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Effects of viscous dissipation on natural convection flow over a sphere with temperature dependent thermal conductivity in presence of heat generation

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Abstract

In this paper, the steady two-dimensional laminar incompressible flow over a sphere in the presence of viscous dissipation and heat generation is considered. Thermal conductivity is assumed as a linear function of temperature. The governing equations are solved numerically by numerical solution strategy as per requirement and suitability. The obtained self similar equations are then solved numerically by an implicit, tri-diagonal, finite-difference method with Keller Box scheme. Favorable comparison with previously published work is performed. Computations are performed for a wide range of the governing flow parameters such as thermal conductivity variation parameter γ , heat generation parameter Q, Prandtl number Pr and Eckert number Ec. The computational findings for the dimensionless velocity, temperature profiles as well as for the skin-friction coefficient and heat transfer rate are presented in tabular form and graphically.

Keywords: Variable thermal conductivity; Heat generation; Eckert number

1. Introduction

The study of convective flow, heat transfer gets much interest of researchers for nature and industrial application.

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Chen and Mucoglu [1,2] have studied mixed convection over a sphere with uniform surface temperature and uniform surface heat flux for very large Reynolds Re and Grashof numbers Gr , using the boundary layer approximations.

Nomenclature

a	Radius of the sphere
C_f	Skin friction coefficient
C_p	Specific heat at constant pressure
f	Dimensionless stream function
Gr	Grashof number
g	Acceleration due to gravity
k	Thermal conductivity
k_f	Thermal conductivity of the fluid
k_∞	Thermal conductivity of the ambient fluid
Ec	Eckert number
Nu	Local Nusselt number
Pr	Prandtl number
q_w	Heat flux at the surface
Q	Heat generation parameter
Q_0	Constant
r	Radial distance from the symmetric axis to the surface
T	Temperature of the fluid in the boundary layer
T_∞	Temperature of the ambient fluid
T_w	Temperature at the surface
u	Dimensionless velocity component along x direction
v	Dimensionless velocity component along y direction
U	Velocity component along the surface
V	Velocity component normal to the surface
X	Axis in the direction along the surface
Y	Axis in the direction normal to the surface

Greek Symbols

ξ	Dimensionless coordinate along to the surface
η	Dimensionless coordinate normal to the surface
ψ	Stream function
τ_w	Shearing stress
ρ	Density of the fluid
μ	Viscosity of the fluid
ν	Kinematics viscosity of the fluid
θ	Dimensionless temperature function
β	Coefficient of thermal expansion
β_0	Strength of magnetic field
γ	Thermal conductivity variation parameter
γ^*	Constant
σ_0	Electric conductivity

Viscous dissipation effects on natural convection flow along a sphere with radiation heat loss are examined by Alim *et al*. [3]. Viscous dissipation effects on natural convection flow along a sphere with heat generation is studied by Salina *et al*. [4]. Natural convection flow on a sphere through porous medium in presence of heat source/sink near a stagnation point was considered by Mukhopadhyay [5]. Magneto hydrodynamic natural convection flow on a sphere in presence of heat generation was investigated by Molla *et al.* [6]. Alam *et al*. [7] has been investigated the free convection from a vertical permeable circular cone with pressure work and non-uniform surface temperature. Viscous dissipation effects on MHD natural convection flow over a sphere in the presence of heat generation was introduced by Alam *et al*. [8]. Mixed convection boundary layer flow about a solid sphere with Newtonian heating was analyzed by Salleh *et al*. [9]. Rahman *et al*. [10] analyzed the effects of temperature dependent thermal conductivity on magnetohydrodynamic (MHD) free convection flow along a vertical flat plate with heat conduction. Combined effects of viscous dissipation and temperature dependent thermal conductivity on MHD free convection flow with conduction and joule heating along a vertical flat plate was studied by Nasrin and Alim [11]. Safiqul Islam *et al.* [12] investigated the effects of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate with heat generation and joule heating. Effects of variable viscosity and thermal conductivity on unsteady MHD flow of non-Newtonian fluid over a stretching porous sheet was presented by Abdel Rahman [13]. Borah and Hazarika [14] studied the effects of variable viscosity & thermal conductivity on steady free convection flow along a semi-infinite vertical plate (in presence of uniform transverse magnetic field). He solved the governing boundary layer equations by taking series expansion of the stream function and temperature function. Uddin and Kumar [15] examined the effect of temperature dependent properties on MHD free convection flow and heat transfer near the lower stagnation point of a porous isothermal cylinder.

