

(i) To calculate decay constant

Formula used $\lambda =$

Quantities given

$t = 100$ min $\qquad a = 100 \qquad a - x = 25$

Substitution of values

$\lambda = \dfrac{2.303}{100 \text{ min}} \log \dfrac{100}{25}$

$= 0.02303 \text{ min}^{-1} \times \log 4$

$= 0.02303 \text{ min}^{-1} \times 0.6021$

$= \mathbf{0.01387 \text{ min}^{-1}}$

(ii) To calculate Half Life period

Formula used $= \dfrac{0.693}{\lambda}$

$\therefore \quad t_{½} = \dfrac{0.693}{0.01386\ \text{min}^{-1}}$

$= \mathbf{49.96\ min}$

SOLVED PROBLEM 4. Calculate the number of α and β particles emitted in the conversion of thorium $_{90}Th^{232}$ to $_{82}Pb^{208}$.

SOLUTION :

Let x and y be the number of λ and β particles emitted during the change

$$_{90}Th^{232} \longrightarrow {}_{82}Pb^{208} + x\ {}_{2}He^{4} + y\ {}_{-1}e^{0}$$

Comparing the mass numbers, we have

$232 = 208 + 4x + y \times 0$

or $4x = 24$

or $x = \mathbf{6}$

Now comparing the atomic numbers, we have

$90 = 82 + 2x - 1y$

or $90 = \qquad\qquad (\quad x = 6)$

or $y = 94 - 90$

$= \mathbf{4}$

$82 + 2 \times 6 - y$

$\dfrac{0.693}{t_{½}}$

SOLVED PROBLEM 5. Half life period of thorium is 24.5 minutes. How much thorium would be left after 30 minutes if the initial amount of thorium is one gram ?

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$\lambda =$

Quantities given

$t_{½} = 24.5$ min

Substitution of values

$\lambda = \dfrac{0.693}{24.5\ \text{min}}$

$= \mathbf{0.02828\ min^{-1}}$

(ii) To calculate the amount of Thorium left

Formula used

$$\lambda = \frac{1}{t} \log \frac{a}{a-x}$$

Quantities given

$\lambda = 0.02828\ \text{min}^{-1}$ $\quad t = 30$ minutes $\quad a = 1$ g $\quad a - x = 1 - x$ g

Substitution of values

$$0.02828\ \text{min}^{-1} = \frac{1}{30\ \text{min}} \log \frac{1}{1-x}$$

or $$\log \frac{1}{1-x} = 30 \times 0.02828$$

$$= 0.8484$$

or $$= \text{Antilog } 0.8484$$

$$= 2.3359$$

or $$1 = (1-x)\ 2.3359$$

$$1 = 2.3359 - 2.3359\,x$$

or $$x = \frac{2.3359-1}{2.3359}$$

$$= 0.572\ \text{g}$$

$\therefore$ Amount left $= 1-x$ $= 1 - 0.572\ \text{g}$

$$= \mathbf{0.428\ g}$$

SOLVED PROBLEM 6. How many α and β particles will be emitted by an element ${}_{84}A^{218}$ is changing to a stable isotope of ${}_{82}B^{206}$?

SOLUTION :

Let x and y be the number of α and β particles respectively emitted during the change

$${}_{84}A^{218} \longrightarrow {}_{82}B^{206} + x\ {}_{2}He^{4} + y\ {}_{-1}e^{0}$$

Comparing the mass number

$$218 = 206 + 4x + 0y$$

or $$4x = 12$$

or $$\boldsymbol{x} = \mathbf{3}$$

Now comparing the atomic numbers

$$84 = 82 + 2x - y$$

$$84 = 82 + 2 \times 3 - y \qquad [\ x = 3]$$

or $$\boldsymbol{y} = \mathbf{4}$$

SOLVED PROBLEM 7. Calculate the decay constant for Ag^{108}, if its half life is 2.31 minutes.

