

NUMERICAL INVESTIGATION OF NONLINEAR ACOUSTIC WAVE PHENOMENA IN STRONGLY AND WEAKLY COUPLED PLASMAS



By

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Declaration

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Dedicated to

*My beloved Spouse
and
all other family members*

List of Publications

- [1] M. N. Islam, M. G. Hafez, and U. K. Deb, “Burgers equation with cubic and quartic nonlinearity: Dynamics of heavy ion-acoustic coherent structures in unmagnetized strongly coupled plasma,” *Physica Scripta*, vol. 100, no. 4, p. 045249, 2025.
- [2] M. N. Islam, M. G. Hafez, and R. Sakthivel, “Effects of the polarization force and coupling parameter on dust acoustic shocks around the critical and supercritical values in a strongly coupled dusty plasma,” *Indian Journal of Physics*, vol. 99, no. 1, pp. 301–317, 2025.
- [3] M. N. Islam and M. G. Hafez, “Propagation of dust acoustic shocks and oscillatory waves in a coupled complex plasma described by nonlinear evolution equations,” *Partial Differential Equations in Applied Mathematics*, vol. 9, p. 100628, 2024.
- [4] M. N. Islam, M. G. Hafez, and U. K. Deb, “Heavy ion-acoustic soliton and dressed soliton in an unmagnetized weakly and strongly coupled plasma,” *Brazilian Journal of Physics*, vol. 52, no. 5, p. 158, 2022.
- [5] M. N. Islam, M. G. Hafez, and M. S. Alam, “An unmagnetized strongly coupled plasma: heavy ion acoustic shock wave excitations,” *Physica Scripta*, vol. 96, no. 12, p. 125610, 2021.

Approval/Declaration by the Supervisor(s)

This is to certify that Md. Nazrul Islam has carried out this research work under our supervision, and that he has fulfilled the relevant Academic Ordinance of the Chittagong University of Engineering and Technology, so that he is qualified to submit the following Thesis in the application for the degree of DOCTOR of PHILOSOPHY. Furthermore, the Thesis complies with PLAGIARISM and ACADEMIC INTEGRITY regulation of CUET.

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Abstract

This thesis investigates nonlinear acoustic wave phenomena in both strongly and weakly coupled collisionless unmagnetized plasmas using analytical and numerical methods. By developing mathematical models for different plasma environments, the study employs the reductive perturbation technique (RPT) to derive nonlinear evolution equations (NLEEs). These equations are then used to analyze the propagation of shock waves, solitons, and periodic waves in various plasma configurations.

In Chapter 2, Burgers equations involving quadratic, cubic, and combined quadratic-cubic nonlinearities are derived for a coupled complex plasma system consisting of Boltzmann-distributed electrons, nonthermal ions, and charged dust particles. The generalized Riccati equation mapping method (GREMM) is employed to obtain both shock and oscillatory wave solutions from these equations, while parametric effects on wave characteristics are systematically examined.

Chapter 3 investigates heavy ion-acoustic shock waves (HIASWs) in a plasma comprising inertial heavy ions, Maxwellian light ions, and (α, q) -distributed electrons, by deriving the Burgers, modified Burgers, and mixed modified Burgers equations. The study presents stationary shock wave solutions for these equations and examines how plasma parameters influence shock wave characteristics.

Chapter 4 investigates the dust acoustic (DA) shock wave phenomena in an strongly coupled dusty plasma. By deriving Burgers equations with quadratic, cubic, and quartic nonlinearities, we analyze shock wave behavior near critical values (CVs) and super-critical values (SCVs), revealing how polarization force and coupling parameter modifies shock wave excitations.

Chapter 5 analyzes heavy ion-acoustic (HIA) solitons and dressed solitons in weakly/strongly coupled plasmas with nonthermal electrons.

Chapter 6 advances the Chapter 3 framework by examining HIASWs near CVs and SCVs, deriving a non-integrable NLEE with combined cubic-quartic nonlinearities that necessitates numerical solution via Runge-Kutta-Fehlberg analysis.

Thus, the thesis enhances understanding of nonlinear coherent structures observed in various space and astrophysical environments.

বিমূর্ত

এই থিসিসটি অ্যানালিটিক্যাল এবং সংখ্যাগত পদ্ধতি ব্যবহার করে শক্তিশালী এবং দুর্বলভাবে যুক্ত সংঘর্ষহীন অচুষকিয় প্লাজমার ননলিনিয়ার অ্যাকোস্টিক তরঙ্গ ঘটনা তদন্ত করে। বিভিন্ন প্লাজমা পরিবেশের জন্য গাণিতিক মডেল তৈরি করে, গবেষণাটি রিডাক্টিভ পার্টারবেশন টেকনিক (RPT) ব্যবহার করে ননলিনিয়ার ইন্ডোলিউশন সমীকরণগুলো (NLEEs) উদ্ভূত করে। পরবর্তীতে এই সমীকরণগুলি বিভিন্ন প্লাজমা কনফিগারেশনে শক ওয়েভ, সলিটন এবং পর্যায়ক্রমিক তরঙ্গের বিস্তার বিশ্লেষণ করতে ব্যবহৃত হয়।

অধ্যায় ২-এ, বোল্টজমান-বিতরণ ইলেকট্রন, ননথার্মাল আয়ন এবং চার্জড ধূলিকণা সমন্বিত একটি যুক্ত জটিল প্লাজমা সিস্টেমের জন্য দ্বিঘাত, ঘন এবং সম্মিলিত দ্বিঘাত-ঘন ননলিনিয়ারিটি বার্জাস সমীকরণ নির্ণয় করা হয়। জেনারেলাইজড রিকাটি সমীকরণ ম্যাপিং পদ্ধতি (GREMM) ব্যবহার করে এই সমীকরণগুলি থেকে শক এবং অসিলেটরি ওয়েভ সমাধান বের করার পাশাপাশি তরঙ্গ বৈশিষ্ট্যের উপর প্যারামেট্রিক প্রভাবগুলি পদ্ধতিগতভাবে পরীক্ষা করা হয়।

৩য় অধ্যায় এ জড়তাযুক্ত ভারী আয়ন, ম্যাক্সওয়েলিয়ান হালকা আয়ন এবং (α, q) -বিতরণের ইলেকট্রন সমন্বিত প্লাজমায় বার্জাস, পরিবর্তিত বার্জাস এবং মিশ্র পরিবর্তিত বার্জাস সমীকরণ উদ্ভূত করে ভারী আয়ন-অ্যাকোস্টিক শক ওয়েভ (HIASWs) এর তদন্ত করা হয়। গবেষণাটি এই সমীকরণগুলির জন্য স্থির শক ওয়েভ সমাধান উপস্থাপন করে এবং প্লাজমা প্যারামিটারগুলি কীভাবে শক ওয়েভ বৈশিষ্ট্যকে প্রভাবিত করে তা পরীক্ষা করে।

৪র্থ অধ্যায় এ একটি শক্তিশালীভাবে যুক্ত ধূলি প্লাজমায় ডাস্ট অ্যাকোস্টিক (DA) শক ওয়েভ ঘটনা তদন্ত করে। দ্বিঘাত, ঘন এবং চতুর্ঘাত ননলিনিয়ারিটি সহ বার্জাস সমীকরণ নির্ণয় করে, আমরা ক্রিটিক্যাল মান (CVs) এবং সুপার-ক্রিটিক্যাল মান (SCVs) এর নিকটে শক ওয়েভ আচরণ বিশ্লেষণ করি, যা দেখায় কিভাবে পোলারাইজেশন ফোর্স এবং কাপলিং প্যারামিটার শক ওয়েভ উত্তেজনাকে পরিবর্তন করে।

৫ম অধ্যায় এ, ননথার্মাল ইলেকট্রন সহ দুর্বল/শক্তিশালীভাবে যুক্ত প্লাজমায় ভারী আয়ন-অ্যাকোস্টিক (HIA) সলিটন এবং ড্রেসড সলিটন বিশ্লেষণ করে।

৬ষ্ঠ অধ্যায় ৩য় অধ্যায় এর কাঠামোকে এগিয়ে নিয়ে CVs এবং SCVs এর নিকটে HIASWs পরীক্ষা করে, একটি নন-ইন্টিগ্রেবল NLEE উদ্ভূত করা হয় যাতে সম্মিলিত ঘন-চতুর্ঘাত ননলিনিয়ারিটি থাকে যা রুগ্জে-কুট্রা-ফেলবার্গ বিশ্লেষণের মাধ্যমে সংখ্যাগত সমাধান করা হয়।

এইভাবে, থিসিসটি বিভিন্ন মহাকাশ এবং জ্যোতিঃপদার্থবিজ্ঞান পরিবেশে পর্যবেক্ষণকৃত ননলিনিয়ার সুসংগত কাঠামোকে বুঝতে সাহায্য করে।

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Nomenclature

Index of Abbreviation

VDF	Velocity Distribution Function
IAW	Ion Acoustic Wave
HIAW	Heavy Ion Acoustic Wave
HIASW	Heavy Ion Acoustic Shock Wave
NLEEs	Nonlinear Evolution Equations
DAW	Dust Acoustic Wave
RPT	Reductive Perturbation Technique
SW	Shock Wave
SC (SCR)	Strongly Coupled (Strongly Coupled Regime)
WC (WCR)	Weakly Coupled (Weakly Coupled Regime)
SWEs	Shock Wave Excitations
BE	Burgers Equation
mBE	Modified Burgers Equation
mmBE	Mixed Modified Burgers Equation
CV	Critical Value
SCV	Super-Critical Value
SCP (WCP)	Strongly Coupled Plasma (Weakly Coupled Plasma)
SAEs	Space and Astrophysical Environments
GREMM	Generalized Riccati Equation Mapping Method
MLIs	Maxwellian Light Ions
BGS	British Geological Survey
HON	Higher Order Nonlinearity
NHO	Nonlinear Heteroclinic Orbit
NPO	Nonlinear Periodic Orbit
VRT	Viscosity Relaxation Time

PL	Plasma Laboratory
CDP	Coupled Dusty Plasma
VC	Viscosity Coefficient
NIs	Negative Ions
KdVB	Korteweg-de Vries and Burgers
TR	Temperature Ratio
DR	Density Ratio
HTN	Hybrid Tsallis Nonthermal
NCDGs	Negatively Charged Dust Grains
BDEs	Boltzmann Distributed Electrons
ACVs	Around the Critical Values
NHO	Nonlinear Heteroclinic Orbit
RK45	Runge-Kutta Fehlberg
ODE	Ordinary Differential Equation
DLs	Double Layers
DIAWs	Dust Ion Acoustic Waves
HIs	Heavy Ions
HIA	Heavy Ion Acoustic
SN	Strength of Nonextensivity

Chapter 1: INTRODUCTION

1.1 BACKGROUND AND MOTIVATION

The three well-known states of matter are solid, liquid, and gas. As the temperature increases, a solid changes to a liquid, and with further heating, it changes to a gaseous state. Plasma, regarded as the fourth state of matter, emerges when a gaseous medium undergoes additional heating, causing individual atoms to ionize [1–4]. Consequently, plasma consists of free charged particles and collectively maintains electrical neutrality [1–4]. However, despite this neutral property, plasma particles interact with long-range electromagnetic forces, exhibiting numerous intriguing phenomena. This prevalent form of plasma in the universe attracts the attention of researchers not only for fundamental research but also for its diverse applications across various scientific domains [1–6]. Plasma constitutes around 99% of the observable matter in the universe. Plasmas can be broadly categorized based on their existence, such as laboratory plasma (used in display panels, fluorescent lamps, ion thrusters, textile industries, agriculture, materials processing, wastewater treatment, etc.), terrestrial plasmas (found in lightning, magnetosphere, ionosphere, polar aurora, polar wind, etc.), and space and astrophysical plasmas (observed in stars, nuclear fusion, solar wind, interplanetary medium, interstellar medium, intergalactic medium, accretion discs, etc. [1–9]). Furthermore, plasmas can be classified based on their degree of ionization: partially ionized plasma, where the density of charged particles is much less than that of neutral particles, and completely ionized plasma, where the density of charged particles is approximately equal to that of neutral particles. Additionally, depending on the temperatures of plasma species, a measure of the thermal kinetic energy of the particles, plasmas can be categorized into non-thermal plasma, where the electron temperature exceeds the ion or gas temperature, and thermal plasma, where the electron temperature is approximately equal to the ion or gas temperature.

The study of dust species dynamics within a plasma offers insight into various universal behaviors. Dust particles, typically ranging in mass from 10^{-15} to 10^{-10} kg, exist alongside the conventional electron-ion plasma [2]. The dust particles vary in shape and size and tend to acquire charge from electron and ion fluxes on their surfaces. Typically, electron

flux dominates, resulting in a net negative charge on the dust species. However, under specific conditions such as secondary electron emission, dust grains may become positively charged. Dusty plasmas hold significant implications in industrial and space physics applications [2], [10–13]. Dust particles can play a beneficial role in the formation of astronomical bodies, but they can also act as contaminants in fusion plasma devices and industrial processes.

The study of linear and nonlinear wave propagation in multi-ion plasmas [14–31] is important for understanding propagation phenomena. Such plasmas are observed in Space and Astrophysical Environments (SAEs) [32–35] as well as in Laboratory Plasma (LP) [36–43]. SAEs, including the Earth’s ionosphere and magnetosphere, often contain both positive and negative ion species, along with electrons.

Strongly and weakly coupled plasmas exhibit distinct features in plasma physics, showing characteristic behaviors due to variations in density, temperature, and interaction strength. Strongly coupled plasmas are characterized by high particle density and low temperature, leading to strong interactions among particles [44],[45]. Theoretical descriptions of strongly coupled plasmas often rely on methods such as molecular dynamics simulations, Monte Carlo techniques, and density functional theory. Experimental investigations utilize techniques such as laser spectroscopy, X-ray scattering, and optical diagnostics to probe the properties of strongly coupled plasmas. Recent studies have revealed intriguing phenomena in strongly coupled plasmas, including non-equilibrium dynamics, phase transitions, and collective excitations such as plasma crystals and liquid-like behavior. In contrast, weakly coupled plasmas are characterized by low particle density and high temperature, resulting in weak interactions among particles. Theoretical models for weakly coupled plasmas often employ fluid equations, kinetic theory, and magnetohydrodynamics to describe their behaviors. Recent research has focused on phenomena such as plasma waves, instabilities, and turbulence in weakly coupled plasmas, shedding light on processes relevant to fusion reactors, space plasmas, and plasma-material interactions. The fundamental differences between strongly and weakly coupled plasmas are reflected in their properties and dynamics. Strongly coupled plasmas exhibit collective behavior and solid-like properties, whereas weakly coupled plasmas behave like classical gases with dominant individual particle interactions. Despite these

differences, both strongly and weakly coupled plasmas are interconnected through common plasma physics principles, offering insights into the diverse range of plasma phenomena observed in nature and in the laboratory. The phenomena concerning strongly and weakly coupled plasmas have numerous applications in various fields. Strongly coupled plasmas find applications in colloidal science, dusty plasmas, and inertial confinement fusion research. Weakly coupled plasmas are essential for understanding processes in magnetically confined fusion devices, space plasmas, and plasma-material interactions in industrial plasma processing. Future research directions include investigating multi-scale plasma phenomena, developing advanced diagnostic techniques, and exploring novel applications of strongly and weakly coupled plasmas in emerging technologies. However, non-Maxwellian velocity distributions [46–48] are common in space and astrophysical environments (SAEs), where long-range interactions violate typical thermodynamic assumptions. Non-extensive statistics [46], proposed by Tsallis, provide a framework for describing such non-equilibrium plasma regimes. Energetic plasma particles, deviating from Maxwellian distributions, have been observed in various SAEs, emphasizing the need for generalized velocity distribution functions to characterize their density functions. By elucidating the distinct characteristics and behaviors of these plasmas, researchers can gain insights into a wide range of phenomena occurring in natural and engineered plasma systems.

In light of these considerations, this research work focuses on nonlinear wave phenomena for dust acoustic shocks, heavy ion acoustic shocks, ion-acoustic solitons, and dressed solitons under different plasma conditions such as polarizing force, critical and super-critical parameters, dust particles, and so on for both weakly and strongly coupled plasmas.

1.2 BASIC PROPERTIES OF PLASMAS

A plasma is a quasi-neutral gas that has been ionized enough to display collective behaviors. The term "quasi-neutral" means that, on a macroscopic scale, the charge density is essentially zero. "Collective behavior" refers to the phenomena that arise in ionized media, which do not occur in gases. In plasmas, the charged particles are influenced by, and also generate, electromagnetic fields, allowing for a wider range of collective behaviors

than what is seen in gases. Plasma science is particularly challenging because plasmas, as electrostatic fluids, interact through long-range magnetic and electric fields, rather than just through particle collisions. This makes plasmas more complex to understand compared to fluid dynamics or electromagnetic behavior in dielectric media.

Quasi-neutrality condition:

The quasi-neutrality in plasma refers to the condition where the positive and negative charges in a plasma are nearly balanced, resulting in an overall neutral electric charge on a macroscopic scale. In this case the number of ions is approximately equal to the number of electrons. That is, $n_e \approx n_i$, where the n_e and n_i are the number densities of electrons and ions respectively. The dimension of the ionized gas should satisfy the quasi-neutrality condition.

Collective behavior:

This criterion arises due to the long-range nature of Coulomb potential, ensuring that local disturbances can influence remote regions of the plasma significantly. Collective behavior is characterized by a large number of plasma particles within Debye's sphere.

Debye length:

The Debye length, named after physicist Peter Debye, plays a crucial role in understanding electrostatic shielding and the screening of electric fields in plasmas. It measures the distance over which the plasma screens out external electric potentials, determining the extent of charge interactions. Mathematically, the Debye length (λ_D) is given by:

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n q^2}},$$

where ϵ_0 is the permittivity of free space, k_B is the Boltzmann constant, T is the plasma temperature, n is the particle density, and q is the elementary charge. A smaller Debye length indicates stronger screening, meaning electric fields and charges are shielded over shorter distances. Conversely, a larger Debye length suggests weaker screening, allowing electric fields to extend further. The Debye length represents the transition from collective plasma behavior to individual particle interactions, bridging micro and macro scales. Understanding its properties is essential for both theoretical and experimental plasma physics research.

Plasma parameter:

The plasma parameter is the ratio of the potential energy to the translational kinetic energy of a plasma particle within the Debye sphere, where the electric field of the particle is screened by surrounding charged particles. It is a key indicator of the plasma's coupling strength, with a value greater than 1 suggesting strong coupling between particles, where electrostatic interactions dominate over thermal motion. In this regime, particles exhibit collective behavior, and phenomena like plasma crystals and phase transitions may occur. The strongly coupled plasma is often seen in dusty plasmas and low-temperature environments. Conversely, when the plasma's coupling strength is closer to or less than 1, the kinetic energy of particles dominates, leading to weakly coupled, gas-like behavior, typical of high-temperature plasmas. Thus, the plasma parameter helps characterize whether a plasma exhibits collective interactions or behaves more like an ideal gas, influencing the study and applications of various plasma systems.

Electromagnetic force:

Electromagnetic forces occur when an electromagnetic field interacts with electrically charged particles, such as those that make up a plasma (i.e. electrons, positrons and other ions). It includes the electric force, which produces electric fields between charged forces, and the magnetic force, which manifests itself as magnetic fields wherever there are moving charges. Plasmas interact strongly with electromagnetic forces that results in complexity in structure and motion that far exceeds that found in gases, liquids, and solids. This is exemplified by solar flares and the solar wind, which overcome the Sun's magnetic field, accelerate away, and out into interplanetary space. Electromagnetic forces acting on plasma particles should dominate over collision forces.

1.3 PLASMA WAVES

Plasma waves and oscillations occur when charged particles (electrons and ions) move collectively, creating disturbances in the plasma's electromagnetic and electrostatic fields, which propagate as waves. These waves are generated by the interactions between charged particles, which experience restoring forces when displaced from equilibrium positions. These forces result from electrostatic (electric fields) and magnetostatic (magnetic fields) effects.

Plasmas, as complex fluids, support various wave modes driven by dynamic pressure, electrical, and magnetic forces. The propagation of these waves depends on their frequency and the angle of propagation relative to the background magnetic field. Plasma waves can be electrostatic or electromagnetic, depending on the presence of oscillating magnetic fields. They include modes like ion acoustic waves, dust ion acoustic waves, and electron acoustic waves. Plasma waves are crucial in space and astrophysical environments and contribute to various physical phenomena. In strongly coupled plasmas, particles interact in periodic patterns, generating wave phenomena. These plasmas, which include electrons, ions, and other species, exhibit collective behavior through electric and magnetic field interactions. Wave propagation depends on the medium's dispersive or dissipative nature and is important for plasma diagnostics, as wave modes reveal plasma characteristics. In most plasmas, the temperature of lighter species (such as electrons) is higher than that of ions, causing them to move faster. Wave modes depend on the species' mass and temperature, with modes like the lower hybrid wave relying on both electron and ion inertia. Plasmas can also support nonlinear waves, where dispersion or dissipation leads to complex disturbances.