In all the aforementioned study the effects of viscous dissipation on natural convection flow over a sphere with temperature dependent thermal conductivity in presence of heat generation has not been considered yet. The governing partial differential equations are reduced to locally non-similar partial differential forms by adopting some appropriate transformations. The transformed boundary layer equations are solved numerically using implicit finite difference scheme together with Keller box technique. Numerical results have been obtained in terms of local skin friction, rate of heat transfer for a selection of relevant physical parameters are shown graphically.

2. Mathematical Formulation

The steady two-dimensional natural convection boundary layer flow of an incompressible viscous and electrically conducting fluid over a sphere of radius a has been considered. In this analysis T_w is assumed as the constant temperature at the surface of the sphere and T_∞ being the ambient temperature of the fluid. Whereas T is the temperature of the fluid in the boundary layer.

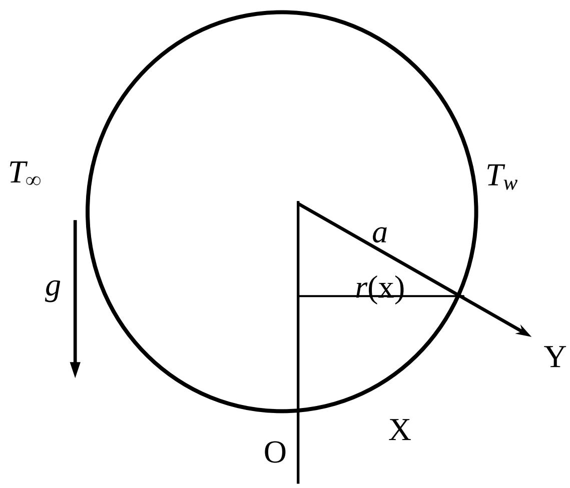

Fig. 1: Physical model and coordinate system

The conservation equations for the flow characterized with steady, laminar and two dimensional boundary layers, the continuity, momentum and energy equations can be written as:

$$\frac{\partial}{\partial X}(rU)+\frac{\partial}{\partial X}(rV)=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\nu\frac{\partial^2 U}{\partial Y^2}+g\beta\left(T\text{-}T_\infty\right)\sin\left(\frac{X}{a}\right) \tag{2}$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{\rho C_p}\frac{\partial}{\partial Y}\left(k_f\frac{\partial T}{\partial Y}\right)+\frac{\nu}{C_p}\left(\frac{\partial U}{\partial Y}\right)^2 \tag{3}$$

The boundary conditions for the governing equations are

$$\left.\begin{aligned} &U=V=0,\quad T=T_w \quad on \quad Y=0\\ &U\to 0, T\to T_\infty \quad at \quad Y\to\infty\end{aligned}\right\} \tag{4}$$

$$r(x)=a\sin\left(\frac{X}{a}\right) \tag{5}$$

where r is the radial distance from the symmetrical axis to the surface of the sphere, $k(T)$ is the thermal conductivity of the fluid depending on the fluid temperature T. Here we will consider the form of the temperature dependent thermal conductivity which is proposed by Charraudeau [16], as follows

$$k=k_\infty\left(1+\gamma^*(T-T_\infty)\right) \tag{6}$$

where k_∞ is the thermal conductivity of the ambient fluid and γ^* is constant which is defined as

$$\gamma^*=\frac{1}{k_f}\left(\frac{\partial k}{\partial T}\right)_f \tag{7}$$

The above equations are non-dimensional as usual manner by the following substitutions:

$$\xi=\frac{X}{a},\eta=Gr^{1/4}\frac{Y}{a},u=\frac{U}{u_0}=\frac{a}{\nu}Gr^{-1/2}\,\mathrm{U},\ v=\frac{a}{\nu}Gr^{-1/4}V,\theta=\frac{T-T_\infty}{T_w-T_\infty},\theta_w=\frac{T_w}{T_\infty} \tag{8}$$

Where, $u_0=\frac{\nu}{a}Gr^{1/2}$ is the characteristic velocity of the fluid.