SOLUTION :

Formula used

$$\lambda =$$

Quantities given

$$t_{1/2} = 2.31\ \text{minutes}$$

Substitution of values

$$\lambda = \frac{0.693}{2.31\ \text{min}}$$

$$= \mathbf{0.3\ min^{-1}}$$

SOLVED PROBLEM 8. A radioactive isotope has half life period of 20 days. What is the amount of the isotope left over after 40 days if the initial concentration is 5 g ?

SOLUTION :

(i) To calculate the decay constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantities given

$$t_{1/2} = 20 \text{ days}$$

Substitution of values

$$\lambda = \frac{0.693}{20 \text{ days}} = \mathbf{0.03465\ day^{-1}}$$

(ii) To calculate amount of radioactive isotope

Formula used

$$\lambda = \frac{1}{t} \log \frac{a}{a-x}$$

Quantities given

$t = 40$ days $\quad a = 5$ g $\quad a - x = 5 - x$ g

Substitution of values

$$0.03465\ \text{day}^{-1} = \frac{1}{40 \text{ days}} \log \frac{5}{5-x}$$

or $$\log \frac{5}{5-x} = 1.386$$

$$\frac{5}{5-x} = \text{Antilog } 1.386$$

$$= 3.9988$$

$$5 = (5-x) \times 3.9988$$

or $$x = 3.75 \text{ g}$$

$\therefore$ Amount left = Initial concentration – Amount used

$$= 5 - 3.75 \text{ g}$$

$$= \mathbf{1.25\ g}$$

$$\text{Ra} \xrightarrow{\alpha} \text{Rn} \xrightarrow{\alpha} \text{Po} \text{ or } \xrightarrow{\alpha} \text{Pb} \xrightarrow{\beta} \text{Bi}$$

SOLVED PROBLEM 9. Calculate the mass number, atomic number and group in the periodic table for Bi in the following disintegration series.

(Ra : Mass No. 226, at. no. 88, group no. II A)

SOLUTION :

We can write

$$_{88}Ra^{226} - 3\ _{2}He^{4} - {}_{-1}e^{0} = {}_{Z}Bi^{A}$$

Comparing the mass numbers

$$226 - 3 \times 4 - 0 = A$$

or $$A = \mathbf{214}$$

Comparing the atomic numbers

$$88 - 3 \times 2 - 1 \times (-1) = Z$$

or $$Z = \mathbf{83}$$

Group = **VA**

SOLVED PROBLEM 10. The half life of Th^{233} is 1.4 10^{10} years. Calculate the disintegration constant.

SOLUTION :

Formula used

$$\lambda =$$

Quantities given

$$t_{1/2} = 1.4 \times 10^{10} \text{ years}$$

Substitution of values

$$\lambda =$$

$$= 0.495 \times 10^{10} \text{ year}^{-1}$$

$$= \mathbf{4.95 \times 10^{-11} \text{ year}^{-1}}$$

SOLVED PROBLEM 11. A radioactive substance having half life of 3.8 days, emitted initially alpha particles per second. In what time will its rate of emission reduces to alpha particles per second ?

SOLUTION :

(i) To calculate disintegration constant

Formula used

$$\lambda =$$

Quantities given

$$t_{1/2} = 3.8 \text{ days}$$

Substitution of values

$$\lambda = \frac{0.693}{3.8 \text{ days}}$$

$$= \mathbf{0.1824 \text{ day}^{-1}}$$

(ii) To calculate the time t

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N}$$

or

$$t = \frac{2.303}{\lambda} \log \frac{N^0}{N}$$

Quantities given

$\lambda = 0.1824 \text{ days}^{-1}$ $\quad N^0 = 7 \times 10^4 \text{ dps}$ $\quad N = 2 \times 10^4 \text{ dps}$

Substitution of values

$$t = \frac{2.303}{0.1824 \text{ day}^{-1}} \log \frac{7 \times 10^4 \text{ dps}}{2 \times 10^4 \text{ dps}}$$

$$= \frac{2.303}{0.1824 \text{ day}^{-1}} \log 3.5$$

$$= \frac{2.303}{0.1824 \text{ day}^{-1}} \times 0.5440$$

$$= \mathbf{6.872 \text{ days}}$$

SOLVED PROBLEM 12. A freshly cut piece of plant gives 20.4 counts per minute per gram. A piece of wood antique gives 12.18 counts per minute per gram. What is the age in years of antique ? It is assumed that the radioactivity is entirely due to C^{14}. The half life period of C^{14} is 5760 years.