This thesis focuses on studying nonlinear ion, dust, and dust-ion acoustic waves in both unmagnetized strongly and weakly coupled plasmas observed in some astrophysical and space environments. Below is a brief overview of some important plasma wave phenomena.

Ion Acoustic Waves (IAWs):

IAWs are electrostatic waves where the restoring force is provided by the pressure of lighter species (e.g., electrons, positrons), and the inertia is provided by the mass density of the ion species. These purely longitudinal waves are generated due to compressions and rarefactions of the medium, propagating through the plasma similar to ordinary sound waves in neutral gas. Unlike sound waves, IAWs may propagate in a collision-less medium as ions interact with electrostatic or electromagnetic fields at long distances. The motion of electrons affects wave dispersion, with electrons quickly following ion motion to preserve charge neutrality. Baumjohann and Treumann [49] derived the dispersion relation, considering the motion of both electrons and ions, revealing that the group velocity of IAWs is equal to the phase velocity. Various types of IAWs exist in plasmas, such as solitary waves, shock waves, double layers, each contributing to the understanding of plasma

physics. Besides, both dust acoustic waves (DAWs) and dust ion acoustic waves (DIAWs) are types of waves that can propagate in dusty plasmas, which are plasmas containing charged dust particles.

Dust Acoustic Waves (DAWs):

DAWs are low-frequency waves that propagate in a dusty plasma where the dust particles are negatively charged and much heavier than the ions and electrons. These waves are driven by the inertia of the dust particles and the pressure of the electrons and ions in the plasma. DAWs are similar to acoustic waves in neutral gases but with modifications due to the presence of charged dust particles. They play a significant role in the dynamics and stability of dusty plasmas and have been observed in laboratory experiments and astrophysical environments [2].

Dust Ion Acoustic Waves (DIAWs):

DIAWs are another type of wave that propagates in dusty plasmas but under different conditions. Unlike DAWs, DIAWs occur in plasmas where the dust particles are positively charged, typically due to absorbing electrons and becoming negatively charged. These waves are driven by the interactions between the ions and electrons in the plasma and the charged dust particles. DIAWs have characteristics similar to ion acoustic waves in traditional plasmas but with modifications due to the presence of charged dust particles. They are also important in understanding the dynamics and behavior of dusty plasmas and have been observed in laboratory experiments and space environments. Both types of waves are essential for understanding the collective behavior of charged particles in dusty plasmas, which have applications in various fields such as astrophysics, fusion research, and industrial plasma processing [2],[50].

Solitons and Shocks:

Solitons, or solitary waves, are nonlinear wave phenomena that maintain their shape and size while propagating at a constant speed [51]. These stable waves result from the balance between nonlinearity and dispersion effects, with negligible dissipation. In a non-dispersive medium, solitons travel without changing shape, while in a dispersive medium, they spread due to differences in wave velocities. Solitary waves are important for understanding particle and energy transport in plasmas and are central to nonlinear plasma physics [2]. They can transfer energy from high-intensity lasers to plasma through excitation

of nonlinear plasma waves, manifesting as modulated wave packets. Solitary waves are observed in various phenomena, including shallow water waves and astrophysical as well as space environments [2],[12],[52],[53].

In contrast, shock waves occur when waves travel faster than the local speed of sound, causing an abrupt change in pressure, temperature, and density. Unlike solitons, shock waves dissipate energy and become weaker with distance. They result from the steepening of ordinary waves, and as the steepness increases in plasmas, dispersion effects cause nonlinear deformation. Shock waves are found in both collisional and collisionless plasmas, with collisionless shocks being common in dilute hot gases or fully ionized plasmas. Research on collisionless shock waves, first discovered in 1965, has applications in astrophysics and laboratory plasma physics [1],[2], [54–57].

1.4 PLASMA COUPLING

Plasma coupling refers to the strength of interactions between particles within a plasma, affecting its behavior. The degree of coupling determines whether the plasma behaves more like a gas, liquid, or solid. The coupling is quantified by the plasma coupling parameter, which is the ratio of potential energy (due to Coulomb interactions) to kinetic energy (due to thermal motion). This parameter is given by:

$$\Gamma = \frac{Z^2 e^2}{4\pi\epsilon_0 a_0 k_B T},$$

where, $Z, e, \epsilon_0, a_0, k_B$ and T are the ion charge state, fundamental charge, vacuum permittivity, inter particle spacing, Boltzmann's constant and temperature, respectively. This parameter helps categorize plasmas as either weakly or strongly coupled, significantly impacting plasma behavior and wave dynamics.

Weakly Coupled Plasma: When $\Gamma < 1$, the plasma is considered weakly coupled or ideal. In this state, the interactions between particles are relatively weak compared to their kinetic energy, and the plasma behaves more like a gas [44],[45]. The plasma particles move freely, and collisions between particles are relatively infrequent. The plasma tends to exhibit behaviors similar to an ideal gas, and the Debye shielding (where charged particles are shielded by surrounding particles) is effective in preventing long-range interactions.

Strongly Coupled Plasma: For $\Gamma > 1$, interactions dominate over thermal motion, causing the plasma to behave like a liquid or solid [44],[45]. This results in collective behavior, such as crystallization or solitons, and nonlinear plasma waves. Strong coupling is seen in dense astrophysical objects, laboratory plasmas, and dusty plasmas. Strong coupling is key to understanding phenomena in high-density environments, such as stellar interiors and certain laboratory conditions, and for studying complex wave behaviors like solitons and shocks.

1.5 HYBRID VELOCITY DISTRIBUTION

The Velocity Distribution Function (VDF) provides a statistical description of plasma particles' velocities, including electrons, ions, and neutrals. By analyzing the VDF's shape and parameters, researchers can determine key plasma properties such as temperature, mean velocity, and velocity spread, which are crucial for understanding plasma dynamics and energy distribution. The VDF is used to extract important plasma parameters, like temperature, density, and flow velocity, which are essential for modeling and predicting plasma behavior in applications such as fusion reactors, space plasmas, and industrial plasma processing.

Plasma waves and instabilities arise from collective interactions between particles, which are influenced by the VDF. By studying the VDF, researchers can identify and analyze plasma waves such as Langmuir waves, ion acoustic waves, and electron cyclotron waves. Understanding the relationship between the VDF and plasma waves is essential for investigating instabilities and controlling plasma behavior. In summary, the VDF plays a crucial role in characterizing plasma kinetics, determining plasma parameters, studying waves and instabilities, predicting transport processes, and designing plasma diagnostics. A thorough understanding of the VDF is vital for advancing plasma physics and developing plasma-based technologies.

The distribution functions $f(x, v, t)$ are key components of plasma systems, describing the spatial and velocity distributions of particles. These functions are outlined in several references [46–48], [58]. Plasma particle velocity distributions often deviate from the Maxwellian distribution, particularly in Space and Astrophysical Environments (SAEs) where long-range interactions like gravitational or Coulomb forces occur. These

interactions lead to non-thermal behaviors, such as subthermality and superthermality, and the particles may exist in different phase spaces [59]. In such environments, energetic plasma particles often show deviations from Maxwellian distribution, as demonstrated by numerous studies. This deviation can be described using nonextensive statistics, introduced by Tsallis [47],[58], which generalizes the Boltzmann-Gibbs statistics to account for systems far from equilibrium exhibiting power-law distributions. Tsallis entropy has gained significant attention for its relevance to various astrophysical and space-related phenomena, such as dark matter halos, the upper ionosphere of Mars, the vicinity of the Moon, and the magnetospheres of Jupiter and Saturn [11],[60–62].

Additionally, observations from the Viking spacecraft [63] and the Freja satellite [64] have detected electrostatic wave structures in the magnetosphere, revealing density depressions. Cairns et al. [48] proposed a distribution model to explain the existence of rarefactive ion-acoustic (IA) structures, which were inconsistent with previous observations of energetic tails. Various studies [65–71] have explored different types of IA waves, assuming either electrons, ions, or both to exhibit nonextensive or nonthermal behavior due to the influence of long-range interactions. By the mixture of Tsallis and Cairns models, Tribeche et al. [72] have proposed the following hybrid Tsallis–nonthermal velocity distribution function for electrons:

$$f_e(V_x) = C_{q,\alpha} \left(1 + \alpha \frac{V_x^4}{V_{te}^4} \right) \left[1 - (q-1) \frac{V_x^2}{2V_{te}^2} \right],$$

where $V_{te} = \sqrt{T_e/m_e}$ is the thermal velocity of electrons, q is the strength of nonextensivity, α is measuring the population of nonthermal electrons and $C_{q,\alpha}$ is the normalized constant (see details in Ref. [72]). They have provided that the hybrid distribution, so-called (α, q) –velocity distribution is a better fit of the space observations due to the flexibility provided by the nonextensivity parameter. They have also mentioned that nonthermality and nonextensivity may act concurrently on the nature (rarefactive or compressive) of the IA structures. Latter, the proper ranges of the nonextensive and nonthermal parameters are defined in Refs. [73–76] subject to the physical cutoff imposed the monotonicity conditions $q \geq 5/7$ and $\alpha_{max} = (2q-1)/4$ for nonthermal-nonextensive case. Hence, one can use the (α, q) –velocity distribution to examine the physical issues, such as particle

transport, wave-particle interactions, and energy dissipation observed in many SAEs by considering the appropriate ranges of α and q , that is, for (i) isothermal ($q \rightarrow 1$ and $\alpha = 0$), (ii) superthermal ($\alpha = 0$, $0.33 < q < 1$) and (iii) subthermal ($\alpha = 0$, $q > 1$) but also (iv) nonthermal ($q \rightarrow 1$ and $0 < \alpha < 0.25$) lighter plasma particles [73–76].

1.6 FLUID MODEL EQUATIONS

To study acoustic wave phenomena, one derives the continuity and momentum equations based on mass and momentum conservation laws. The plasma environment is bounded by Poisson’s equation, derived from Maxwell’s equations, since plasma particles are connected to electric or electromagnetic fields. Detailed derivations of the fluid model equations, such as continuity, momentum, Poisson’s equation, and the equation of state, are available in the literature [1–4], [44],[45]. This section highlights limitations of previous models through a specific example and emphasizes the need for further research to refine or propose new models. On the other hand, [57], [67], [77–95] explored acoustic wave phenomena using fluid model equations in weakly or strongly coupled plasma regimes. Find past from like [81], [91], [93–95] studied linear and acoustic waves in strongly coupled (SC) plasmas. However, results in [91],[93–96] may suffer from truncation errors due to incorrect coefficients in nonlinear evolution equations and improper handling of physical variables. Additionally, they ignored the coupling parameter Γ and overlooked nonlinear features of shock and solitary wave excitations around critical values (CVs). Furthermore, there has been no research on strongly coupled plasmas with super-critical values (SCVs) of plasma parameters. Thus, there are many opportunities for further research on fluid model equations in various plasma environments. Based on this, we propose fluid model equations to study acoustic wave propagation in different plasma assumptions, including their relevance in astrophysical environments. The fluid model equations used in this work are described in detail.

The equation of continuity in plasma physics elucidates the conservation principles of mass and charge within the plasma medium. It articulates how the density and velocity of charged particles evolve over time, stipulating that the change in particle density within a defined volume equals the net flux of particles crossing the boundary of that volume. The

mathematical form of equation of continuity is given by

$$\frac{\partial N_s}{\partial t} + \vec{\nabla} \cdot (N_s \vec{U}_s) = 0, \quad (1.1)$$

where N_s ($\vec{U}_s$) is the particle density (velocity) of s -species. Besides, classical fluid theory, represented by the Navier-Stokes equation, does not encompass elastic effects. To address this limitation, Frenkel [97] extended classical hydrodynamic theory to include elastic fluids by replacing the viscosity coefficient with a nonlocal operator that incorporates the elastic properties of fluids. The generalized hydrodynamic equations, which describe the dynamics of a fluid as derived by Frenkel [97] and applied to dense plasmas by Kaw and Sen [81], can be formulated for the momentum equation as follows:

$$\left(1 + \tau_R \frac{\partial}{\partial t}\right) \left[m_s N_s \left(\frac{\partial}{\partial t} + \vec{U}_s \cdot \vec{\nabla} \right) \vec{U}_s - e N_s \vec{E} + \vec{\nabla} P \right] = \vec{\nabla} \cdot \Pi^0, \quad (1.2)$$

where $\vec{E}$ (P) is the self-consistent electric field (pressure), τ_R is the relaxation time, and Π^0 is the dissipative term. Subsequently, Shukla [98] introduced an additional force intended to depict the polarizing interaction between strongly coupled ions and electrons into the generalized momentum equation. The formulation of the generalized momentum equation proposed by Shukla is as follows:

$$\left(1 + \tau_R \frac{\partial}{\partial t}\right) \left[m_s N_s \left(\frac{\partial}{\partial t} + \vec{U}_s \cdot \vec{\nabla} \right) \vec{U}_s - (1 - R) e N_s \vec{E} + \vec{\nabla} P \right] = \vec{\nabla} \cdot \Pi^0, \quad (1.3)$$

here R represents the polarization force arising from interactions among the thermal electrons.

Finally, the Poisson equation plays a crucial role in elucidating the dynamics of electric fields within plasma systems, revealing the intricate interactions between charged particles and electromagnetic fields in these complex environments. It is essential for describing the self-consistent electric field in plasma alongside the Vlasov equation, which characterizes the distribution function of electrons and ions. This equation provides a mathematical framework to determine the distribution of electric potential within plasma, arising from the collective behavior of charged particles. Mathematically, the Poisson equation is expressed

as:

$$\nabla^2 \psi = e\rho/\epsilon_0, \quad (1.4)$$

where $\vec{E} = -\vec{\nabla}\psi$ represents the electric field, e is the charge of particles, and ρ is the overall charge density, respectively. In this equation, $\nabla^2 \psi$ denotes the Laplacian of the electric potential ψ , representing spatial variations in potential. On the right-hand side, (ρ/ϵ_0) accounts for the charge density ρ normalized by the vacuum permittivity ϵ_0 . This term quantifies how the spatial distribution of charges influences the electric potential.

1.7 REDUCTIVE PERTURBATION TECHNIQUE

Now, the question arises: how can we solve the fluid equations described above? To address this, one can employ either non-perturbative or perturbative methods. Perturbative methods are ideal for studying phenomena with small but precise amplitude acoustic waves, while non-perturbative methods can be applied regardless of the amplitude. Among the perturbative methods, the Reductive perturbation technique (RPT) stands out as particularly useful [99]. The process of deriving the mathematical physics equations using this method is explained in the following description.

To derive many kind of evolution equations from the proposed model equations, especially for (1+1)-dimensional space-time, one needs to define the appropriate coordinate frame and multiple scale expansion via the general principles of RPT. It is noted that the dispersion or dissipation relation for harmonic waves are determined by linearizing Eqs. (1.1), (1.2) and (1.2) along with the solutions as $N_s, U_s, \psi \sim \exp(i\Omega)$, where $\Omega = kz - \omega(k)t$. Without loss of generality, one can define $k = \epsilon^i K$, where K and i are the new wave number and real number, respectively. For this case, one obtains $\Omega(z, t) = \epsilon^i Kz - \omega(\epsilon^i K)t$. However, the purely dispersive/dissipative media cannot has a mixture of both odd and even terms in k in the Taylor expansion of $\Omega(k)$. In the case of purely dissipative media, the even terms in k are only occurred, and the Taylor expansion for $\omega(k)$ provides $\omega(k) = \omega'(0)\epsilon^i K + \omega''(0)\epsilon^{2i}K^2 + \dots$. This implies that $\Omega(z, t) = K\epsilon^i(z - \omega'(0)t) - K^2\omega''(0)\epsilon^{2i}t$. Then one may scale out for z and t as $\zeta = \epsilon^i(z - V_p t)$ and $\tau = \epsilon^{2i}t$ because $\omega'(0)$ and $\omega''(0)$ are constants. Hence, one can easily derive the mathematical physics equations from the fluid model equations by considering the appropriate starching of space-time and

the suitable expansions of perturb quantities, like the following:

$$H = H_0 + \varepsilon H_1 + \varepsilon^2 H_2 + \dots, A = A_0 + \varepsilon A_1 + \varepsilon^2 A_2 + \dots,$$

where $H_1, H_2, \dots, A_1, A_2, \dots$ are differentiable with respect to space and time coordinates up to p -times and H_0 , and A_0 are constants that depend on the initial assumption and charge neutrality conditions.

1.8 THESIS OUTLINE

After introducing and providing an overview in this chapter, a brief description of remaining chapters of the thesis is outlined below:

In Chapter 2, the Burgers equations involving quadratic, cubic, and both quadratic and cubic nonlinearity are derived by assuming the appropriate stretching to describe the propagation of electrostatic shock and oscillatory waves in a coupled complex plasma having Boltzmann-distributed electrons, hybrid Tsallis-nonthermal velocity distributed ions and negatively charged dust grains. In the long-wave approximation, the dynamics of shock wave excitations are governed by these equations with the presence of viscosity coefficient. The generalized Riccati equation mapping method is implemented to determine not only the shocks but also oscillatory waves of these obtained equations. The parametric effect on characteristic of shock waves and oscillatory waves (amplitude, width, polarity, etc.) is described.

In Chapter 3, the propagation of heavy ion-acoustic shock waves (HIASWs) are studied in an unmagnetized plasma having inertial positively charged heavy ions (HIs) fluid and inertia-less (α, q) -distributed electrons and Maxwellian light ions (MLIs). To examine such phenomena, the Burgers equation (BE), modified Burgers equation (mBE), and mixed mBE (mmBE) are derived without considering the stretching of viscosity coefficient of HIs via the reductive perturbation technique (RPT). In addition, the appropriate stationary shock wave solutions of the nonlinear equations, particularly, mBE and mmBE is formulated. With the changes of plasma parameters, the basic features of HIASWs are examined.

In Chapter 4, the impact of polarization force and coupling parameter, along with other plasma parameters, on the nonlinear propagation of dust acoustic (DA) shocks

is presented in an unmagnetized collisionless strongly coupled dusty plasma having (α, q) -distributed electrons, Maxwellian ions, and charged dust grains fluid. The Burgers equation with quadratic nonlinearity is derived by implementing the reductive perturbation method and it is hence found that DA shocks exist associated with both positive and negative potentials. However, the nonlinear coefficient of Burgers equation with quadratic nonlinearity approaches zero for the critical values (CVs) of some specific parameters. To report the propagation of DA shocks around the CVs, the Burgers equation with cubic nonlinearity is derived by taking the higher-order power of the implemented method. Additionally, the Burgers equation with quartic nonlinearity is derived, which yields the existence of super-critical values (SCVs) of some specific parameters. As a result, progress in understanding the propagation of DA shocks around CVs as well as SCVs is reported.

Chapter 5 investigates the nonlinear propagation of the heavy ion-acoustic (HIA) soliton and dressed soliton in an unmagnetized weakly and strongly coupled plasma having heavy ions fluid, hybrid Tsallis-nonthermal-distributed electrons and Maxwellian lighter ions. To do so, the Korteweg-de Vries (KdV) and inhomogeneous KdV-like equations are derived by implementing the reductive perturbation technique. The characteristics of HIA dressed solitons are presented in this chapter.

Chapter 6 investigates the effects of plasma parameters on shock and periodic waves, along with their dynamic behaviors, in unmagnetized strongly coupled plasmas (SCPs), particularly with the presence of super-critical values (SCVs). To achieve this, the research extends the Burgers equation (BE) by incorporating higher-order nonlinear terms to explore the nonlinear characteristics of heavy ion acoustic (HIA) shock waves and periodic waves, along with their dynamics in SCPs. First, a BE with quartic nonlinearity is derived, which is used to analyze the propagation features of HIA shock waves near and at critical values (CVs) and SCVs. Based on the analytical solutions, the effects of plasma parameters on electrostatic shocks, anti-shocks, and their dynamic behaviors are examined. Second, the study investigates the propagation features of shock and periodic waves and their dynamics, not only near but also at SCVs, by deriving a new nonlinear evolution equation that incorporates both cubic and quartic nonlinearities. Since integrable nonlinear evolution equations are often solvable analytically, the research also assesses the need for numerical methods in studying acoustic wave phenomena in such plasmas. This approach

provides a comprehensive understanding of both the theoretical and practical aspects of wave propagation in SCPs. The findings of this study may aid in understanding and predicting the behavior of HIA wave dynamics in space environments, such as stellar polytropes, hadronic matter, quark-gluon plasma, dark-matter halos, and contribute to future experimental studies in plasma laboratories.