Using the above transformations into equations (1) to (5), we have

$$\frac{\partial}{\partial\xi}(ru)+\frac{\partial}{\partial\eta}(rv)=0 \tag{9}$$

$$u\frac{\partial u}{\partial\xi}+v\frac{\partial u}{\partial\eta}=\frac{\partial^2 u}{\partial\eta^2}+\theta\sin\xi \tag{10}$$

$$u\frac{\partial\theta}{\partial\xi}+v\frac{\partial\theta}{\partial\eta}=\frac{1}{Pr}(1+\gamma\theta)\frac{\partial^2\theta}{\partial\eta^2}+\frac{1}{Pr}\gamma\left(\frac{\partial\theta}{\partial\eta}\right)^2+Ec\left(\frac{\partial u}{\partial\eta}\right)^2 \qquad \text{since } \upsilon\rho=\mu \tag{11}$$

The reduced boundary conditions are

$$u = v = 0,\ \theta = 1\ at\ \xi = 0$$
$$u = v = 0,\ \theta = 1\ at\ \eta = 0,\ \xi > 0 \tag{12}$$
$$u \to 0,\ \theta \to 0\ as\ \eta \to \infty,\ \xi > 0$$
$$r(\xi) = a \sin \xi \tag{13}$$

Here, $Gr = \dfrac{g\beta\left(T_w - T_\infty\right)a^3}{\nu^2}$ is the Grashof number and $\theta(\xi,\eta)$ is the non dimensional temperature function, viscous dissipation parameter $N = \dfrac{\nu^2 Gr}{\rho a^2 C_p\left(T_w - T_\infty\right)}$ is characterized by Eckert number $Ec = \dfrac{u_0^2}{C_p\left(T_w - T_\infty\right)}$, $Pr = \dfrac{\mu C_p}{k_\infty}$ is the Prandtl number, $\gamma = \dfrac{1}{k_f}\left(\dfrac{\partial k}{\partial T}\right)\left(T_w - T_\infty\right)$ is the thermal conductivity variation parameter. To solve equations (10) and (11) subject to the boundary conditions (12), we assume the following variables u and v where $\psi = \xi\, r(\xi)\, f(\xi,\eta)$ and $\psi(\xi,\eta)$ is the non-dimensional stream function which is related to the velocity components in the usual way as

$$u = \frac{1}{r}\frac{\partial \psi}{\partial \eta} \quad \text{and} \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial \xi} \tag{14}$$

Putting the above value in equation (10) and (11), we have

$$\frac{\partial^3 f}{\partial \eta^3} + \left(1 + \frac{\xi}{\sin\xi}\cos\xi\right) f \frac{\partial^2 f}{\partial \eta^2} - \left(\frac{\partial f}{\partial \eta}\right)^2 + \theta \frac{\sin\xi}{\xi} = \xi\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \eta \partial \xi} - \frac{\partial f}{\partial \xi}\frac{\partial^2 f}{\partial \eta^2}\right) \tag{15}$$

$$\frac{1}{Pr}(1 + \gamma\theta)\frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{Pr}\gamma\left(\frac{\partial \theta}{\partial \eta}\right)^2 + \left(1 + \frac{\xi}{\sin\xi}\cos\xi\right) f \frac{\partial \theta}{\partial \eta} + Ec\,\xi^2\left(\frac{\partial^2 f}{\partial \eta^2}\right) = \xi\left(\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial \xi} - \frac{\partial f}{\partial \xi}\frac{\partial \theta}{\partial \eta}\right) \tag{16}$$

The corresponding boundary conditions are

$$\left.\begin{aligned} & f = \frac{\partial f}{\partial \eta} = 0, \quad \theta = 1\ at\ \eta = 0 \\ & f = \frac{\partial f}{\partial \eta} = 0, \quad \theta = 1\ at\ \eta = 0, \xi > 0 \\ & \frac{\partial f}{\partial \eta} \to 0, \theta \to 0\ as\ \eta \to \infty, \xi > 0 \end{aligned}\right\} \tag{17}$$

It can be seen that near the lower stagnation point of the sphere i.e. $\xi \approx 0$ Equations (15) and (16) reduces to the following ordinary differential equations:

$$f''' + 2 f f'' - \left(f'\right)^2 + \theta = 0 \tag{18}$$

$$\frac{1}{Pr}(1 + \gamma\theta)\theta'' + \frac{1}{Pr}\gamma\left(\theta'\right)^2 + 2 f\theta' = 0 \tag{19}$$

Where primes denote the differentiation of the function with respect to η .