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantities given

$$t_{1/2} = 5760 \text{ years}$$

Substitution of values

$$\lambda = \frac{0.693}{5790 \text{ years}}$$

$$= \mathbf{1.203 \times 10^{-4} \ year^{-1}}$$

(ii) To calculate the age of antique

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{N^0}{N}$$

or

$$t = \frac{2.303}{\lambda} \log \frac{N^0}{N}$$

Quantities given

$\lambda = 2.303 \times 10^{-4}$ year^{-1} $N^0 = 20.4$ counts per minutes per g

$N = 12.18$ counts per minutes per g

Substitution of values

$$t = \frac{2.303}{2.303 \times 10^{-4} \text{ year}^{-1}} \log \frac{20.4}{12.18}$$

$$= \frac{2.303}{2.303 \times 10^{-4} \text{ year}^{-1}} \log 1.6749$$

$$= \frac{2.303}{2.303 \times 10^{-4} \text{ year}^{-1}} \times 0.2339$$

$$= \mathbf{4287.43 \text{ years}}$$

SOLVED PROBLEM 13. 2 g of a radioactive element degraded to 0.5 g in 60 hours. In what time will it be reduced 10% of its original amount ?

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{a}{a-x}$$

Quantities given

$t = 60$ hours $\quad a = 2$ g $\quad a - x = 0.5$ g

Substitution of values

$$\lambda = \frac{2.303}{60 \text{ hours}} \times \log \frac{2}{0.5}$$

$$= \frac{2.303}{60 \text{ hours}} \times \log 4$$

$$= \frac{2.303}{60 \text{ hours}} \times 0.6021$$

$$= \mathbf{2.311 \quad 10^{-2} \ hour^{-1}}$$

(ii) To calculate the time t

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{a}{a-x}$$

or $$t = \frac{2.303}{\lambda} \log \frac{a}{a-x}$$

Quantities given

$\lambda = 2.311 \times 10^{-2}$ hour^{-1} $\quad a = 2$ g $\quad a - x =$ 90% of $a = 1.8$ g

Substitution of values

$$t = \frac{2.303}{2.311 \times 10^{-2} \text{ hour}^{-1}} \times \log \frac{2}{1.8}$$

$$= \frac{2.303}{2.311 \times 10^{-2} \text{ hour}^{-1}} \times \log 1.11$$

$$= \frac{2.303}{2.311 \times 10^{-2} \text{ hour}^{-1}} \times 0.0453$$

$$= \mathbf{4.51 \text{ hours}}$$

SOLVED PROBLEM 14. A radioactive isotope has half life of 20 days. What is the amount of isotope left over after 40 days if the initial amount is 5 g ?

SOLUTION :

(i) To calculate the disintegration constant

Formula used

$$\lambda = \frac{0.693}{t_{1/2}}$$

Quantity given

$$t_{1/2} = 20 \text{ days}$$

Substitution of values

$$\lambda = \frac{0.693}{20 \text{ days}}$$

$$= \mathbf{0.03465 \ day^{-1}}$$

(ii) To calculate the amount decayed, *x*

Formula used

$$\lambda = \frac{2.303}{t} \log \frac{a}{a-x}$$

Quantities given

$t = 40$ days $\quad \lambda = 0.03465$ day^{-1} $\quad a = 5$ g

Substitution of values

$$0.03465 \text{ day}^{-1} = \frac{2.303}{40 \text{ days}} \log \frac{5}{5-x}$$

or $$\log \frac{5}{5-x} = \frac{0.03465 \text{ day}^{-1} \times 40 \text{ days}}{2.303}$$

$$= 0.6018$$

or $$\frac{5}{5-x} = \text{Antilog } 0.6018$$

$$= 3.9978$$

or $$5 = (5-x) \times 3.9978$$