Chapter 7 concludes with final remarks and explores potential future avenues for investigating acoustic wave phenomena further.

Chapter 2: PROPAGATION OF DUST ACOUSTIC SHOCKS AND OSCILLATORY WAVES IN A COUPLED COMPLEX PLASMA DESCRIBED BY NONLINEAR EVOLUTION EQUATIONS

2.1 INTRODUCTION

Nonlinear evolutions equations (NLEEs) are one kinds of partial differential equations that are widely used to describe the propagation characteristics of wave phenomena in various environments of modern sciences and engineering. To illustrate it, the dust-acoustic waves (DAWs) are supported in strongly and weakly coupled dusty plasmas (CDPs) as observed in many space and astrophysical environments (SAEs)[2],[44],[81–83],[100–105] as well as in plasma laboratory (PL) [106],[107]. As a result, huge amount of researchers [67],[81–83],[85–90],[105] have investigated the basic features of linear and nonlinear characteristics of DAWs in such plasmas. Rao et al. [85] are the pioneer to provide theoretically the production of DAWs. Sukla and Mamun [86] have reported the soliton propagation by formulating Korteweg-de Vries (KdV) equation, while the shock waves (SWs) phenomena are reported by formulating KdV-Burgers equation with unrealistic manner in strongly CDPs. Because, the viscosity coefficient (VC) is stretched as $\eta = O(\varepsilon^{1/2})$ to derive KdV-Burgers (KdVB) equation (see in Ref. [86]). Mamun et al. [87] have studied the SW phenomena in strongly CDPs having the non-isothermal vortex-like ion distribution, Maxwellian electron distribution and strongly correlated grains in a liquid like-state. Ghosh [88] has been investigated the arbitrary SW characteristics in the regime of kinetic strongly CDPs. He has also been reported the influence of deferred charging dust gain on DAWs. Lin et al. [90] have investigated the DAWs with the presence of nonthermal ions (NIs). Pakzad and Javidan [91] have examined the nonlinear soliton and SW propagation in strongly CDPs having NIs dust gains. To do it, they are used $\eta_1 = O(\varepsilon^{1/2})$ for VC and $\tau_m = O(\varepsilon^{1/2})$ for viscoelastic relaxation time (VRT) by the unscientific manner. Since, Mamun [108] has been clearly mentioned that no one can expand the unperturbed quantities by introducing the proper fraction. It is therefore

important to do further research work on DAW propagation with the realistic concepts in CDPs with the presence not only isothermal, subthermal and superthermal but also nonthermal interialess charged particles.

However, no research work has been done previously to report the DA SWs propagation in CDPs not only around critical values (ACVs) or at the composition of CVs of a specific parameter, e.g. density ratio (DR), temperature ratio (TR), strength of nonextensivity (SN), nonthermatity index, etc. To overcome the aforementioned complexities, one needs to modify the starching coordinates for formulating NLEEs involving higher order nonlinearity (HON). Also, one needs to determine the appropriate solutions of NLEEs involving HON by using the suitable mathematical methods [109–116]. The generalized Riccati equation mapping method (GREMM) [115],[116] is one of suitable techniques that can be easily applicable to determine the analytical solutions of NLEEs. Very recently, researchers [117],[118] have only reported the propagation of heavy ion solitons, dressed solitons and SWs in coupled plasmas with the appropriate solutions of NLEEs involving HON. However, the problem studied herein by considering CDP having hybrid Tsallis nonthermal (HTN)-distributed ions. Thus, the work highlights the following by considering an unmagnetized CDP having Boltzmann distributed electrons (BDEs), HTN velocity distributed ions and negatively charged dust grains (NCDGs) in this chapter:

- (i) The derivation the NLEEs involving HON by considering the appropriate stretching.
- (ii) The determination of the useful exact solutions of NLEEs involving HON by implementing the GREMM.
- (iii) A novel investigation on DA SWs not only ACVs but also at the composition of CV in the considered CDP, where the superthermal/subthermal/isothermal/ nonthermal ions, BDEs, and NCDGs dynamics play an important role.
- (iv) The effect of plasma parameters on the propagation of electrostatic DA SWs.

2.2 MODEL EQUATIONS

Let us consider an unmagnetized CDP having BDEs, HTN velocity distributed ions and NCDGs, where the ions and electrons are assumed to be weakly coupled rather than to the dust grains. Based on the above assumption, the continuity and momentum equations are

obtained as follows [91]:

$$\frac{\partial N_d}{\partial t} + \frac{\partial}{\partial x}(N_d U_d) = 0, \quad (2.1)$$

$$\left(1 + \tau_m \frac{\partial}{\partial t}\right) \left[N_d \left\{ \left(\frac{\partial}{\partial t} + U_d \frac{\partial}{\partial x} \right) U_d - \frac{\partial \psi}{\partial x} \right\} \right] = \mu \frac{\partial^2 U_d}{\partial x^2}, \quad (2.2)$$

where

$$\tau_m = \mu \frac{T_e}{T_d} [1 - \mu_d + \frac{4}{15} U(\Gamma)]^{-1}, \mu_d = \left[1 + \frac{1}{3} U(\Gamma) + \frac{\Gamma}{9} \frac{\partial U(\Gamma)}{\partial \Gamma} \right]. \quad (2.3)$$

The plasma system is closed based on the equilibrium condition by the following

$$\frac{\partial^2 \psi}{\partial x^2} = N_d + \frac{\rho_{ei}}{1 - \rho_{ei}} \exp(\sigma_{ie} \psi) - \frac{1}{1 - \rho_{ei}} [1 - (q - 1) \psi]^{\frac{(q+1)}{2(q-1)}} \times [1 - \Sigma_1 \psi + \Sigma_2 \psi^2] = -\rho, \quad (2.4)$$

where the following HTN velocity distribution function [72] is used for describing the motion of NIs:

$$f_e(v_x) = C_{q,\alpha} \left(1 + \alpha \frac{v_x^4}{V_{te}^4} \right) \left[1 - (q - 1) \frac{v_x^2}{2V_{te}^2} \right], \quad (2.5)$$

In the above equations, N_d (number density of dust particles), U_d (velocity of the dust fluid) and ψ (electrostatic wave potential) are normalized by N_{d0} (unperturbed dust particle number density), $C_d = (Z_d T_i / m_d)^{1/2}$ (dust acoustic speed) and T_i / e , respectively, $U(\Gamma)$ is applicable to measure the excess internal energy, τ_m (VRT) is normalized $\omega_{dp}^{-1} = (m_d / 4\pi N_{d0} Z_d e^2)^{1/2}$ (dust plasma period) q is a parameter measuring the nonextensivity of ions, α is a parameter indicating the number of fast (nonthermal) ions, and $\mu = \left(1 / m_d N_{d0} \omega_{dp} \lambda_{dp}^2 \right) [\mu_b + (4/3) \zeta_b]$ (normalized viscosity coefficient), , where T_i , e , Z_d , m_d , λ_{dp} , μ_b and ζ_b are the ions temperature, electron charge, charged number of dust particles, mass of dust particles, dust Debye length, transport coefficients of shear and bulk viscosities, respectively. Due to the normalization of Eqs. (2.1), (2.2), and (2.4) and maintaining of charge neutrality condition, $\sigma_{ie} = T_i / T_e$ (T_e is the electrons temperature) and $\rho_{ei} = N_{e0} / N_{i0}$ (N_{e0} and N_{i0} are the unperturbed densities of electrons and ions, respectively) are obtained.

2.3 DESCRIPTION OF THE GREMM

NLEEs involving quadratic, cubic and composition of quadratic and cubic nonlinearity can be defined in the following form:

$$f\left(\psi^{(1)}, \frac{\partial \psi^{(1)}}{\partial \tau}, \frac{\partial \psi^{(1)}}{\partial \xi}, \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}\right) = 0. \quad (2.6)$$

By using the transformation $\psi^{(1)}(\xi, \tau) \sim \psi^{(1)}(\zeta)$, where $\zeta = \xi - s_0 \tau$, Eq. (2.6) can be converted to

$$F\left(\psi^{(1)}, \frac{d\psi^{(1)}}{d\zeta}, \frac{d^2\psi^{(1)}}{d\zeta^2}\right) = 0. \quad (2.7)$$

Based on the GREMM, the exact solution of Eq. (2.7) can be defined in a series as

$$\psi^{(1)} = \sum_{r=0}^N h_r (\phi^{(1)})^r \quad (2.8)$$

where N can be evaluated by balancing highest order derivative and nonlinear term of Eq. (2.7) and $\phi^{(1)}$ satisfies the following auxiliary equation:

$$\frac{d\phi^{(1)}}{d\zeta} = k_0 + k_1 \phi^{(1)} + k_2 (\phi^{(1)})^2. \quad (2.9)$$

By fulfilling the constraint conditions based on the values of k_0 , k_1 , and k_2 , Eq. (2.9) yields various types of solutions. The constraint conditions and the twenty seven solutions of Eq. (2.9) are given in Ref. [115],[116]. In this work, two useful solutions of Eq. (2.7) are only considered for simplicity and given below [115],[116]:

When $\Delta = k_1^2 - 4k_0k_2 > 0$ and $k_1k_2 \neq 0$ or $k_0k_2 \neq 0$, then

$$\phi^{(1)} = -\frac{1}{2k_2} \left[k_1 + \sqrt{\Delta} \tanh\left(\frac{\sqrt{\Delta}}{2}\zeta\right) \right]. \quad (2.10)$$

When $\Delta = k_1^2 - 4k_0k_2 < 0$ and $k_1k_2 \neq 0$ or $k_0k_2 \neq 0$, then

$$\phi^{(1)} = \frac{1}{2k_2} \left[-k_1 + \sqrt{-\Delta} \tan\left(\frac{\sqrt{-\Delta}}{2}\zeta\right) \right]. \quad (2.11)$$

To determine the useful solutions, one needs to consider the the particular values of k_0 , k_1 and k_2 by fulfilling the constraint conditions. The above two solutions are represented shocks and oscillatory waves, respectively and very useful for laboratory verification.

2.4 BURGERS EQUATION INVOLVING QUADRATIC NONLINEARITY

Based on the reductive perturbation technique, one may scale out for x and t (see details in [67],[119]) as

$$\xi = \varepsilon^r(x - s_p t), \tau = \varepsilon^{2r} t \quad (2.12)$$

and the expansions of unknown variables are assumed as

$$\begin{bmatrix} N_d \\ U_d \\ \psi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \sum_i^\infty \varepsilon^i \begin{bmatrix} N_d^{(i)} \\ U_d^{(i)} \\ \psi^{(i)} \end{bmatrix}, \quad (2.13)$$

where s_p and ε are the linear phase speed and a proper fraction quantity measuring the weakness of dissipation. Implementing Eq. (2.12) and (2.13) into Eqs. (2.1), (2.2), and (2.4) along with $r = 1$ in a systematic way, one can formulate the set of equations by depending on the several power of ε .

For $O(\varepsilon^2)$, yields

$$N_d^{(1)} = -\frac{1}{s_p^2} \psi^{(1)}, U_d^{(1)} = -\frac{1}{s_p} \psi^{(1)}, s_p = \sqrt{\frac{1 - \rho_{ei}}{\rho_{ei} \sigma_{ie} + \frac{q+1}{2} - \frac{16q\alpha}{3-14q+15q^2+12\alpha}}}. \quad (2.14)$$

For $O(\varepsilon^3)$, yields

$$-\frac{1}{s_p^2} \frac{\partial \psi^{(1)}}{\partial \tau} - s_p \frac{\partial N_d^{(2)}}{\partial \xi} + \frac{\partial U_d^{(2)}}{\partial \xi} + \frac{2}{s_p^3} \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} = 0, \quad (2.15)$$

$$-\frac{1}{s_p} \frac{\partial \psi^{(1)}}{\partial \tau} + \frac{\partial \psi^{(2)}}{\partial \xi} - s_p \frac{\partial U_d^{(2)}}{\partial \xi} + \frac{1}{s_p^2} \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} = \frac{\mu}{s_p} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \quad (2.16)$$

$$0 = h_1 \frac{\partial \psi^{(2)}}{\partial \xi} + 2h_2 \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} + \frac{\partial N_d^{(2)}}{\partial \xi}, \quad (2.17)$$

where

$$\begin{aligned} h_1 &= \frac{1}{1 - \rho_{ei}} \left[\rho_{ei} \sigma_{ie} + \frac{q+1}{2} - \frac{16q\alpha}{3 - 14q + 15q^2 + 12\alpha} \right], \\ h_2 &= \frac{1}{1 - \rho_{ei}} \left[\frac{\rho_{ei} \sigma_{ie}^2}{2} - \frac{(q+1)(3-q)}{8} + \frac{8q(q+1)\alpha}{3 - 14q + 15q^2 + 12\alpha} \right] \\ &\quad - \frac{1}{1 - \rho_{ei}} \left[\frac{16q(2q-1)\alpha}{3 - 14q + 15q^2 + 12\alpha} \right]. \end{aligned} \quad (2.18)$$

Simplifying Eqs. (2.15)–(2.17), the following Burgers equation is derived:

$$\frac{\partial \psi^{(1)}}{\partial \tau} + P \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} = Q \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \quad (2.19)$$

where

$$P = -\frac{3}{2s_p} - h_2 s_p^3, Q = \frac{\mu}{2}. \quad (2.20)$$

The useful exact SW solution of Eq. (2.19) is defined based on the GREMM as

$$\psi^{(1)} = A_0 \left\{ 1 - \tanh \left(\frac{\zeta}{W_0} \right) \right\}, \quad (2.21)$$

where $A_0 = (s_0/P)$ and $W_0 = (2Q/s_0)$ are respectively the amplitude and width of DA SWs and $\zeta = \xi - s_0 \tau$ (s_0 is the constant velocity of reference frame). Whereas, the useful exact oscillatory wave solution of Eq. (2.19) is obtained as

$$\psi^{(1)} = A_0 \left\{ 1 + \tan \left(\frac{\zeta}{W_0} \right) \right\}, \quad (2.22)$$

which is in good agreement with the earlier investigation [119]. It is predicted that Eq. (2.19) yields the positive and negative electrostatic shocks by depending on $P > 0$ and $P < 0$, respectively. But, the Burgers equation involving quadratic nonlinearity is not applicable to describes propagation of shocks when $P \rightarrow 0$, which beaks down the validity of the reductive perturb method. Such situation arises only for some values of a specific parameter, that is so-called the critical values (CVs). One can easily evaluate CVs say α_c by setting $P \sim 0$ via the computational software like Maple along with the fixed values of other plasma parameters within the given ranges.

2.5 BURGERS EQUATION INVOLVING CUBIC NONLINEARITY

To examine the nature of DA SWs ACVs, the old variables can be stretched by taking the higher-order correction of perturbative approach, that is $r = 2$. For $r = 2$, the $O(\epsilon^3)$ equations yield the similar solutions as in Eq. (2.14).

For $O(\epsilon^4)$, one obtains

$$N_d^{(2)} = -\frac{1}{s_p^2} \psi^{(2)} + \frac{3}{2s_p^4} \left\{ \psi^{(1)} \right\}^2, U_d^{(2)} = -\frac{1}{s_p} \psi^{(2)} + \frac{1}{2s_p^3} \left\{ \psi^{(1)} \right\}^2 \quad (2.23)$$

and

$$-\left[\frac{3}{2s_p^4} + h_2 \right] \times \left\{ \psi^{(1)} \right\}^2 = \frac{1}{s_p^3} \times P \times \left\{ \psi^{(1)} \right\}^2 = 0. \quad (2.24)$$

Finally, the $O(\epsilon^5)$ yields

$$\frac{1}{s_p^2} \frac{\partial \psi^{(1)}}{\partial \tau} + s_p \frac{\partial N_d^{(3)}}{\partial \xi} - \frac{\partial U_d^{(3)}}{\partial \xi} + \frac{6}{s_p^5} \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} - \frac{2}{s_p^3} \frac{\partial}{\partial \xi} \left[\psi^{(1)} \psi^{(2)} \right] = 0, \quad (2.25)$$

$$\frac{1}{s_p} \frac{\partial \psi^{(1)}}{\partial \tau} - s_p \frac{\partial U_d^{(3)}}{\partial \xi} + \frac{\partial \psi^{(3)}}{\partial \xi} + \frac{3}{2s_p^4} \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} - \frac{1}{s_p^2} \frac{\partial}{\partial \xi} \left[\psi^{(1)} \psi^{(2)} \right] = \frac{\mu}{s_p} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \quad (2.26)$$

$$0 = h_1 \frac{\partial \psi^{(3)}}{\partial \xi} + 3h_3 \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} + \frac{\partial N_d^{(3)}}{\partial \xi} + 2h_2 \frac{\partial}{\partial \xi} \left[\psi^{(1)} \psi^{(2)} \right], \quad (2.27)$$

where,

$$h_3 = \frac{1}{1 - \rho_{ei}} \left[\frac{\rho_{ei} \sigma_{ie}^3}{6} + \frac{(q+1)(3-q)(5-3q)}{48} - \frac{2q(q+1)(3-q)\alpha}{3-14q+15q^2+12\alpha} \right] + \frac{1}{1 - \rho_{ei}} \left[\frac{8q(q+1)(2q-1)\alpha}{3-14q+15q^2+12\alpha} \right]. \quad (2.28)$$

Simplifying Eqs. (2.25)-(2.27) along with the consideration $P = 0$, the Burgers equation involving cubic nonlinearity is obtained as

$$\frac{\partial \psi^{(1)}}{\partial \tau} + P' \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} = Q \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \quad (2.29)$$

where

$$P' = \frac{15}{4s_p^3} - \frac{3}{2} s_p^3 h_3. \quad (2.30)$$

The exact SW solution of Eq. (2.29) can be written based on the GREMM as

$$\psi^{(1)} = \sqrt{A_1 \left\{ 1 - \tanh \left(\frac{\xi}{W_1} \right) \right\}}, \quad (2.31)$$

where $A_1 = (3s_0/2P')$ and $W_1 = (Q/s_0)$ are respectively the amplitude and width of DA SWs ACVs. Whereas, the exact oscillatory wave solution of Eq. (2.29) can be written as

$$\psi^{(1)} = \sqrt{A_1 \left\{ 1 + \tanh \left(\frac{\xi}{W_1} \right) \right\}}. \quad (2.32)$$

2.6 BURGERS EQUATION INVOLVING BOTH QUADRATIC AND CUBIC NONLINEARITY

To report the nonlinear propagation DA SWs not only ACVs but also at the composition of CVs, one can express the nonlinear coefficient P for CV, say σ_c of any parameter say σ_{ie} as

$$P_0 = \Omega \left(\frac{\partial P}{\partial \sigma_{ie}} \right)_{\sigma_{ie}=\sigma_c} |\sigma_{ie} - \sigma_c| = \Omega \varepsilon K, \quad (2.33)$$

where $|\sigma_{ie} - \sigma_c| \equiv \varepsilon$ (because $|\sigma_{ie} - \sigma_c|$ is small quantity), $K = \left(\frac{\partial P}{\partial \sigma_{ie}} \right)_{\sigma_{ie}=\sigma_c}$ and $\Omega = 1(-1)$ for $\sigma_{ie} > \sigma_c (\sigma_{ie} < \sigma_c)$. As a result, one can reformulate Eq. (2.27) by introducing $\rho^{(2)} \equiv -\varepsilon^3 \Omega \frac{1}{2} K \left\{ \psi^{(1)} \right\}^2$ in Eq. (2.4) as

$$0 = h_1 \frac{\partial \psi^{(3)}}{\partial \xi} + \Omega K \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} + 3h_3 \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} + \frac{\partial N_d^{(3)}}{\partial \xi} + 2h_2 \frac{\partial}{\partial \xi} \left[\psi^{(1)} \psi^{(2)} \right]. \quad (2.34)$$