Subject to the boundary conditions

$$\left.\begin{aligned} & f(0) = f'(0) = 0, \theta(0) = 1 \\ & f' \to 0, \theta \to 0 \ as\ \eta \to \infty \end{aligned}\right\} \tag{20}$$

In practical application, the physical quantities of principal interest are the heat transfer and the skin- friction coefficient, which can be written in non- dimensional form as

$$Nu = \frac{aGr^{-1/4}}{k\left(T_w - T_\infty\right)} q_w \text{ and } C_f = \frac{Gr^{-3/4} a^2}{\mu \nu} \tau_w \tag{21}$$

Where $q_w = -k_f \left(\frac{\partial T}{\partial Y}\right)_{Y=0}$ and $\tau_w = \mu \left(\frac{\partial U}{\partial Y}\right)_{Y=0}$, k_f being the thermal conductivity of the fluid. Using the new variables (8) along with the boundary conditions (17), we have the simplified form of the heat transfer and the skin-friction coefficient as

$$Nu = -\theta'(\xi,0) \text{ and } C_f = \xi f''(\xi,0) \tag{22}$$

3. Result and discussion

In order to gain physical insight the velocity and temperature profiles as well as skin friction coefficient and rate of heat transfer have been discussed by assigning numerical values to the parameter encountered in the problem in which the numerical results are displayed with the graphical illustrations. Solutions are obtained for the fluid having Prandtl number Pr = 1.0, viscous dissipation parameter N which is characterized by Eckert number Ec (= 1.0, 1.5, 2.5, 3.5), thermal conductivity variation parameter γ (= 0.50, 1.50, 2.50, 3.50) and heat generation parameter Q (= 0.01, 0.15, 0.25, 0.35) against η at any position of ξ .

Figs. 2(a) and (b) display results for the velocity and temperature profiles, for different values of Eckert number Ec with thermal conductivity variation parameter γ = 0.50, heat generation parameter Q= 0.01 and Prandtl number Pr =1.0. From Fig. 2(a), it can be observed that the velocity goes significantly upward with the increase of the Eckert number Ec . From Fig. 2(b), it is seen that when the values of Eckert number Ec increases in the region $0 \le \eta \le 6.5$, the temperature distribution also increases. Figs. 3(a) and (b) show how variations in Ec affect the flow on skin-friction coefficient and the rate of heat transfer. It is observed that at ξ = 0.87266, the skin friction coefficient C_f increases by 10.03% and the Nusselt number Nu decreases by 77.74% as Ec increases from 1.0 to 3.5. Figs. 4(a) and (b) illustrate the velocity and temperature distribution against the variable η for different values of the thermal conductivity variation parameter γ while Pr =1.0, Ec = 1.0 and Q = 0.01. It is found that both the velocity and temperature distribution increases with the increasing values of the thermal conductivity variation parameter γ . It should be noted that at each value of the thermal conductivity variation parameter γ , the velocity profile has a local maximum value within the boundary layer. The maximum values of the velocity are 0.37182, 0.40869, 0.44253 at η = 1.11440 and final maximum value is 0.47266 at η = 1.17520 for γ = 0.50, 1.50, 2.50, 3.50 respectively. The velocity increases by 27.12% as γ increases from 0.50 to 3.50. It is obvious that the velocity boundary layer and the thermal boundary layer thickness enhance for large values of γ .