Eliminating the higher order perturb quantities from Eqs. (2.25), (2.26), and (2.34), the following Burgers equation involving both quadratic and cubic nonlinearity is obtained:

$$\frac{\partial \psi^{(1)}}{\partial \tau} + \Omega Q' \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} + P' \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} = Q \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \quad (2.35)$$

where

$$Q' = \frac{s_p}{2} K. \quad (2.36)$$

The above equation is validated to study propagation of electrostatic DA SWs not only ACVs but also at the composition of CVs. The correct exact SW solution of the above type of equation is recently provided in Ref. [117]. Thus, the useful stationary SW solution of Eq. (2.35) can be written as

$$\psi^{(1)} = \sqrt{A_2 \left\{ 1 - \tanh \left(\frac{\zeta}{W_2} \right) \right\}} - \frac{\Omega Q'}{2P'}, \quad (2.37)$$

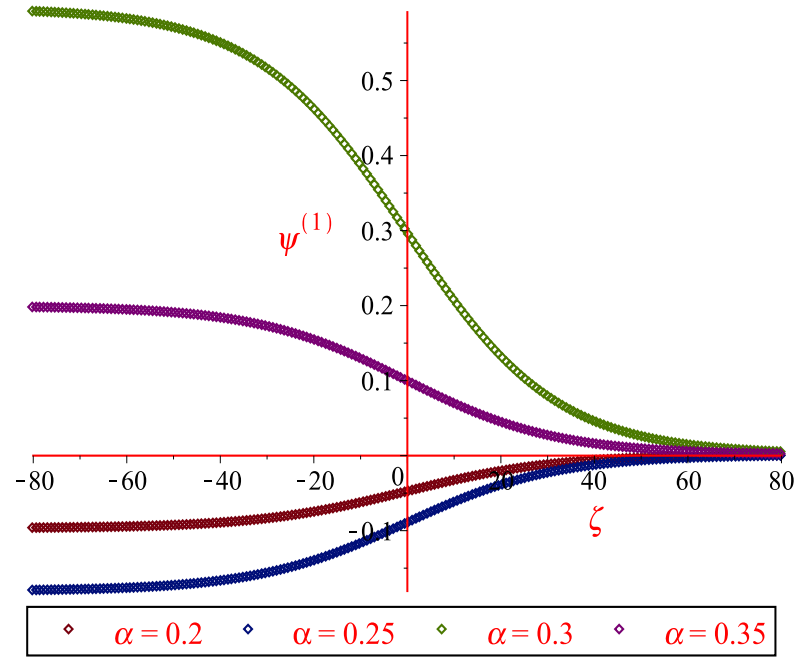
where $A_2 = \frac{3}{2P'}(s_0 + \frac{\Omega^2(Q')^2}{4P'})$ and $W_2 = \frac{Q}{2(s_0 + \frac{\Omega^2(Q')^2}{4P'})}$ are the amplitude and width of shocks not only ACVs but also at the composition of CVs. Whereas, the exact oscillatory wave of Eq. (2.35) can be written based on the GREMM as

$$\psi^{(1)} = \sqrt{A_2 \left\{ 1 + \tanh \left(\frac{\zeta}{W_2} \right) \right\}} - \frac{\Omega Q'}{2P'}. \quad (2.38)$$

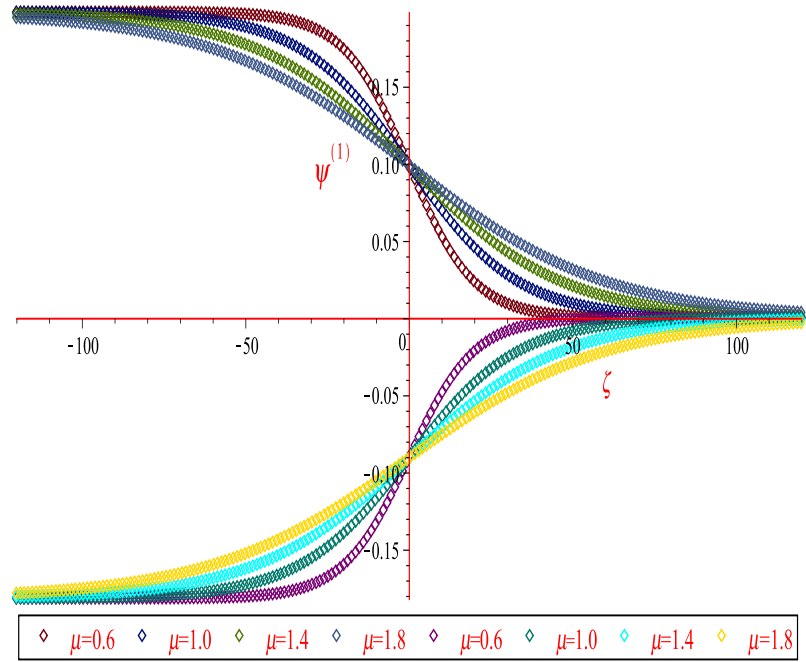
2.7 RESULTS AND DISCUSSION

To study whether or not the electrostatic DA SWs and oscillatory waves of positive and negative polarity and double layer support, a CDP having HTN velocity distributed ions, BDEs, and NCDGs has been assumed. To archive our goals, the Burgers equations having quadratic, cubic and both of quadratic and cubic nonlinearity have been formulated. The effect of α (nonthermality index), q (nonextensivity index), ρ_{ei} (electron to ions density ratio), σ_{ie} (ion to electron temperature ratio), μ (viscosity coefficient) and s_0 (constant speed of reference frame) on the electrostatic DA SWs and double layer are discussed below. In the presented outcomes, the values of parameters are considered within the proper ranges.

The Figure 2.1 presents the effect of α and μ on the propagation of DA shocks described by Burgers equation involving quadratic nonlinearity without considering nonextensivity, whereas the Figure 2.2 presents the changes of the amplitudes of DA shocks described by Burgers equation involving quadratic nonlinearity with regards to ρ_{ie} and σ_{ie} along with the influences of q only. The Figure 2.3 presents the changes of DA oscillatory wave described by Burgers equation involving quadratic nonlinearity with regards to α and μ by

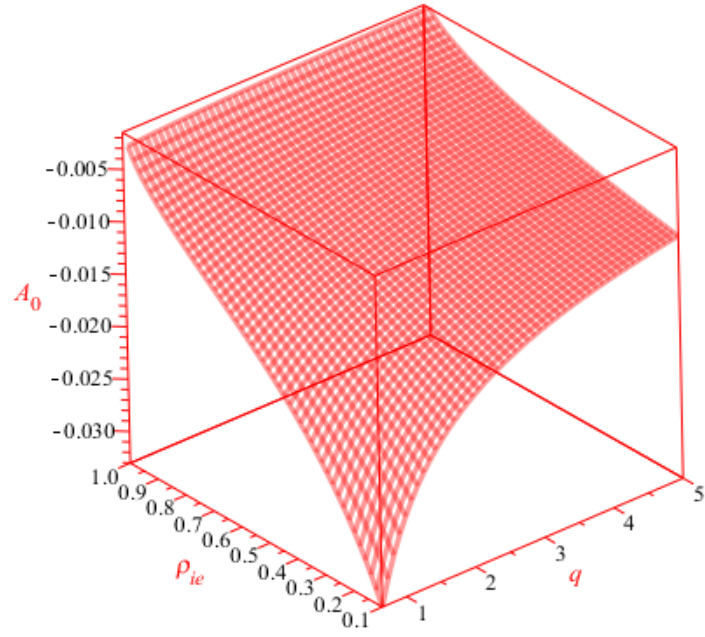


(a)

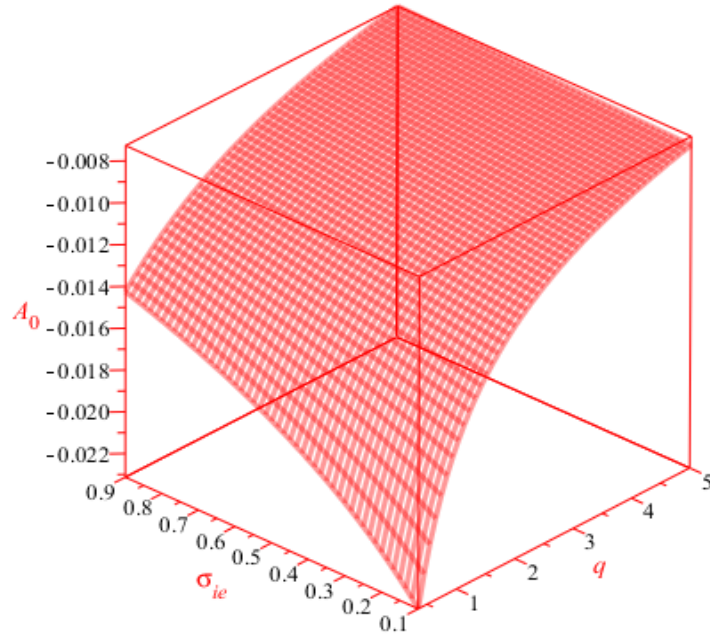


(b)

Fig. 2.1 The variation of DA SWs potential ($\psi^{(1)}$) for different values of (a) α with $\rho_{ie} = 0.4$, $\sigma_{ie} = 0.2$, $\mu = 0.96$, $s_0 = 0.03$, $q = 1$, and (b) μ with $\rho_{ie} = 0.4$, $\sigma_{ie} = 0.2$, $\alpha = 0.35$, $s_0 = 0.03$, $q = 1$.

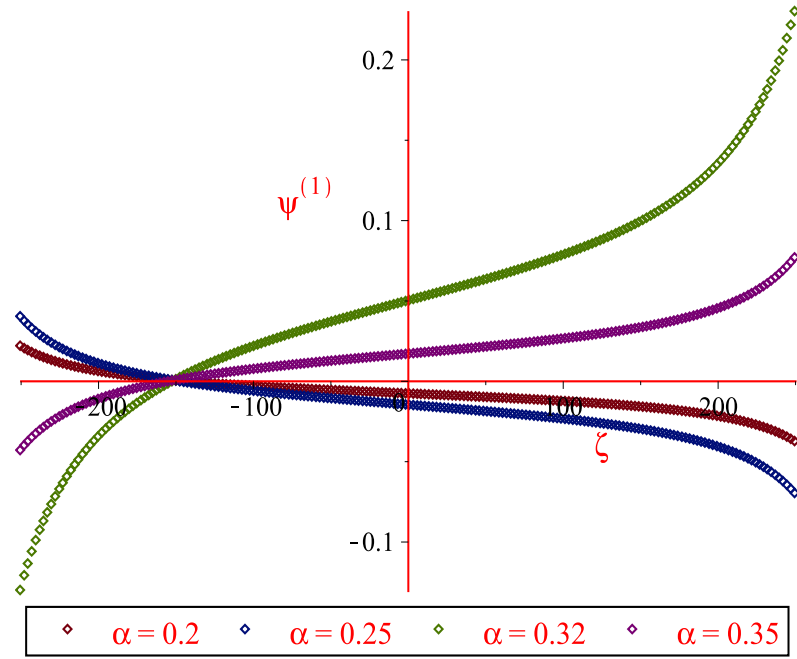


(a)

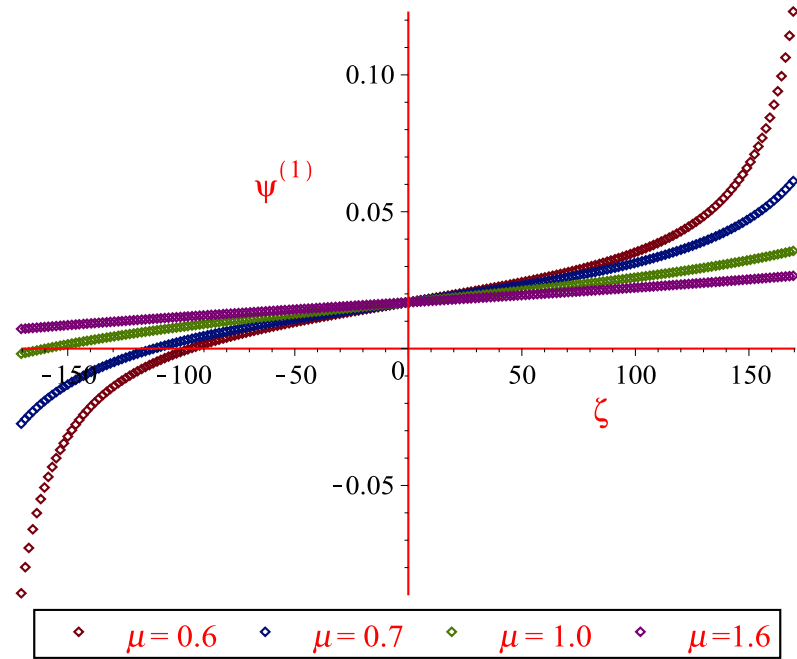


(b)

Fig. 2.2 The variation of amplitude of DA SWs potential ($\psi^{(1)}$) with regard to (a) q and ρ_{ie} with $\sigma_{ie} = 0.4$, and (b) q and σ_{ie} with $\rho_{ie} = 0.5$. The other parameters are considered as $\alpha = 0$, and $s_0 = 0.03$.



(a)



(b)

Fig. 2.3 The variation of DA oscillatory waves ($\psi^{(1)}$) for different values of (a) α with $\rho_{ie} = 0.4$, $\sigma_{ie} = 0.2$, $\mu = 0.96$, $s_0 = 0.005$, $q = 1$, and (b) μ with $\rho_{ie} = 0.4$, $\sigma_{ie} = 0.2$, $\alpha = 0.35$, $s_0 = 0.005$, $q = 1$.

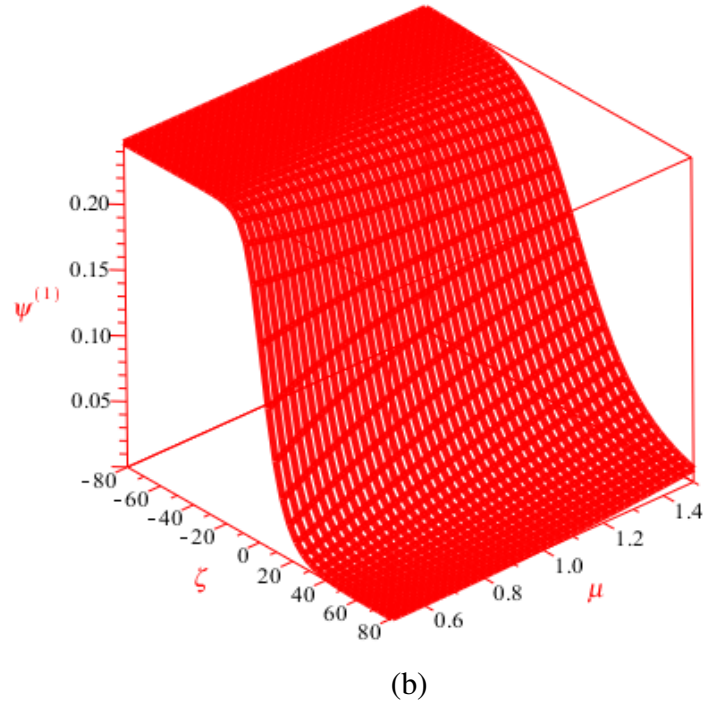
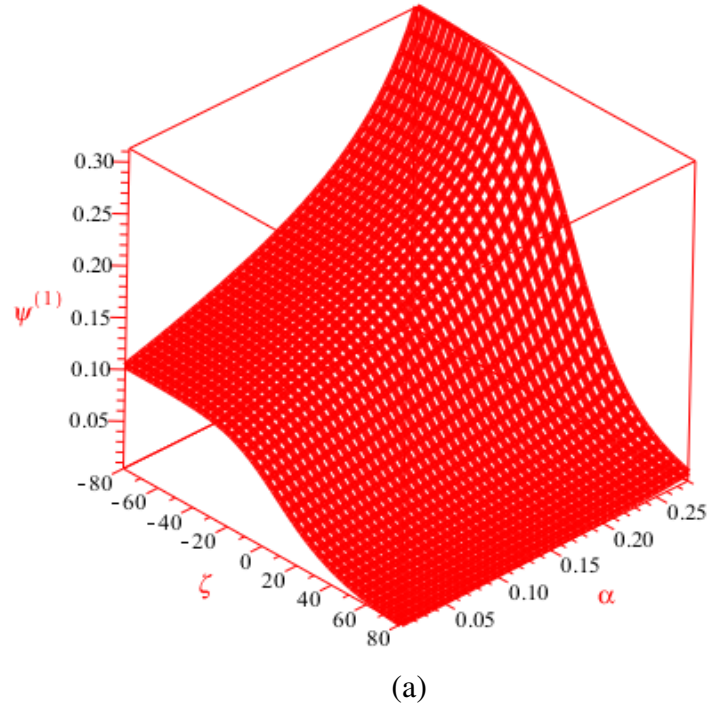
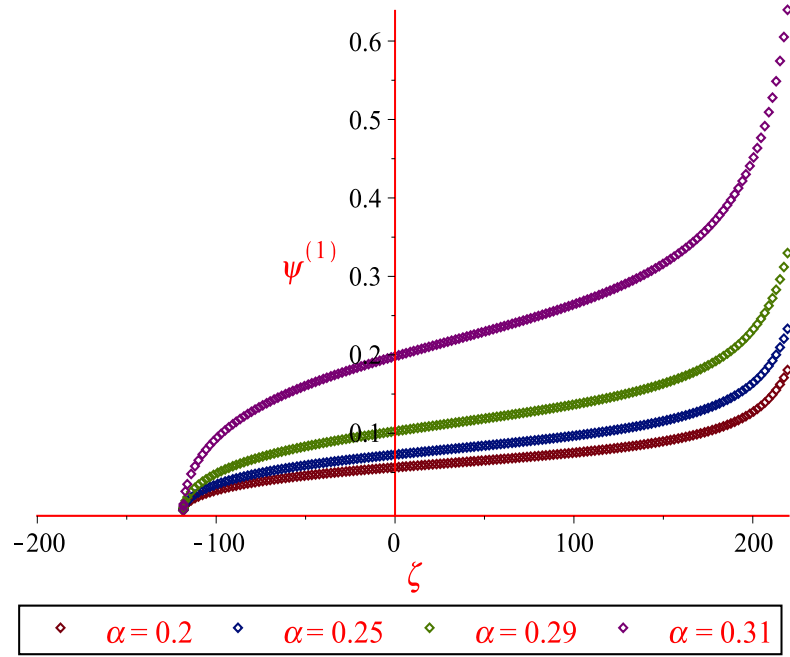
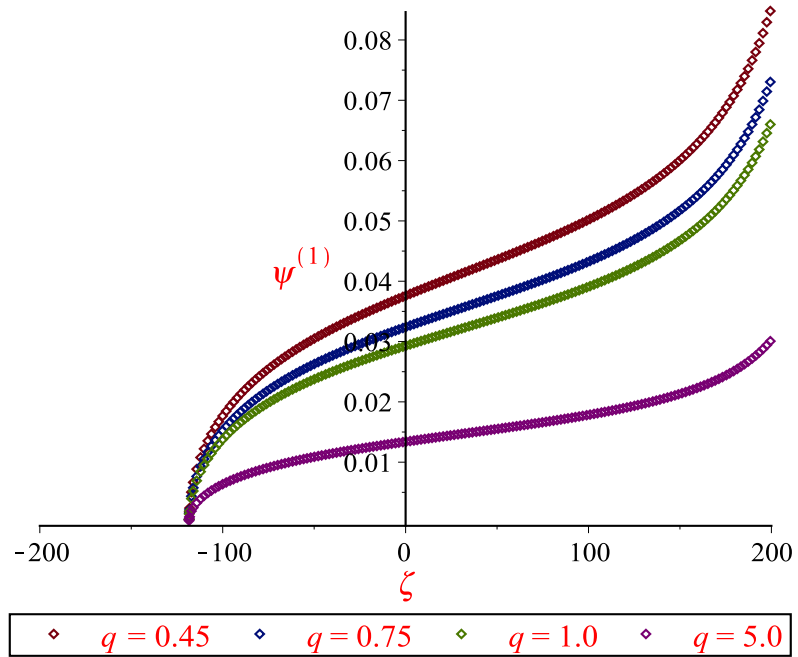


Fig. 2.4 The variation of DA SWs potential ($\psi^{(1)}$) (a) around CV, that is $\alpha < \alpha_c = 0.3043614619$ ($\mu = 1.5$) and (b) with the changes of viscosity coefficient around cv, that is $\alpha = 0.25 < \alpha_c = 0.3043614619$. The values of other parameters are $\rho_{ie} = 0.4$, $\sigma_{ie} = 0.2$, $q = 1$ and $s_0 = 0.03$.



(a)



(b)

Fig. 2.5 The variation of DA oscillatory waves ($\psi^{(1)}$) (a) around CV, that is $\alpha < \alpha_c = 0.3043614619$ and $\alpha > \alpha_c = 0.3043614619$ ($\mu = 1.5$) with $\rho_{ie} = 0.4$, $\sigma_{ie} = 0.2$, $q = 1$ and $s_0 = 0.005$, and (b) with the changes of nonextensivity ($\alpha = 0$) with $\sigma_{ie} = 0.2$, and $s_0 = 0.005$.

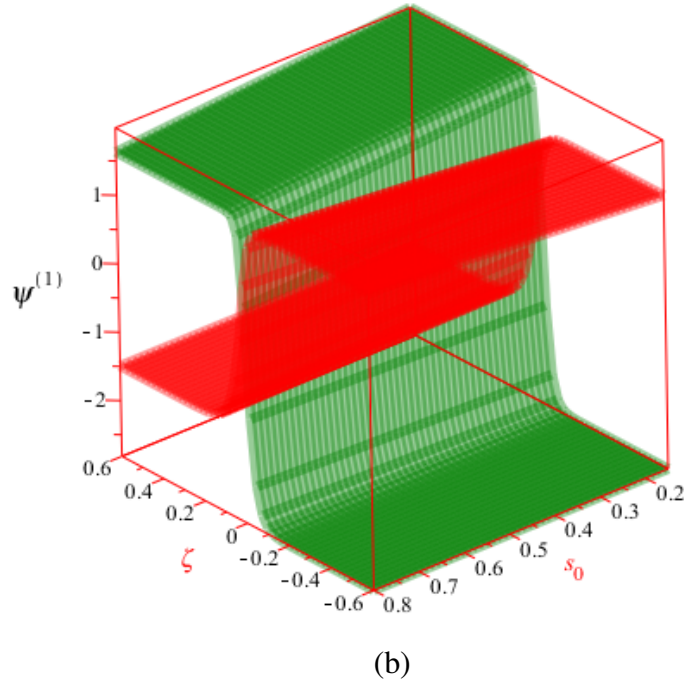
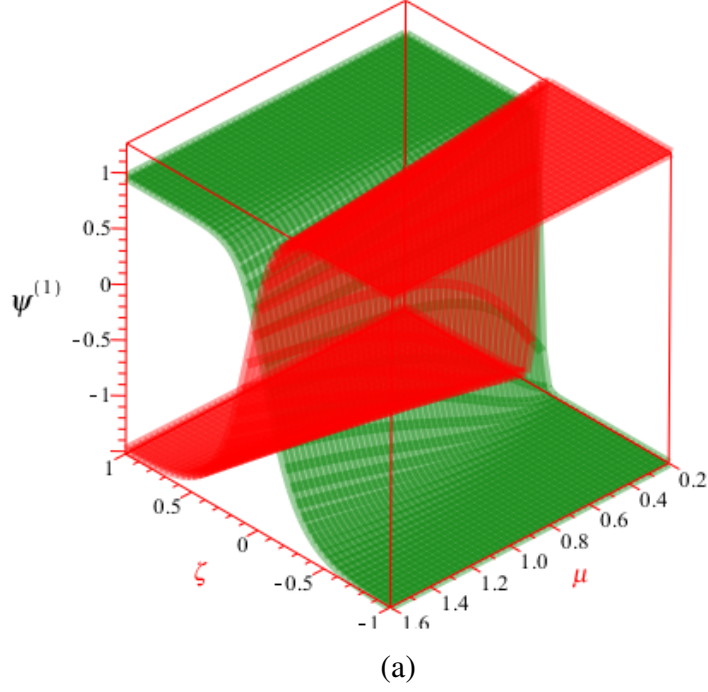
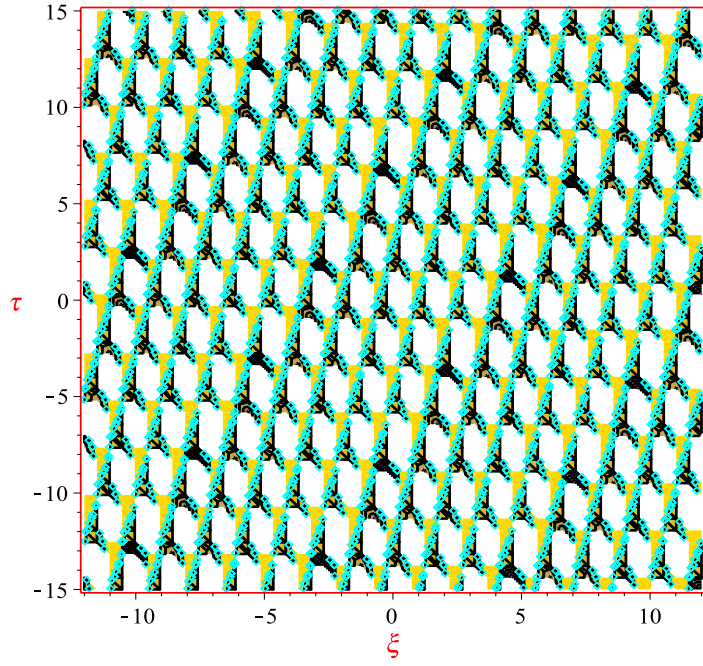
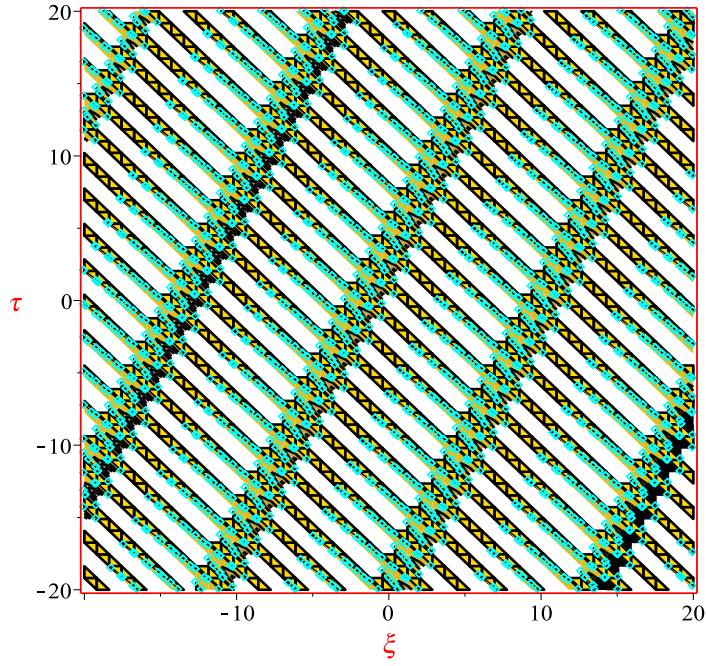


Fig. 2.6 The electrostatic DA SWs potential ($\psi^{(1)}$) described by Burgers equation involving of both quadratic and cubic nonlinearity in (a) (ζ, μ) -plane for $\sigma_{ie} = 0.6 > \sigma_c = 0.4$ (red surface) and $\sigma_{ie} = 0.25 < \sigma_c = 0.4$ (green surface) with $\alpha = 0.3468292023$, $q = 1$, $\rho_{ie} = 0.34$, $s_0 = 0.3$; and (b) (ζ, s_0) -plane for $\sigma_{ie} = 0.6 > \sigma_c = 0.4$ (red surface) and $\sigma_{ie} = 0.25 < \sigma_c = 0.4$ (green surface) with $\alpha = 0.3468292023$, $q = 1$, $\rho_{ie} = 0.34$ and $\mu = 0.4$.



(a)



(b)

Fig. 2.7 Contour plot of the electrostatic DA oscillatory waves ($\psi^{(1)}$) described by Burgers equation involving of both quadratic and cubic nonlinearity in (ξ, τ) -plane for (a) $\sigma_{ie} = 0.56 > \sigma_c = 0.4$ and (b) $\sigma_{ie} = 0.25 < \sigma_c = 0.4$ with $\alpha = 0.3468292023$, $q = 1$, $\rho_{ie} = 0.34$, $s_0 = 0.5$ and $\mu = 0.6$.

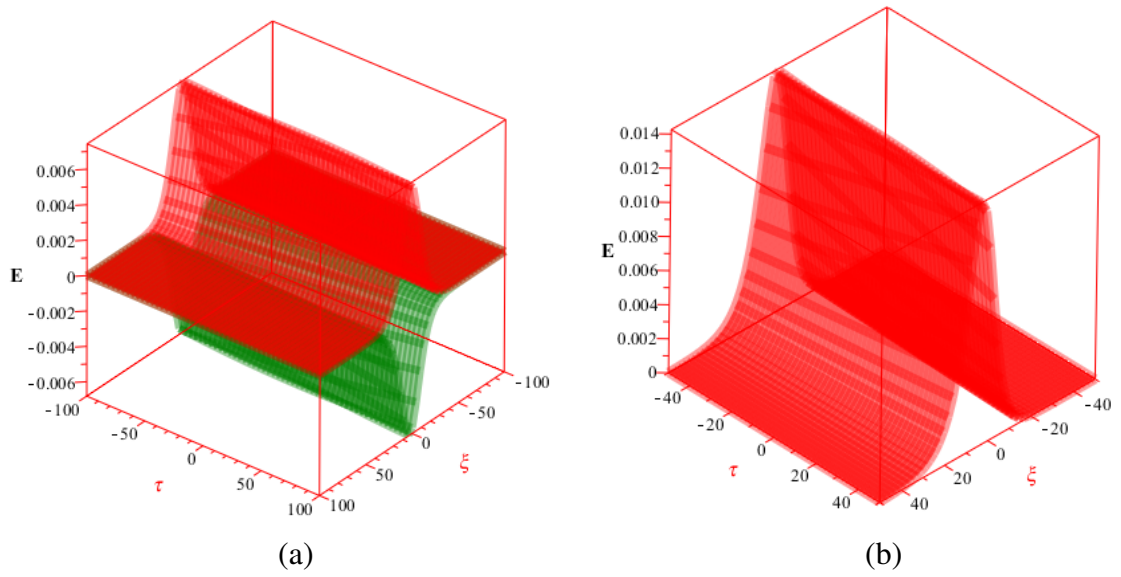


Fig. 2.8 The shape of normalized electric field ($\mathbf{E}$) (a) described by Burgers equation involving of quadratic nonlinearity for $\alpha = 0.35$ (red surface) and $\alpha = 0.25$ (green surface) with $q = 1$, $\rho_{ie} = 0.4$, $\sigma_{ie} = 0.2$, $\mu = 0.4$, and $s_0 = 0.03$; and (b) described by Burgers equation involving of cubic nonlinearity for $\alpha = 0.25$ with $q = 1$, $\rho_{ie} = 0.4$, $\sigma_{ie} = 0.2$, $\mu = 0.4$, and $s_0 = 0.03$.

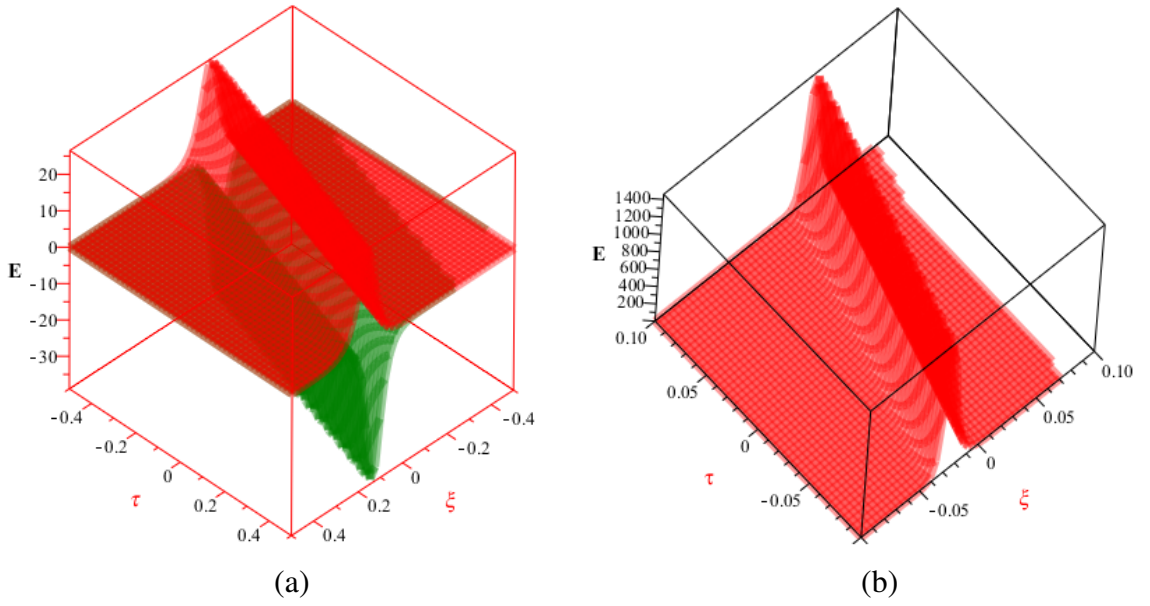


Fig. 2.9 The shape of normalized electric field (E) described by Burgers equation involving of both quadratic and quadratic nonlinearity: (a) ACVs, that is, $\sigma_{ie} = 0.6 > \sigma_c = 0.4$ (red surface) and $\sigma_{ie} = 0.25 < \sigma_c = 0.4$ (green surface) with $\alpha = 0.3468292023$, $q = 1$, $\rho_{ie} = 0.34$, $s_0 = 0.3$; and (b) at the composition of CV, that is $\sigma_{ie} = 0.4$, $\alpha = 0.3468292023$, $q = 1$, $\rho_{ie} = 0.34$ and $\mu = 0.4$.

considering the remaining parameters constant (The influences of other parameters on DA oscillatory wave ignored for simplicity). It is clearly observed from Figures 2.1, 2.2, and 2.3 that (i) the amplitudes of electrostatic DA shocks decrease (increase) by increasing values of α (q , ρ_{ie} and σ_{ie}), while the amplitude of electrostatic DA oscillatory wave periodically decreases with the increase of α , (ii) the width of shocks monotonically increases but the amplitudes remain unchanged by increasing values of μ , (iii) the maximum energy gains with the presence of subthermal rather than isothermal, superthermal and nonthermal ions, and (iv) the considered plasma is supported both of compressive and rarefactive shocks and oscillatory waves with the presence of nonthermality, whereas only positive shocks are produced with the presence of nonextensivity. This clearly indicates that Burgers equation involving quadratic nonlinearity is not useful to study DA shocks(DAs) and oscillatory waves when the nonlinear coefficient of it is approaching to zero for a specific value, so called CV of α along with the fixed values of the other parameters within the valid ranges. At this stage, one must be needed other nonlinear evolution equation involving more higher order nonlinearity. That is why, the higher-order Burgers equations are formulated to study the nonlinear propagation of DA shocks(DAs) and oscillatory waves ACVs and at the composition of CV's in the considered plasmas. The effects of α and μ on the propagation of DA shocks ACV are displayed in Figs. 2.4(a) and 2.4(b), respectively described by the Burgers equation involving cubic nonlinearity based on the useful solution of it. Figure 2.4(a) that the shocks are existed when the value of α is less than its CV and the amplitudes of shocks increase when the values of α increase up to less than its CV. Figure 2.4(b) is indicated that the monotonically shocks are also obtained ACV. It is interesting to be found form the Otherwise the complexity arises in the considered plasmas. Additionally, the effect of α and q on the propagation of DA shocks ACV described by the Burgers equation involving cubic nonlinearity are displayed in Figs. 2.5(a) and 2.5(b), respectively. It is interesting to be found from the Figure 2.5(a) and 2.5(b) that the oscillatory waves are supported when the value of α is less or greater than its CV and with the presence of superthermal electrons ($\alpha = 0$). To overcome not only such difficulty but also to study shocks at the composition of CV's, another evolution equation, so called the Burgers equation involving both quadratic and cubic nonlinearity is derived. By the useful solution of it, the DA SW structures in (ζ, μ) , and (ζ, s_0) -plane are displayed

by Figures 2.6(a) and 2.6(b), respectively. The figure 2.6 is obviously shown that the electrostatic not DA SWs and DA double layer are supported by fulfilling the condition $\sigma_{ie} > \sigma_c$ and $\sigma_{ie} < \sigma_c$, respectively. It is observed that the maximum growth of amplitudes for DA SWs are occurred at CVs. Since, the CVs are independent of μ and s_0 , the effect of such parameters on DA SWs and double layer are included in the Figure 2.6. It is found that both amplitude and width of DA SWs (double layer) increase (decrease) with the increase of s_0 , but only the width of DA SWs (double layer) monotonically increase (decrease) with the increase of μ . In addition, the Figures 2.7 (a) and 2.7(b) show the better visualization of space and time variation due to the DA oscillatory wave by fulfilling the condition $\sigma_{ie} > \sigma_c$ and $\sigma_{ie} < \sigma_c$, respectively. It is found from figure 2.7 that Burgers equation involving both quadratic and cubic nonlinearity are supported parallel oscillatory waves having periodicity for $\sigma_{ie} > \sigma_c$ (left to right propagation) and $\sigma_{ie} < \sigma_c$ (right to left propagation). Finally, the normalized electric field $E = -grad(\psi)$ described by the Burgers equation involving quadratic nonlinearity and the Burgers equation involving cubic nonlinearity is displayed in Figs. 2.8(a) and 2.8(b), respectively. Whereas, Figures 2.9(a) and 2.9(b) present the normalized electric field $E = -grad(\psi)$ described by Burgers equation involving both quadratic and cubic nonlinearity. It is found from Figures 2.8 and 2.9 that (i) Burgers equation involving quadratic nonlinearity is supported both of hump (bright) and dip (dark) shaped electric field based on compressive and rarefactive DA SWs, respectively, (ii) Burgers equation involving cubic nonlinearity is supported only hump shaped electric field, and (iii) Burgers equation involving both quadratic and cubic nonlinearity is supported both of hump and dip shaped electric field based on DA shocks and DA double layer, respectively. From the above discussion, the plasma parameters including the reference speed play an vital role to the variation of shock wave excitations, whereas the viscosity coefficient of NIs is only responsible to form shock wave excitations in the considered plasmas. The Burgers equation involving cubic nonlinearity is only applicable to study electrostatic shocks ACVs and provided only compressive shocks. But, the Burgers equation involving both quadratic and cubic nonlinearity is applicable to study nonlinear propagation of electrostatic DA shocks not only ACVs but also at the CVs and provided both shock wave excitations and double layer in the plasmas. The SWs structure becomes steeper (monotonic-type) with the presence of the stronger dissipation. The SWs

and double layer also becomes more steeper at the CVs. From the physical side, the growth of amplitude of SWs is indicated that the electrostatic potential drop across the double enhances. This is a possibility that more plasma particles may accelerate in such plasmas. It is predicted from the physical point of view that with the increasing values of ion temperature, electron density and strength of nonextensivity provides to more potential drop and accelerates dust and NIs/electrons in oppositely.

2.8 CONCLUDING REMARKS

The nonlinear propagation of DA SWs as well as oscillatory waves that propagate periodically and double layer in an unmagnetized coupled dusty plasma has been studied. The NLEEs involving various nonlinearity have been derived. The effective exact shock and oscillatory wave solutions have been determined by employing the GREMM. The influences of parameters on the nonlinear propagation of DA SWs, oscillatory waves and double layer have been investigated. The obtained solutions have been shown that the GREMM is an effective, very useful mathematical technique and easily applicable to determining the solutions of NLEEs. Because one can determine the solutions involving amplitude and width parametric functions of the propagating wave by this method. Thus, it can be concluded that the outcomes would be beneficial for examining the nature of shock wave excitations and oscillatory wave phenomena where the considered plasma exists and in laboratory experiments. In the future, one may derive the new NLEEs from the model equations and apply any other suitable mathematical techniques to determine the useful form of analytical solutions of NLEEs.