Figs. 5(a) and 5(b) deal with the effect of thermal conductivity variation parameter γ associated with the heat generation parameter Q = 0.01and Eckert number Ec =1.0 and Prandlt number Pr = 1.0. From Figs. 5(a) and 5(b) we observed that the skin friction co-efficient C_f increase sharply, on the contrary heat transfer rate decrease monotonically for selected value of γ . It is seen that skin friction co-efficient and heat transfer rate increases by 16.27% and decreases by 80.52% for distinct value of γ at ξ = 0.87266.The effect of heat generation parameter Q

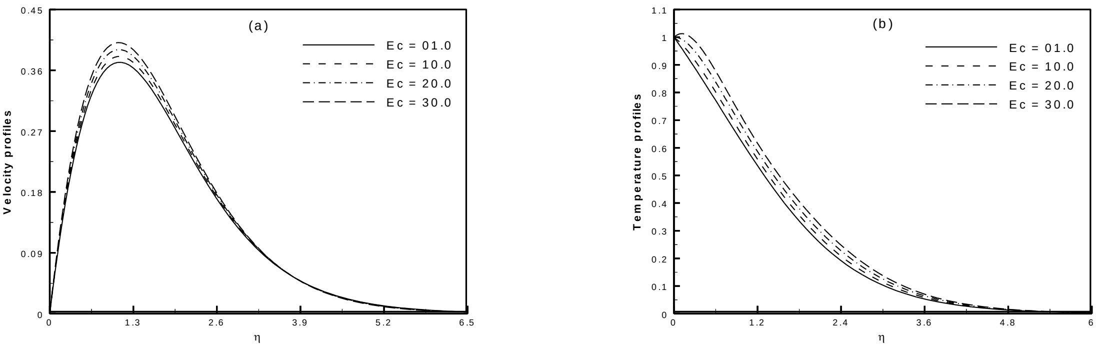

Fig. 2. (a)Velocity profiles and (b) temperature profiles for different values of *Ec* while γ = 0.50, *Q* = 0.01 and *Pr* =1.0.

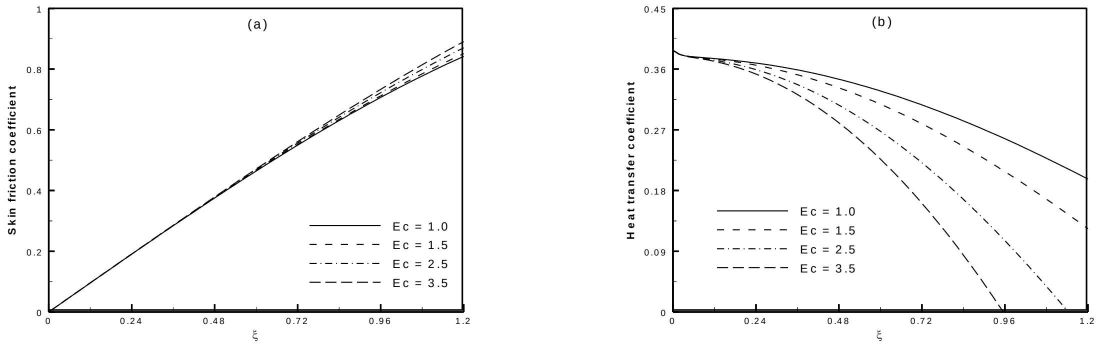

Fig. 3. (a) Skin friction coefficient and (b) rate of heat transfer for different values of *Ec* while γ = 0.50, *Q* = 0.01 and *Pr* =1.0.

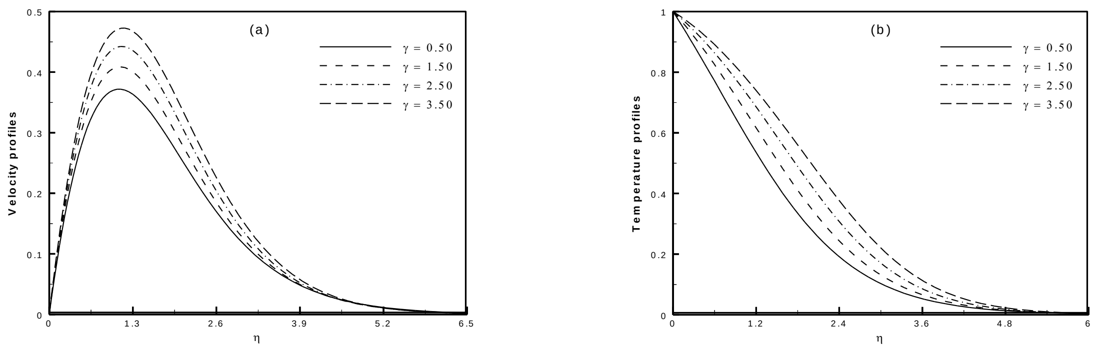

Fig. 4. (a)Velocity profiles and (b) temperature profiles for different values of γ while *Ec* = 1.0, *Q* = 0.01 and *Pr* =1.0.