Chapter 3: AN UNMAGNETIZED STRONGLY COUPLED PLASMA: HEAVY ION ACOUSTIC SHOCK WAVE EXCITATIONS

3.1 INTRODUCTION

Now-a-days, researchers have been dedicated their efforts to the study of linear and nonlinear wave propagation in multi-ions plasmas. The endurance of such plasma is now confirmed in both space and astrophysical environments (SAEs) as well as in plasma laboratory (PL)[2],[22], [31], [52],[53], [85],[102],[120]. In many SAEs, particularly, in the Earths ionosphere and magnetosphere, bow shock in front of the magnetopause boundary layers, cometary comae, solar wind, etc., the plasma system contains both positive, and negative ions species along with electrons. The existence of heavy charged particles is also confirmed as a prestellar stage of the evolution of the universe for the case of all objects are constricted to immensely high density and correspondingly have high temperature [103]. However, the strongly coupled (SC) plasma is an multi-component plasma with many charged particles. It has mainly produced due to the average value of the potential energy of the system is in the order of the average value of the kinetic energy or more. On account of the large charge number and lower temperature of dust, the dusty plasma enters the SC regime. The ions having high charge densities are also becomes SC to each other via their mutual Coulomb interaction. By depending on the coupling parameter $\Gamma = (Z_i^2 Q_i^2 / I_i k_B T_i)$ (Q_i , I_i , Z_i and T_i are respectively the charge, inter spacing, charge state and temperature of ions), one can characterize the strength of interaction as $\Gamma > 1$ for strongly coupled regime (SCR) and $\Gamma < 1$ for weakly coupled regime (WCR). In refs.[104],[121], authors have experimentally reported that the SC behavior is supported not only in dusty plasma but also in ions plasma due to the comparatively large charge and mass of ions. By composing of SC ions and degenerate electrons, Shukla et al. [80] have already reported the ion acoustic (IA) waves in an unmagnetized quantum plasma. Lyon et al. [122] have experimentally shown that the SC plasma is produced from laser-heated solids. They have presented an analysis of ion temperatures in laser-produced plasmas by generating from solids with several initial lattice structures. It is confirmed form the

above description that one can study the basic characteristics of the SC plasmas having SC multi-ions and electrons.

On the other hand, the plasma particle velocity distribution does not exactly follow the Maxwellian distribution because of many objects in SAEs (especially in the magnetosphere of earth, Mercury, Saturn ring, Solar wind, stellar polytropes, hadronic matter, and quark-gluon plasma, etc. [11],[60–62]) are interacted via long-ranges, think of gravitational or Coulomb interactions, where the characteristics of not only subthermailtiy, superthermality but also nonthermality of plasma particles are often violated. In addition, the change particles have existed in different phase space that confirmed in Ref. [59]. SAEs have also witnessed the production of energetic plasma particles which show deviation far from the Maxwellian distribution as reported in many previous studies. As a result, one can consider the constitution of nonextensive statistics proposed by Tsallis [58] as a generalization of BGS statistics when plasma regimes far off from equilibrium having power-law distribution. Tsallis entropy has also received enormous attention because of its outstanding features dealing with SAEs and cosmological scenarios (e.g. dark matter halos, upper ionosphere of Mars, vicinity of the Moon, magnetospheres of Jupiter and Saturn, teller polytropes, hadronic matter and quark-gluon plasma, proton-neutron stars,etc.). However, the Viking spacecraft [63] and Freja satellite [64] have witnessed the existence of electrostatic wave structures in the magnetosphere with density depressions. Accordingly, Cairns et al. [48] have provided the existence of rarefactive IA structures by postulating a distribution model in terms of a parameter α incongruent with the observations of Freja satellite and Viking spacecraft for demonstrating for demonstrating the higher energetic tails. In many refs. [65–71], researchers have examined several types of IA waves by assuming either electron or ion or both to be nonextensive or nonthermal due to the existence of plasma particles in different phase space as well as their long-range interactions. Combining Tsallis and Cairns models, Tribeche et al. [72] have proposed the following hybrid Tsallis–nonthermal velocity distribution function for electrons:

$$f_e(V_x) = C_{q,\alpha} \left(1 + \alpha \frac{V_x^4}{V_{te}^4} \right) \left[1 - (q-1) \frac{V_x^2}{2V_{te}^2} \right],$$

where $V_{te} = \sqrt{T_e/m_e}$ is the thermal velocity of electrons, q is the strength of nonextensivity,

α is measuring the population of nonthermal electrons and $C_{q,\alpha}$ is the normalized constant (see details in Ref. [72]). They have provided that the hybrid distribution, so-called (α, q) -velocity distribution is a better fit of the space observations due to the flexibility provided by the nonextensivity parameter. They have also mentioned that nonthermality and nonextensivity may act concurrently on the nature (rarefactive or compressive) of the IA structures. Latter, the proper ranges of the nonextensive and nonthermal parameters are defined in Refs. [73–76] subject to the physical cutoff imposed the monotonicity conditions $q \geq 5/7$ and $\alpha_{max} = (2q - 1)/4$ for nonthermal-nonextensive case. Hence, one can use the (α, q) -velocity distribution to examine the physical issues observed in many SAEs by considering the appropriate ranges of α and q , that is, for (i) isothermal ($q \rightarrow 1$ and $\alpha = 0$), (ii) superthermal ($\alpha = 0, 0.33 < q < 1$) and (iii) subthermal ($\alpha = 0, q > 1$) but also (iv) nonthermal ($q \rightarrow 1$ and $0 < \alpha < 0.25$) lighter plasma particles [73–76].

Further, the adiabatic plasma research has found a great deal of interest in understanding the electrostatic disturbances not only in SAEs but also in PL, which is considered to be the key to conceptualization in thermodynamics. That why, influence of adiabatically on dust acoustic waves in a SC dusty plasma have reported by Rahman and Mamun [93]. Hossen et al. [104],[123],[124] is considered a good case heavy static ions in the degenerative plasma and strongly investigate the underlying features alone and the shock properties. Besides, the HIs are significantly affected on the astrophysical visibility (e.g. white dwarfs, neutron stars, black holes, etc.). Because the adiabaticity, nonextensivity or nonthermality, and HIs dynamics play an vital role in the formation and stability of existing waves in such environments. Due to the existence of SC inertial HIs in the astrophysical plasmas, Ema et al. [96] have examined the linear and nonlinear HI acoustic waves in a SC plasma by HI fluids, nonextensive distributed electrons, and Maxwellian light ions (MLIs). However, we believe that their investigations leads to truncation errors because the coefficient of the nonlinear term in nonlinear evolution equations are not properly defined. They have overlooked the nonlinear features of shock and solitary wave excitations around the critical values (CVs) and at the CVs due to the difficult mathematical calculations. Very recently, Patidar and Sharma [71] have reported IA solitary and shock waves in SC degenerate plasma. They have derived Kortewegde Vries Burgers equation by expanding the viscosity coefficient $\eta_l = \varepsilon^{1/2}\eta_0$ without any proper explanation to study the shock wave excitation

(SWE). It does not reflect the actual effect of viscosity coefficient on the SWEs. Whereas, References [22],[108] have clearly been shown that one should not be used the stretching of kinematic viscosity coefficient parameters. As a result, one can derive the BE, mBE and mmBE by considering the valid stretching to study the SWEs in the plasmas. Also, no research work has been reported what would be the effect of strongly correlated adiabatic HIs and nonextensivity or nonthermality on the SWEs described by mBE and mmBE with their appropriate solutions around CVs and at the CVs in such plasmas. Additionally, the shock wave solutions of mBE and mmBE are not properly defined in the previous literatures. It is, therefore, still now required to further investigation in the SC plasmas by composing of HI fluid, MLIs and nonextensive or nonthermal or Maxwellian electrons.

Thus, the aim of this chapter is to explore the nonlinear features of heavy ion acoustic waves shock wave (HIASW) excitations in an unmagnetized SC plasma having HIs fluid, (α, q) -distributed electrons and MLIs. It is also worthwhile to represent a novel study on HIASWs not only around CVs but also at the CVs via the useful stationary shock wave solutions of the nonlinear equations, where adiabaticity, superthermal / subthermal/ isothermal/ nonthermal electrons, MLIs, and HIs dynamics play an significant role.

3.2 GOVERNING FLUID EQUATIONS

An unmagnetized collisionless SC plasma is suggested by the mixture of inertial positively charged HIs fluid, inertialess (α, q) -distributed electrons and MLIs along with the neutrality condition $Z_i N_{i0} + Z_h N_{h0} = N_{e0}$. It is noted that one may consider HIs fluid is to be strongly coupled with MLIs because of heavy mass (m_h), immense electric charge (Z_h) and very low temperature (T_h). Besides, one can use Tsallis distribution to formulate the electrons density function as $N_e = N_{e0} [1 + (q - 1) (e\psi/k_B T_e)]^{\frac{(q+1)}{2(q-1)}}$, where ψ , N_{e0} , T_e and $k_B(e)$ are the electrostatic potential, unperturbed electron density, electron temperature and Boltzmann constant (electric charge), respectively for the cases of subthermal and superthermal electrons [119]. Ema et al. [96] have already reported the effects of electron nonextensivity on HIASWs in the considered SC plasmas. But, the results reported in Ref. [96] may lead to truncation errors due to incorrectly defined the coefficient of nonlinear evolution equations. Also, they have not mentioned how they studied the linear and nonlinear properties of HISWs by ignoring the influence of coupling parameter Γ .

In addition, the nonextensive distribution function is unable to investigate the physical scenarios for the nonthermal electron populations. Hence, one can consider model an electron distribution with a population of fast particle by taking the Cairns distribution and yields $N_e = N_{e0} \left[1 - (4\alpha/1 + 3\alpha) (e\psi/k_B T_e) + (4\alpha/1 + 3\alpha) (e\psi/k_B T_e)^2 \right] \exp(e\psi/k_B T_e)$. To study all the cases of energetic electrons, one can use (α, q) -velocity distribution function to find the density function for electrons. As a result, one can easily formulate the density function for electrons by integrating $f_e(V_x)$ over volume space (see details in ref. [72]) as

$$N_e = N_{e0} \left[1 + (q-1) \frac{e\psi}{T_e} \right]^{\frac{(q+1)}{2(q-1)}} \times \left[1 + \Sigma_1 \left(\frac{e\psi}{T_e} \right) + \Sigma_2 \left(\frac{e\psi}{T_e} \right)^2 \right], \quad (3.1)$$

where

$$\Sigma_1 = -16q\alpha / (3 - 14q + 15q^2 + 12\alpha), \Sigma_2 = -\Sigma_1 (2q - 1).$$

To study the basic features of HIASWs by deriving nonlinear equations, one can formulate the normalized model equations by taking the mass and momentum conservation law for HIs fluid in the following forms [96]:

$$\frac{\partial N_h}{\partial t} + \frac{\partial (N_h U_h)}{\partial z} = 0, \quad (3.2)$$

$$\left(1 + \tau_R \frac{\partial}{\partial t} \right) \left[N_h \left\{ \left(\frac{\partial}{\partial t} + U_h \frac{\partial}{\partial z} \right) U_h + \frac{\partial \psi}{\partial z} \right\} + c_f \frac{\partial N_h}{\partial z} \right] = \mu \frac{\partial^2 U_h}{\partial z^2}, \quad (3.3)$$

with

$$\mu = (\omega_{ph}/M_h N_{h0} \lambda_{Dh}^2) [\mu_t + (4/3)\mu_b], c_f = \mu_h k_B T_h \gamma_h,$$

where $\mu_t(\mu_b)$ is transport coefficients of shear (bulk) viscosity. Based on the ref. [96],[125], one can define τ_R as

$$\tau_R = \mu \frac{T_e}{T_h} [1 - \mu_h + \frac{4}{15} U(\Gamma)]^{-1}, \quad (3.4)$$

with the compressibility [96],[125] condition as

$$\mu_h = \frac{1}{k_B T_h} \frac{\partial P_h}{\partial N_h} = \frac{1}{k_B T_h} \left[1 + \frac{1}{3} U(\Gamma) + \frac{\Gamma}{9} \frac{\partial U(\Gamma)}{\partial \Gamma} \right]. \quad (3.5)$$

It is noted that $U(\Gamma)$ is used for measuring the excess internal energy and is determined as $U(\Gamma) \equiv -(\sqrt{3}/2)\Gamma^{3/2}$ for WC plasmas. By neglecting small correction term due to finite number of particles, $U(\Gamma) \equiv -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81$ for $1 < \Gamma < 100$ [126]. Also, the dependence of μ_h on Γ cannot be represented in the above analytical form because it actually becomes very complex. But, one can employ various statistical schemes to find the tabulated/graphical results of their functional behavior derived from the molecular dynamic simulations [125]. The considered plasma system is closed by

$$\frac{\partial^2 \psi}{\partial z^2} = r_1 [1 + (q-1)\psi]^{\frac{(q+1)}{2(q-1)}} \times [1 + \Sigma_1 \psi + \Sigma_2 \psi^2] - r_2 \exp(-\sigma_{ei}\psi) - N_h = -\rho. \quad (3.6)$$

It is noted that the meaning of the physical variables along with their characteristics, physical parameters, ratio's with proper ranges are included in the Table 3.1.

Table 3.1 Physical variables along with their characteristics, physical parameters, ratio's with proper ranges

Physical variables/ parameters	Characteristic scale	Notation/ Parameter ranges
Length	$\lambda_D =$	z
	$= \sqrt{k_B T_e / 4 Z_h^2 \pi N_{h0} e^2}$	
Time	$\omega_{hf} =$	t
	$\sqrt{4 \pi Z_h^2 N_{h0} e^2 / m_h}$	
Number density	N_{s0}	N_s
HI's velocity	$\sqrt{Z_h k_B T_e / m_h}$	U_h
Electrostatic potential	$(k_B T_e / e)$	ψ
Electron to		$r_1 = \frac{N_{e0}}{Z_h N_{h0}}$
HI density ratio		$= 0.1 - 0.7$ [96],[127]
LI to HI density ratio		$r_2 = \frac{Z_i N_{i0}}{Z_h N_{h0}}$
HI density ratio		$= 0.1 - 0.6$ [96],[127]
Electron to		$\sigma_{ei} = \frac{T_e}{T_i}$
LI temperature ratio		$= 0.01$ [96],[127]
Adiabatic index		$\gamma_h = 3$ [96],[127]
Viscosity coefficient		$\mu = 04 - 0.8$ [96],[127]
Unperturbed density		$N_{s0}, s = h(HI),$ $e(electron), i(MLI)$

3.3 FORMATION OF BE WITH STATIONARY WAVE SOLUTION

To derive many kind of evolution equations from the proposed model equations, one needs to define the appropriate coordinate frame and multiple scale expansion via the general principles of reductive perturbation technique (RPT). It is noted that the dispersion or dissipation relation for harmonic waves are determined by linearizing Eq.(3.2),(3.3), and (3.6) along with the solutions as $N_h, U_h, \psi \sim \exp(i\Omega)$, where $\Omega = kz - \omega(k)t$. Without loss of generality, one can define $k = \varepsilon^i K$, where K and i are the new wave number and real number, respectively. For this case, one obtains $\Omega(z, t) = \varepsilon^i K z - \omega(\varepsilon^i K)t$. However, the purely dispersive/dissipative media cannot has a mixture of both odd and even terms in k in the Taylor expansion of $\Omega(k)$. In the case of purely dissipative media, the even terms in k are only occurred, and the Taylor expansion for $\omega(k)$ provides $\omega(k) = \omega'(0)\varepsilon^i K + \omega''(0)\varepsilon^{2i}K^2 + \dots$. This implies that $\Omega(z, t) = K\varepsilon^i(z - \omega'(0)t) - K^2\omega''(0)\varepsilon^{2i}t$. Then one may scale out for z and t as $\zeta = \varepsilon^i(z - V_p t)$ and $\tau = \varepsilon^{2i}t$ because $\omega'(0)$ and $\omega''(0)$ are constants. To study the SWEs, stretching coordinates are considered based on RPT [96],[119] with $i = 1$ as

$$\zeta = \varepsilon(z - \lambda_v t), \tau = \varepsilon^2 t, \quad (3.7)$$

Also, the expansions for perturbed quantities are represented as

$$\begin{bmatrix} N_h \\ U_h \\ \psi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \sum_i \varepsilon^i \begin{bmatrix} N_h^{(i)} \\ U_h^{(i)} \\ \psi^{(i)} \end{bmatrix}, \quad (3.8)$$

where λ_v and ε are the phase speed and a proper fraction quantity measuring the weakness of dissipation. Now, employing Eqs. (3.7) and (3.8) into Eqs. (3.2), (3.3), and (3.6) in a systematic way, one can formulate the set of equations by depending on the several power of ε . From $O(\varepsilon^2)$ order equations, we get following quantities.

$$N_h^{(1)} = \frac{1}{\lambda_v^2 - c_f} \psi^{(1)}, U_h^{(1)} = \frac{\lambda_v}{\lambda_v^2 - c_f} \psi^{(1)}, \quad (3.9)$$

$$\lambda_v = \sqrt{c_f + \frac{1}{\Delta_1}}, \quad (3.10)$$

where

$$\Delta_1 = r_1 \left[\frac{q+1}{2} - \frac{16q\alpha}{3-14q+15q^2+12\alpha} \right] + r_2 \sigma_{ei}. \quad (3.11)$$

Finally, from $O(\varepsilon^3)$ order equations the following relation is obtained.

$$\frac{\partial N_h^{(1)}}{\partial \tau} - \lambda_v \frac{\partial N_h^{(2)}}{\partial \xi} + \frac{\partial U_h^{(2)}}{\partial \xi} + \frac{\partial}{\partial \xi} [N_h^{(1)} U_h^{(1)}] = 0, \quad (3.12)$$

$$\begin{aligned} & \frac{\partial U_h^{(1)}}{\partial \tau} - \lambda_v \frac{\partial U_h^{(2)}}{\partial \xi} - \lambda_v N_h^{(1)} \frac{\partial U_h^{(1)}}{\partial \xi} + U_h^{(1)} \frac{\partial U_h^{(1)}}{\partial \xi} + \frac{\partial \psi^{(2)}}{\partial \xi} + c_f \frac{\partial N_h^{(2)}}{\partial \xi} + N_h^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} \\ & + \tau_R \lambda_v^2 \frac{\partial^2 U_h^{(1)}}{\partial \xi^2} - \tau_R \lambda_v \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} - \tau_R c_f \lambda_v \frac{\partial^2 N_h^{(1)}}{\partial \xi^2} = \mu \frac{\partial^2 U_h^{(1)}}{\partial \xi^2}, \end{aligned} \quad (3.13)$$

$$0 = \Delta_1 \frac{\partial \psi^{(2)}}{\partial \xi} + 2\Delta_2 \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} - \frac{\partial N_h^{(2)}}{\partial \xi}, \quad (3.14)$$

where,

$$\begin{aligned} \Delta_2 = r_1 \left[\frac{(q+1)(3-q)}{8} - \frac{q+1}{2} \times \frac{16q\alpha}{3-14q+15q^2+12\alpha} + \frac{16q(2q-1)\alpha}{3-14q+15q^2+12\alpha} \right] \\ - \frac{r_2 \sigma_{ei}^2}{2}. \end{aligned} \quad (3.15)$$

Simplifying Eqs. (3.12)-(3.14) with the help of Eqs. (3.9) and (3.10), one obtains the following BE:

$$\frac{\partial \psi^{(1)}}{\partial \tau} + A_1 \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} = A_2 \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \quad (3.16)$$

where

$$A_1 = \frac{\lambda_v^2 - c_f}{2\lambda_v} \left[\frac{3\lambda_v^2 - c_f}{(\lambda_v^2 - c_f)^2} - 2\frac{\Delta_2}{\Delta_1} \right], A_2 = \frac{\mu}{2}. \quad (3.17)$$

The useful shock wave solution of Eq. (3.16) is defined as

$$\psi^{(1)} = \psi_B \left\{ 1 - \tanh \left(\frac{\xi}{\psi_{BW}} \right) \right\}, \quad (3.18)$$

where $\psi_B = (\lambda_0/A_1)$ and $\psi_{BW} = (2A_2/\lambda_0)$ are respectively the amplitude and width of HIASWs and $\zeta = \xi - \lambda_0\tau$ (λ_0 is the constant speed of reference frame). It is observed from Eq. (3.18) that the amplitudes of shocks are strongly dependent all the related plasma parameters except viscosity coefficient of HIs, whereas the widths of shocks are only dependent on the viscosity coefficient of HIs and the constant speed of reference frame. Also, Eq. (3.16) provides the positive and negative electrostatic potential based on the condition $A_1 > 0$ and $A_1 < 0$, respectively. The validity of the RPT breaks down for $A_1 = 0$ because the amplitude of shocks become infinite, that is $\psi_B \rightarrow \infty$. This will happen for some particular values, so-called the critical values (CVs), of any related parameters (say J) within the given ranges. It is very difficult to drive a mathematical expression for J by setting $A_1 = 0$. One can determine the possible values of $J = r_1 = \frac{N_{e0}}{Z_h N_{h0}}$ for which $A_1 = 0$ via the computational software. For instance, one can determine $J = r_1 = \frac{N_{e0}}{Z_h N_{h0}} = 0.3150062209$ within $0.1 \leq r_1 \leq 0.7$ by setting $A_1 = 0$ with $\alpha = 0$, $\sigma_{ie} = 0.01$, $q = 1$, $r_2 = \frac{Z_i N_{i0}}{Z_h N_{h0}} = 0.1$ for which ψ_B approaches to infinity in SCR ($\Gamma = 3.5$). Some of other CVs (C_v) for $J = r_1$ are formulated by setting $A_1 = 0$ and given in Table 3.2. In such situations, one can consider only the higher order corrections, that is, $i = 2$ to study SWEs around $J = r_1$. It is noted that the CVs are existed not only in WCR but also in SCR for $\Gamma < 5$.