on velocity and temperature profiles with γ = 0.50, *Ec* = 1.0 and *Pr* = 1.0 are exposed in Figs. 6(a) and 6(b). From Fig. 6(a), it can be stated that the velocity distribution increases as the values of heat generation parameter *Q* increase. It is obvious that when the heat is generated (Q > 0) the buoyancy force increases, which induces the flow rate to increase giving, rise to the increase in the velocity profiles. Again when the heat absorption (Q < 0) intensifies the velocity is found to decrease due the decrease in the buoyancy force. The maximum values of the velocity are 0.37182, 0.39892, 0.41964, 0.44168 for *Q* = 0.01, 0.15 0.25 and 0.35 respectively which occur at η = 1.11440. Here it is observed that the velocity increase by 18.78 % as *Q* increases from 0.01 to 0.35. From Fig. 6(b), it is seen that when the values of heat generation parameter *Q* increase, the temperature distributions also increase.

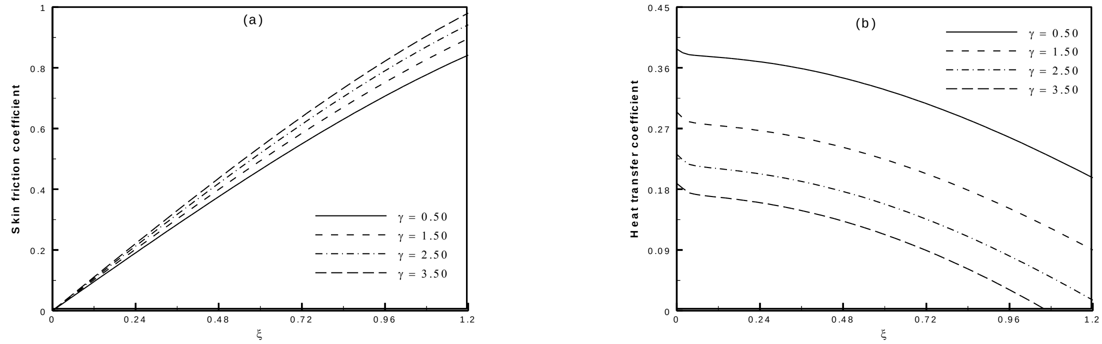

Fig. 5. (a) Skin friction coefficient and (b) rate of heat transfer for different values of of γ while Ec = 1.0, Q = 0.01 and Pr =1.0.

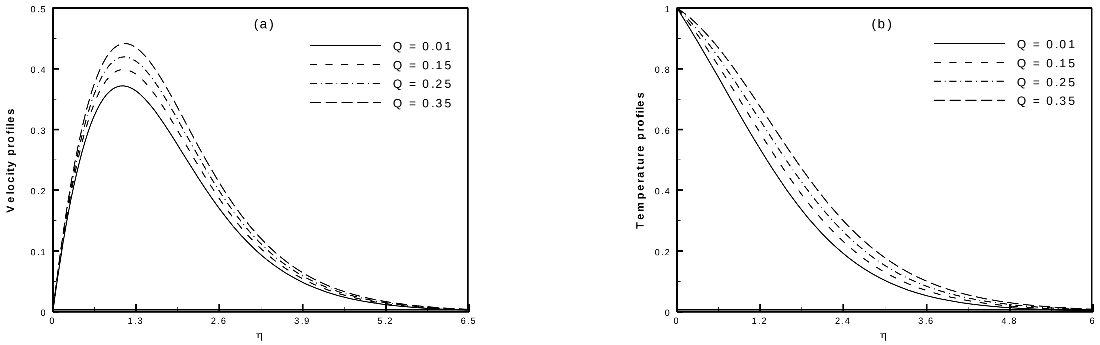

Fig. 6. (a)Velocity profiles and (b) temperature profiles for different values of Q while Ec = 1.0, γ = 0.50 and Pr =1.0.

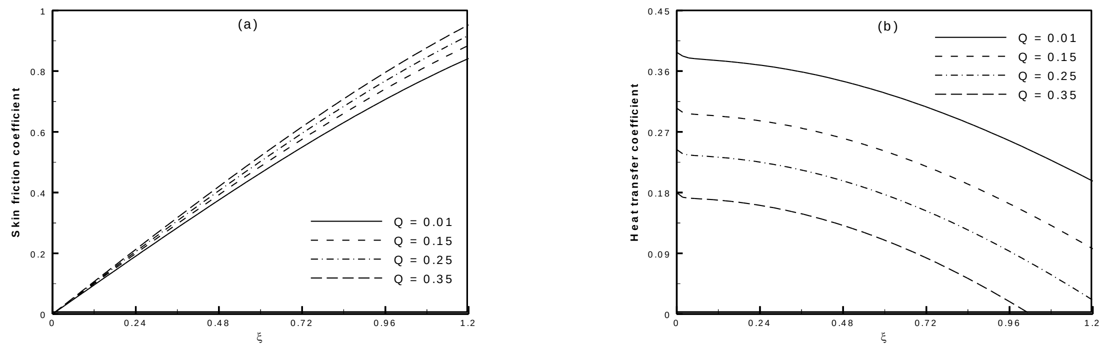

Fig.7. (a) Skin friction coefficient and (b) rate of heat transfer for different values of of Q while Ec = 1.0, γ = 0.50 and Pr =1.0.

The variation of the local skin friction coefficient and the local rate of heat transfer for different values of the heat generation parameter Q are depicted in Figs. 7(a) and 7(b) while γ = 0.50, Ec = 1.0 and Prandtl number Pr =1.0. From the Figs. 7(a) and 7(b), it exhibits that the increase of the heat generation parameter Q leads to an increase in the local skin-friction coefficient C_f and a decrease in the local Nusselt number Nu . Moreover, it is seen that at ξ = 0.87266 the skin friction coefficient C_f increases by 12.44% and the Nusselt number Nu decreases by 84.14% respectively, as Q increases from 0.01 to 0.35.

In order to verify the accuracy of the present work, the values of non dimensional heat transfer parameter Nu for

Ec = 0, γ = 0 and Q = 0 having prandlt number Pr = 0.7 at different position of ξ (in degree) are compared with those reported by Nazar *et al*.[17] and Molla *et al.* [6] as present in table 1. The results are found to be in good agreement.

Table 1: Rate of heat transfer against ξ for Prandlt number Pr = 0.70 with other controlling parameters Ec = 0.0, γ = 0.0 and Q =0.0.

ξ in degree	Nazar *et al*.[17]	Molla [6]	present
0	0.4576	0.4576	0.45762
10	0.4565	0.4564	0.45653
20	0.4533	0.4532	0.45336
30	0.4480	0.4479	0.44808
40	0.4405	0.4404	0.44067
50	0.4308	0.4307	0.43107
60	0.4181	0.4188	0.41920
70	0.4046	0.4045	0.40499
80	0.3879	0.3877	0.38828
90	0.3684	0.3683	0.36891

4. Conclusion

Natural convection heat transfer gained considerable attention because of its numerous applications in the areas of energy conservations cooling of electrical and electronic components, design of solar collectors, heat exchangers, pumps and flow meters and many others. An analysis has been carried out to study the effects of viscous dissipation on natural convection flow over a sphere with temperature dependent thermal conductivity in presence of heat generation. The following observations and conclusions can be drawn:

- The velocity and temperature within the boundary layer increases for increasing values of Eckert number Ec, thermal conductivity variation parameter γ , heat generation parameter Q.
- The local skin friction co-efficient C_f increases for the increasing values of Eckert number Ec, thermal conductivity variation parameter γ , heat generation parameter Q.
- The local Nusselt number Nu decreases for the increasing values of the Eckert number Ec, thermal conductivity variation parameter γ , heat generation parameter Q.
- The effect of increasing values of the thermal conductivity variation parameter γ is to increase the momentum boundary layer as well as the thermal boundary layer thickness.

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