Table 3.2 List of some CVs

C_v for any J	Values of other parameters	$\left[\frac{\partial A_1}{\partial J}\right]_{C_v}$
$r_1 = 0.3150062209$	$\alpha = 0, q = 1, r_2 = 0.1, \Gamma = 3.5$	2.826908822
$r_1 = 0.2320665493$	$\alpha = 0, q = 1, r_2 = 0.1, \Gamma = 0.2$	4.474093144
$r_1 = 0.4459870178$	$\alpha = 0, q = 0.3, r_2 = 0.2, \Gamma = 0.2$	2.762029399
$r_1 = 0.1463985823$	$\alpha = 0, q = 3, r_2 = 0.2, \Gamma = 0.2$	5.893042173
$r_1 = 0.4353905585$	$\alpha = 0.1, q = 1, r_2 = 0.2, \Gamma = 0.2$	2.901033905
$r_1 = 0.4260022692$	$\alpha = 0.01, q = 1, r_2 = 0.1, \Gamma = 4.5$	1.759673215

3.4 FORMATION OF MBE WITH THE FAITHFUL STATIONARY WAVE SOLUTION

To study the SWEs around CVs, the stretching coordinates are assumed by taking the higher-order correction of the RPT, that is $i = 2$ [68–70] as

$$\zeta = \varepsilon^2(z - \lambda_v t), \tau = \varepsilon^4 t. \quad (3.19)$$

Introducing Eqs. (3.19) and (3.8) into Eqs. (3.2), (3.3), and (3.6) in sequentially, the set of equations by depending on the several power of ε can be formulated. From the $O(\varepsilon^3)$ order equations, $N_h^{(1)}$, $U_h^{(1)}$, and λ_v are expressed in the similar form as in Eqs. (3.9) and (3.10). Again, $O(\varepsilon^4)$ order equations yields the following relations:

$$\begin{aligned} N_h^{(2)} &= \frac{1}{\lambda_v^2 - c_f} \psi^{(2)} + \left[\frac{3}{2(\lambda_v^2 - c_f)^2} + \frac{c_f}{(\lambda_v^2 - c_f)^3} \right] \left\{ \psi^{(1)} \right\}^2, \\ U_h^{(2)} &= \frac{\lambda_v}{\lambda_v^2 - c_f} \psi^{(2)} + \left[\frac{\lambda_v}{2(\lambda_v^2 - c_f)^2} + \frac{\lambda_v c_f}{(\lambda_v^2 - c_f)^3} \right] \left\{ \psi^{(1)} \right\}^2, \end{aligned} \quad (3.20)$$

and

$$- \left[\frac{3\lambda_v^2 - c_f}{(\lambda_v - c_f)^2} - 2\frac{\Delta_2}{\Delta_1} \right] \left\{ \psi^{(1)} \right\}^2 = 0. \quad (3.21)$$

Equation (3.21) obviously shows that BE is failed to address HIASW excitations at CVs because the amplitude of shocks is approaching to infinity for the case of $A_1 \rightarrow 0$ since

$$A_1 \sim \left[\frac{3\lambda_v^2 - c_f}{(\lambda_v - c_f)^2} - 2\frac{\Delta_2}{\Delta_1} \right].$$

Finally, $O(\varepsilon^5)$ order equations can be represented in the following forms:

$$\frac{\partial N_h^{(1)}}{\partial \tau} - \lambda_v \frac{\partial N_h^{(3)}}{\partial \xi} + \frac{\partial U_h^{(3)}}{\partial \xi} + \frac{\partial}{\partial \xi} \left[N_h^{(1)} U_h^{(2)} + N_h^{(2)} U_h^{(1)} \right] = 0, \quad (3.22)$$

$$\begin{aligned}
& \frac{\partial U_h^{(1)}}{\partial \tau} - \lambda_v \frac{\partial U_h^{(3)}}{\partial \xi} - \lambda_v N_h^{(1)} \frac{\partial U_h^{(2)}}{\partial \xi} - \lambda_v N_h^{(2)} \frac{\partial U_h^{(1)}}{\partial \xi} + \frac{\partial}{\partial \xi} \left[U_h^{(1)} U_h^{(2)} \right] \\
& + \left(N_h^{(1)} U_h^{(1)} \right) \frac{\partial U_h^{(1)}}{\partial \xi} + \frac{\partial \psi^{(3)}}{\partial \xi} + c_f \frac{\partial N_h^{(3)}}{\partial \xi} + N_h^{(1)} \frac{\partial \psi^{(2)}}{\partial \xi} + N_h^{(2)} \frac{\partial \psi^{(1)}}{\partial \xi} \\
& + \tau_R \lambda_v^2 \frac{\partial^2 U_h^{(1)}}{\partial \xi^2} - \tau_R \lambda_v \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} - \tau_R c_f \lambda_v \frac{\partial^2 N_h^{(1)}}{\partial \xi^2} = \mu \frac{\partial^2 U_h^{(1)}}{\partial \xi^2}, \tag{3.23}
\end{aligned}$$

$$0 = \Delta_1 \frac{\partial \psi^{(3)}}{\partial \xi} + 2\Delta_2 \frac{\partial}{\partial \xi} \left[\psi^{(1)} \psi^{(2)} \right] + 3\Delta_3 \left[\psi^{(1)} \right]^2 \frac{\partial \psi^{(1)}}{\partial \xi} - \frac{\partial N_h^{(3)}}{\partial \xi}, \tag{3.24}$$

where,

$$\Delta_3 = r_1 \left[\frac{(q+1)(3-q)(5-q)}{48} - \frac{2q\alpha(q+1)(3-q)}{3-14q+15q^2+12} + \frac{8q(2q-1)\alpha(q+1)}{3-14q+15q^2+12} \right] + \frac{r_2 \sigma_{ei}^3}{6}. \tag{3.25}$$

From Eqs.(3.22) and (3.23), one obtains

$$\begin{aligned}
& \frac{2\lambda_v}{\lambda_v^2 - c_f} \frac{\partial \psi^{(1)}}{\partial \tau} - (\lambda_v^2 - c_f) \frac{\partial N_h^{(3)}}{\partial \xi} + \frac{\partial \psi^{(3)}}{\partial \xi} + \left[\frac{3\lambda_v^2 - c_f}{(\lambda_v - c_f)^2} \right] \frac{\partial}{\partial \xi} \left[\psi^{(1)} \psi^{(2)} \right] \\
& + \left[\frac{13\lambda_v^2 - 3c_f}{2(\lambda_v - c_f)^3} + \frac{7\lambda_v^2 c_f - c_f^2}{(\lambda_v - c_f)^4} \right] \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} = \frac{\mu \lambda_v}{\lambda_v^2 - c_f} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \tag{3.26}
\end{aligned}$$

Simplifying Eqs.(3.24) and (3.26), the following mBE is derived:

$$\frac{\partial \psi^{(1)}}{\partial \tau} + A_3 \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} = A_2 \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \tag{3.27}$$

where

$$A_3 = \frac{\lambda_v^2 - c_f}{2\lambda_v} \left[\frac{13\lambda_v^2 - 3c_f}{2(\lambda_v - c_f)^3} + \frac{7\lambda_v^2 c_f - c_f^2}{(\lambda_v - c_f)^4} - 3 \frac{\Delta_3}{\Delta_1} \right]. \tag{3.28}$$

It is noted the solution of the mBE

$$\frac{\partial \phi^{(1)}}{\partial \tau} + B \left\{ \phi^{(1)} \right\}^2 \frac{\partial \phi^{(1)}}{\partial \xi} = C \frac{\partial^2 \phi^{(1)}}{\partial \xi^2}$$

is incorrectly defined as

$$\phi^{(1)} = \sqrt{\left(\frac{3u_0}{2B}\right) \left\{ 1 - \tanh \left(\frac{\xi - u_0\tau}{\sqrt{\frac{C}{u_0}}} \right) \right\}},$$

(u_0 is speed of the reference frame) in the previous literatures [68–70] , which is not satisfied of mBE. Especially, the width of shock around CVs leads to truncation errors. This implies that the solution is not useful to understand real features of shocks and further verification in laboratory.

To obtain the correct stationary shock wave solution, Eq. (3.27) is converted by considering $\psi^{(1)}(\xi, \tau) = f(\zeta)$, where $\zeta = \xi - \lambda_0\tau$ to

$$A_2 \frac{df(\zeta)}{d\zeta} = -\lambda_0 f(\zeta) + \frac{A_3}{3} \{f(\zeta)\}^3. \quad (3.29)$$

This implies that

$$\int \frac{A_2}{-\lambda_0 f(\zeta) + \frac{A_3}{3} \{f(\zeta)\}^3} df(\zeta) = \int d\zeta. \quad (3.30)$$

Simplifying Eq. (3.30), one obtains

$$\psi^{(1)} = \sqrt{\psi_{mB} \left\{ 1 - \tanh \left(\frac{\zeta}{\psi_{mBW}} \right) \right\}}, \quad (3.31)$$

where $\psi_{mB} = (3\lambda_0/2A_3)$ and $\psi_{mBW} = (A_2/\lambda_0)$ are respectively the amplitude and width of HIASWs around CVs. It is noted that Eq. (3.31) is verified by putting back in Eq. (3.27) and found correct.

3.5 FORMATION OF MMBE WITH USEFUL WAVE SOLUTION

It is observed from Eq. (3.31) that Eq. (3.27) is still failed to address the nature HIASWs not only at CVs but also very near to the CVs. To overcome such difficulty, one can consider $A_1 = A_1^0$ for not only very near to C_v but also at C_v as

$$A_1^0 = \Lambda \left(\frac{\partial A_1}{\partial J} \right)_{J=C_v} |J - C_v| = \Lambda \epsilon \left(\frac{\partial A_1}{\partial J} \right)_{J=C_v}, \quad (3.32)$$

where $|J - C_v| \equiv \varepsilon$ because $|J - C_v|$ is small quantity and $\Lambda = 1(-1)$ for $J > C_v(J < C_v)$. As a result, one can re-evaluate from Eq. (3.6) by introducing $\rho^{(2)} \equiv -\varepsilon^3 i_{\frac{1}{2}} \left(\frac{\partial A_1}{\partial J} \right)_{J=C_v} \left\{ \psi^{(1)} \right\}^2$ with the $O(\varepsilon^5)$ equation and yields

$$0 = \Delta_1 \frac{\partial \psi^{(3)}}{\partial \xi} + 2\Delta_2 \frac{\partial}{\partial \xi} \left[\psi^{(1)} \psi^{(2)} \right] + \Lambda \frac{1}{2} \left(\frac{\partial A_1}{\partial J} \right)_{J=C_v} \left[\psi^{(1)} \right] \frac{\partial \psi^{(1)}}{\partial \xi} + 3\Delta_3 \left[\psi^{(1)} \right]^2 \frac{\partial \psi^{(1)}}{\partial \xi} - \frac{\partial N_h^{(3)}}{\partial \xi}. \quad (3.33)$$

Simplifying Eqs. (3.26) and (3.33), one obtains

$$\frac{\partial \psi^{(1)}}{\partial \tau} + \Lambda D \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} + A_3 \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} = A_2 \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \quad (3.34)$$

where

$$D = \frac{\lambda_v^2 - c_f}{2\lambda_v} \left(\frac{\partial A_1}{\partial J} \right)_{J=C_v}. \quad (3.35)$$

Here Eq.(3.34) is so-called the mixed modified Burgers equation(mmBE) due to the additional weakly nonlinear term incorporated with the mBE. One can easily convert to mmBE not only mBE but also BE. The main advantage of the above equation is that it is not only supported HIASWs very near at the CVs but also at CVs.

It is noted that no research work has previously been studied the SWEs very near to CVs or at CVs by the properly defined solution of mmBE, like Eq. (3.34). The solution $\phi^{(1)} = [\phi_m^2 \{1 - \tanh(\xi/\Delta)\}]^{1/4}$ as in (31) of the mmBE as in Eq. (3.34) of ref. [69] provided by the authors is incorrect, which is misinterpreted the plasma physics communities. Later, El-Kalaawy [128] has been determined the correct solution of mmBE by the auto-Backlund transformations. But, he has also been failed to address properly the reference speed and width of the shock wave excitations. To examine properly the basic feature of shocks around CVs or at CVs, one needs to define firstly the appropriate solution of Eq. (3.34). To do so, one can transform Eq. (3.34) by considering

$$\psi^{(1)}(\xi, \tau) = \Psi(\xi, \tau) - \frac{\Lambda D}{2A_3}$$

to

$$\frac{\partial \Psi}{\partial \tau} - \left(\frac{\Lambda D}{2\sqrt{A_3}} \right)^2 \frac{\partial \Psi}{\partial \xi} + A_3 \left\{ \psi^{(1)} \right\}^2 \frac{\partial \Psi}{\partial \xi} = A_2 \frac{\partial^2 \Psi}{\partial \xi^2}, \quad (3.36)$$

Hence, the appropriate stationary shock wave solution of Eq. (3.34) with the help of Eq. (3.36) can be defined as

$$\psi^{(1)} = \sqrt{\psi_{mmB} \left\{ 1 - \tanh \left(\frac{\zeta}{\psi_{mmBW}} \right) \right\} - \frac{\Lambda D}{2A_3}}, \quad (3.37)$$

where

$$\psi_{mmB} = \frac{3}{2A_3} \left(\lambda_0 + \frac{\Lambda^2 D^2}{4A_3} \right), \quad \psi_{mmBW} = \frac{A_2}{2 \left(\lambda_0 + \frac{\Lambda^2 D^2}{4A_3} \right)}. \quad (3.38)$$

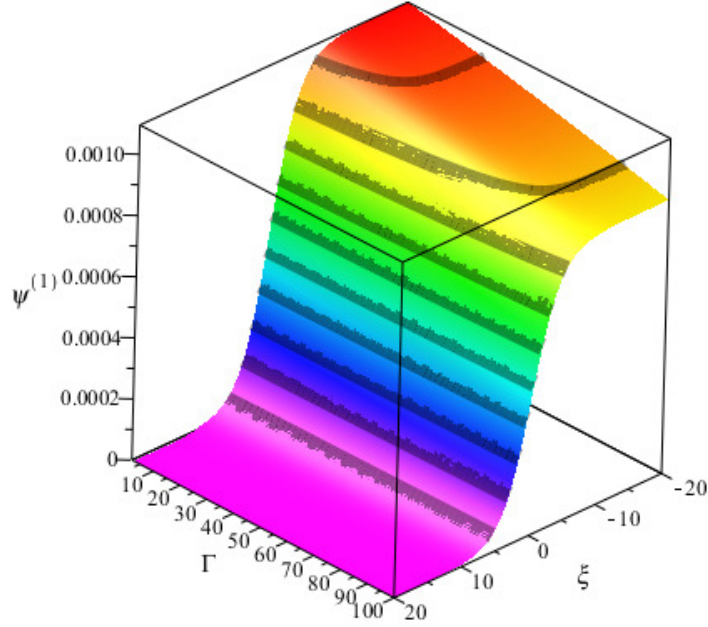


Fig. 3.1 Effect of Γ on the electrostatic potential shock in SCR with $\sigma_{ei} = 0.01$, $\gamma_h = 3$, $r_1 = 0.5$, $r_2 = 0.1$, $\mu = 0.4$, $\alpha = 0.2$, $q = 0.5$ and $\lambda_0 = 0.1$.

3.6 RESULTS AND DISCUSSION

In this section, the effects of plasma parameters on the electrostatic HIASWs has been explored in both of SCR and WCR.

Figure 3.1 shows the effect of coupling parameter (Γ) along with the constant values of the remaining plasma parameters on nonlinear electrostatic potential HIASW ($\psi^{(1)}$)

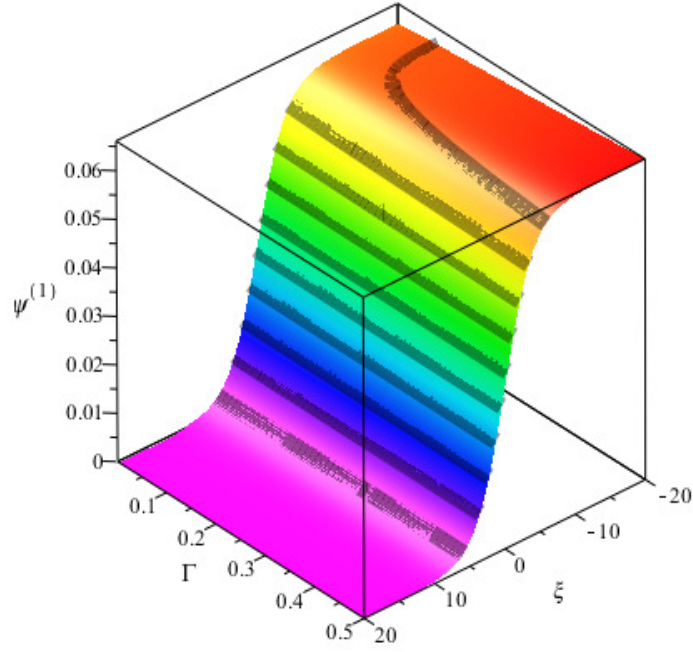


Fig. 3.2 Effect of Γ on the electrostatic potential shock in WCR with $\sigma_{ei} = 0.01$, $\gamma_h = 3$, $r_1 = 0.5$, $r_2 = 0.1$, $\mu = 0.4$, $\alpha = 0.2$, $q = 5$ and $\lambda_0 = 0.1$.

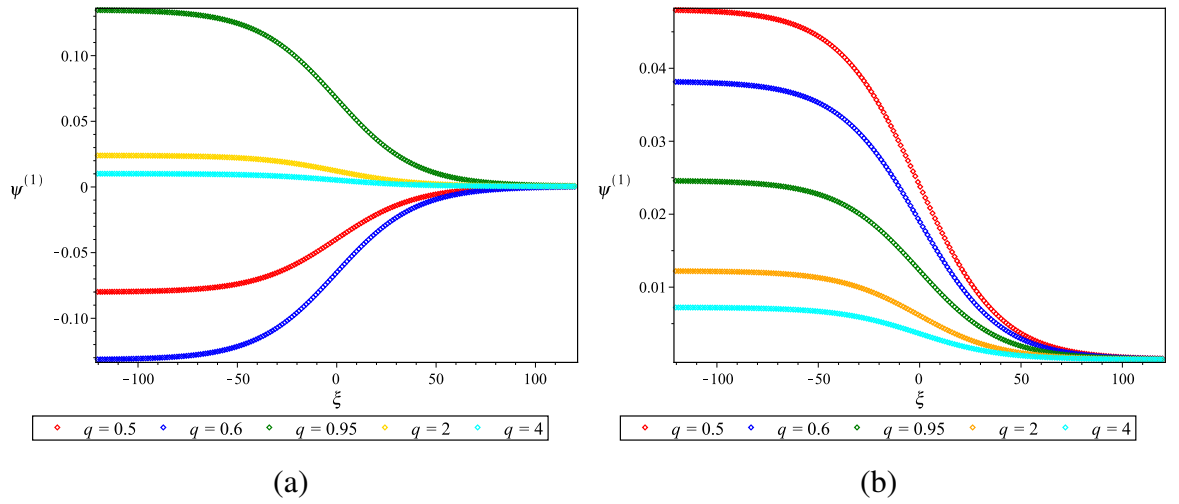


Fig. 3.3 Effect of q (i.e. nonextensivity $\alpha = 0$) on the electrostatic potential shocks (a) SC ($\Gamma = 4.5$) and (b) WC ($\Gamma = 0.001$) regimes with $\sigma_{ei} = 0.01$, $\gamma_h = 3$, $r_1 = 0.5$, $r_2 = 0.3$, $\mu = 0.4$ and $\lambda_0 = 0.01$.

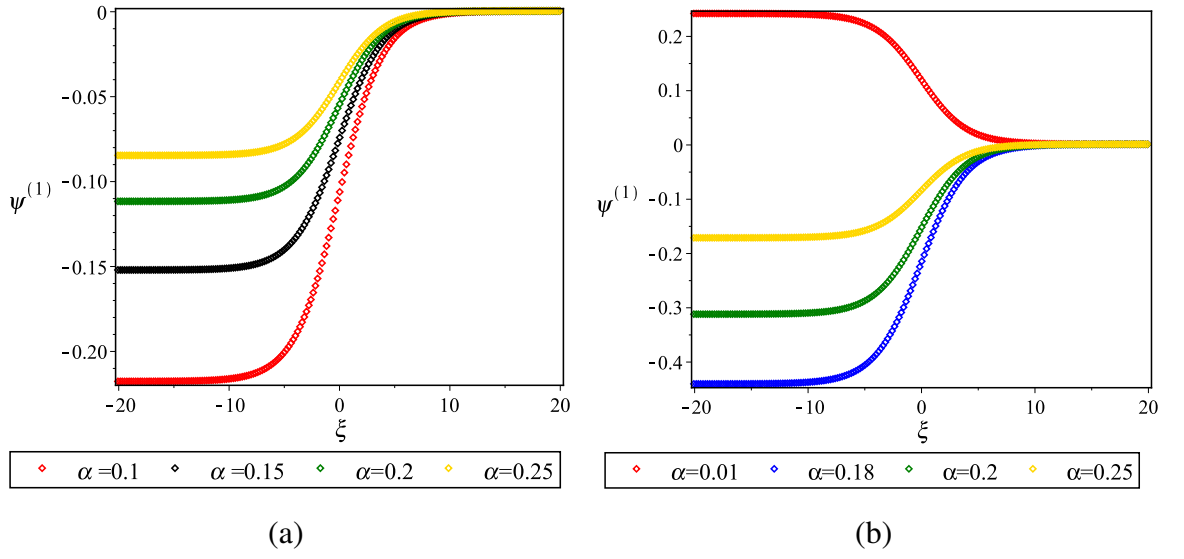


Fig. 3.4 Effect of population of nonthermality i.e. α ($q = 1$) on the electrostatic potential shocks ((a) SC ($\Gamma = 5.6$) and (b) WC ($\Gamma = 0.001$) regimes with $\sigma_{ei} = 0.01$, $\gamma_h = 3$, $r_1 = 0.5$, $r_2 = 0.2$, $\mu = 0.4$ and $\lambda_0 = 0.1$.

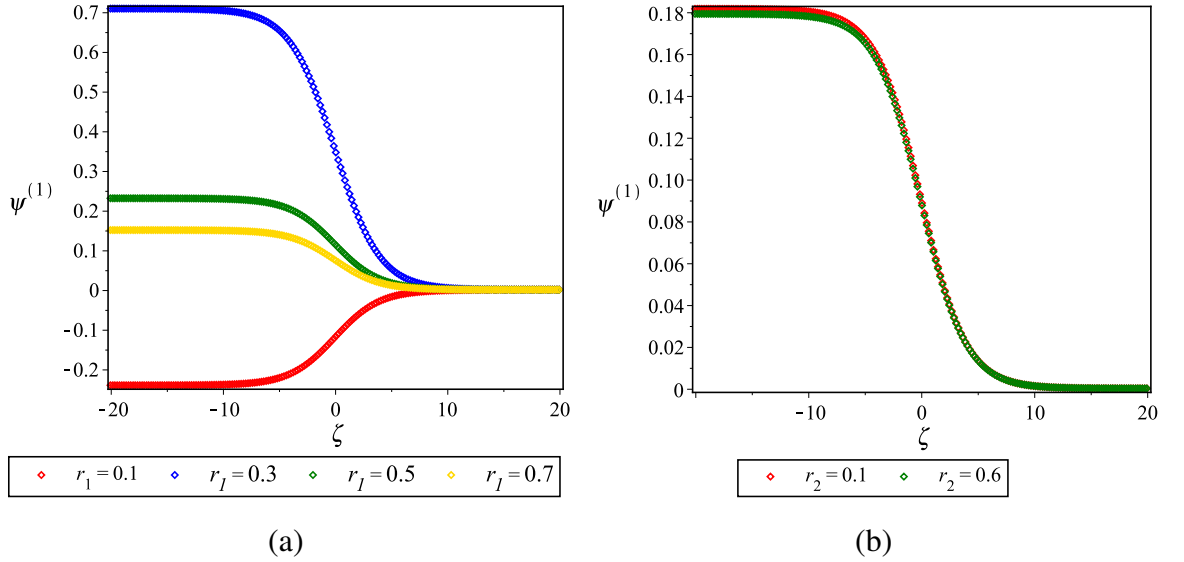


Fig. 3.5 Effect of (a) r_1 ($r_2 = 0.3$) and (b) r_2 ($r_1 = 0.6$) with Maxwellian electrons (i.e. $\alpha = 0, q = 1$) on the electrostatic potential shocks by considering $\Gamma = 0.2$, $\sigma_{ei} = 0.01$, $\gamma_h = 3$, $\mu = 0.4$ and $\lambda_0 = 0.1$.

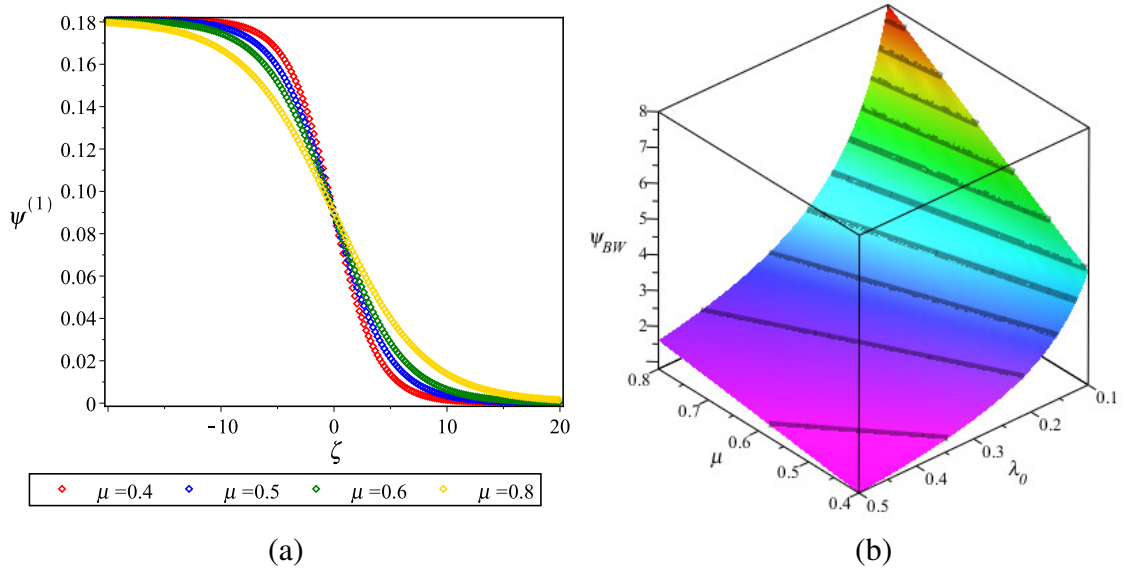


Fig. 3.6 (a) Effect of μ with Maxwellian electrons (i.e. $\alpha = 0, q = 1$) on the electrostatic potential shocks by considering $\Gamma = 0.2$, $\sigma_{ei} = 0.01$, $\gamma_h = 3$, $r_1 = 0.6$, $r_2 = 0.3$, $\mu = 0.4$ and $\lambda_0 = 0.1$; and (b) Variation of width with regards to μ and λ_0 .

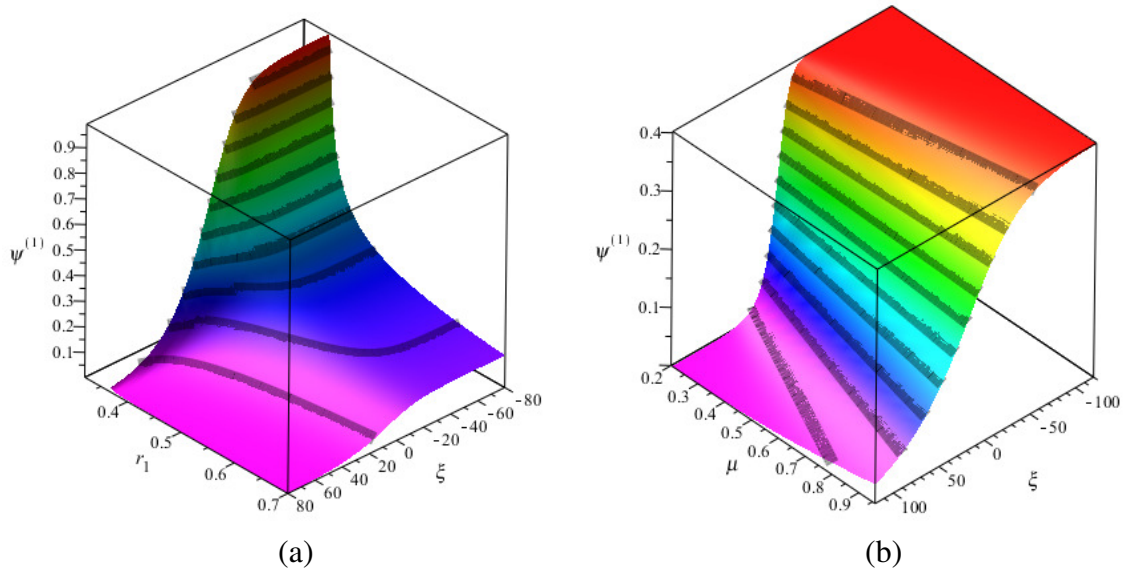


Fig. 3.7 Electrostatic potential HIASWs described by mBE in SCR ($\Gamma = 3.5$) with regards to (a) $r_1 = 0.3$ with $\mu = 0.4$ and (b) μ around $C_v = 0.4459870178$, i.e. $r_1 = 0.4$. The remaining parameters are considered as $\alpha = 0$, $q = 1$, $\sigma_{ei} = 0.01$ and $\lambda_0 = 0.01$.

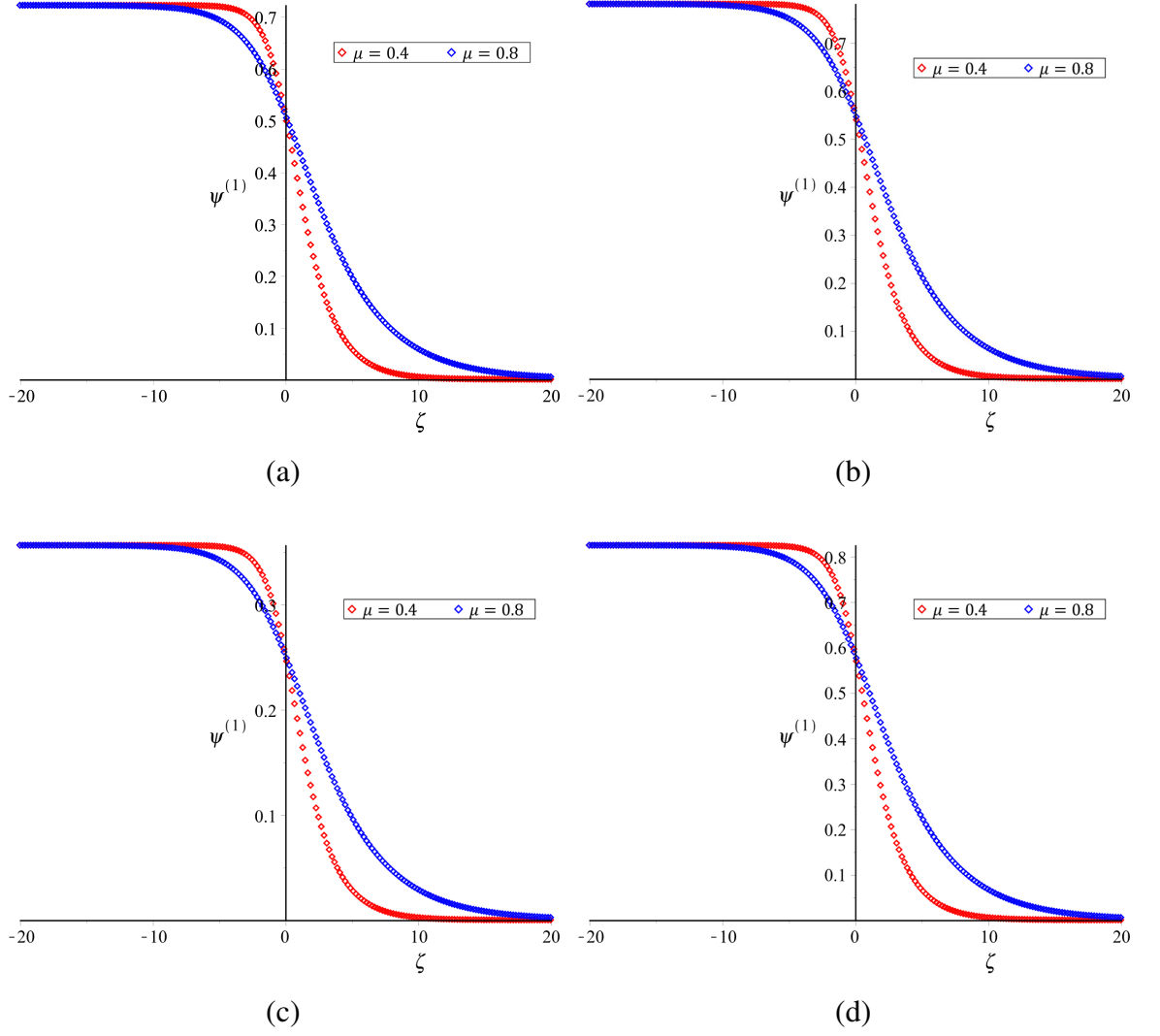


Fig. 3.8 Electrostatic potential HIASWs around (a) $C_v = 0.2320665493$, i.e. $r_1 = 0.3$, (b) $C_v = 0.4459870178$, i.e. $r_1 = 0.5$, (c) $C_v = 0.1463985823$, i.e. $r_1 = 0.2$ and (d) $C_v = 0.4353905585$, i.e. $r_1 = 0.5$ for $\mu = 0.4$ (red color) and $\mu = 0.8$ (blue color).

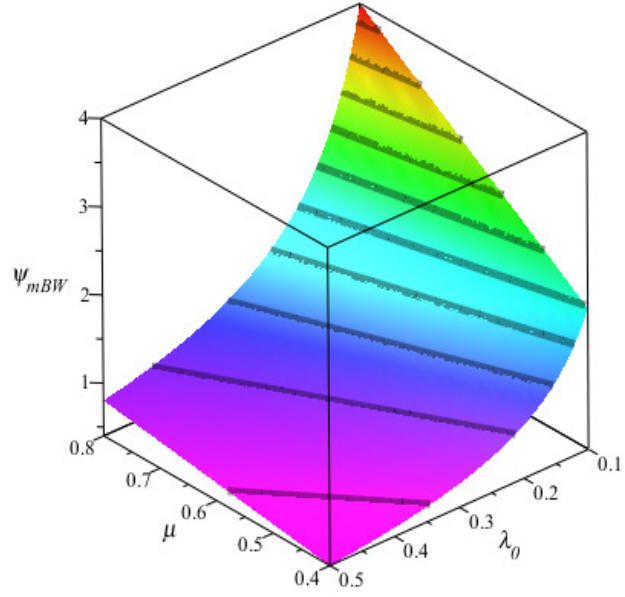


Fig. 3.9 Variation of width around CVs with regards to μ and λ_0 .

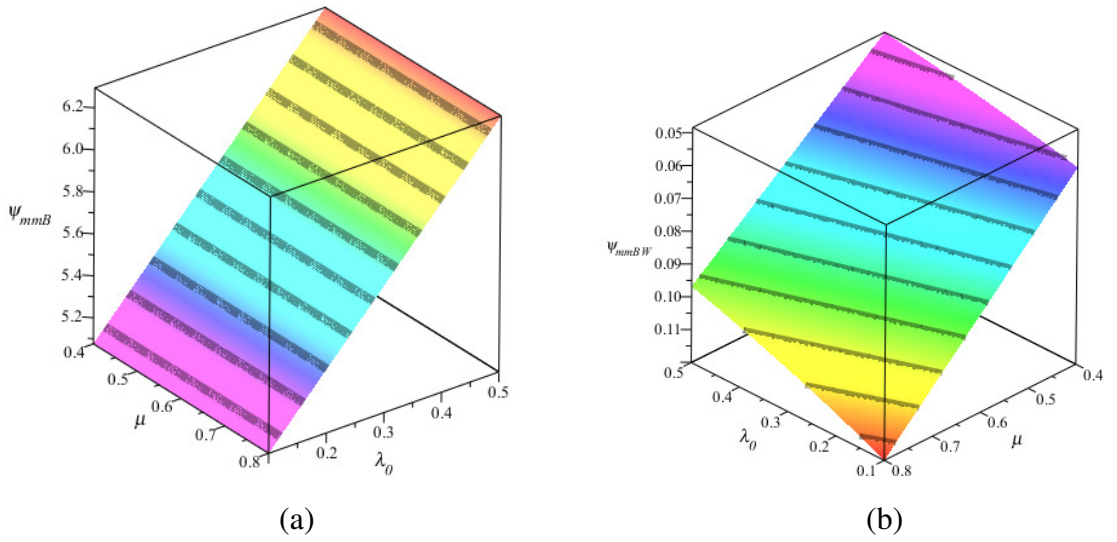


Fig. 3.10 Variation of (a) amplitude and (b) width of shocks described by mmBE with regards to μ and λ_0 with $r_1 = 0.5$, $r_2 = 0.2$, $\Gamma = 0.2$, $\alpha = 0.1$ and $q = 0.3$.

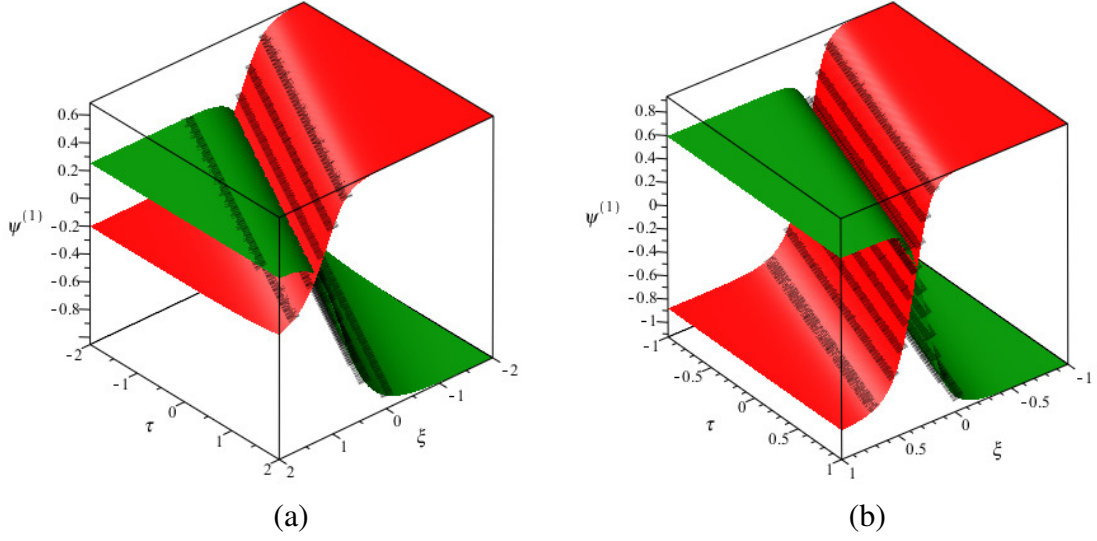


Fig. 3.11 Electrostatic shocks with regards to space and time described by mmBE (a) at $r_1 = 0.19 < C_v = 0.4260022692$ (red surface) and $r_1 = 0.45 > C_v = 0.4260022692$ (green surface) in SCR ($\Gamma = 4.5$) with the population of fast electrons ($\alpha = 0.01$, $q = 1$), $r_2 = 0.1$, $\mu = 0.8$, $\lambda_0 = 0.5$ and (b) at $r_1 = 0.55 > C_v = 0.4459870178$ (red surface) and $r = 0.32 < C_v = 0.4459870178$ (green surface) in WCR ($\Gamma = 0.2$) with the population of fast electrons ($\alpha = 0.2$, $q = 1$), $r_2 = 0.1$, $\mu = 0.5$, $\lambda_0 = 0.3$.

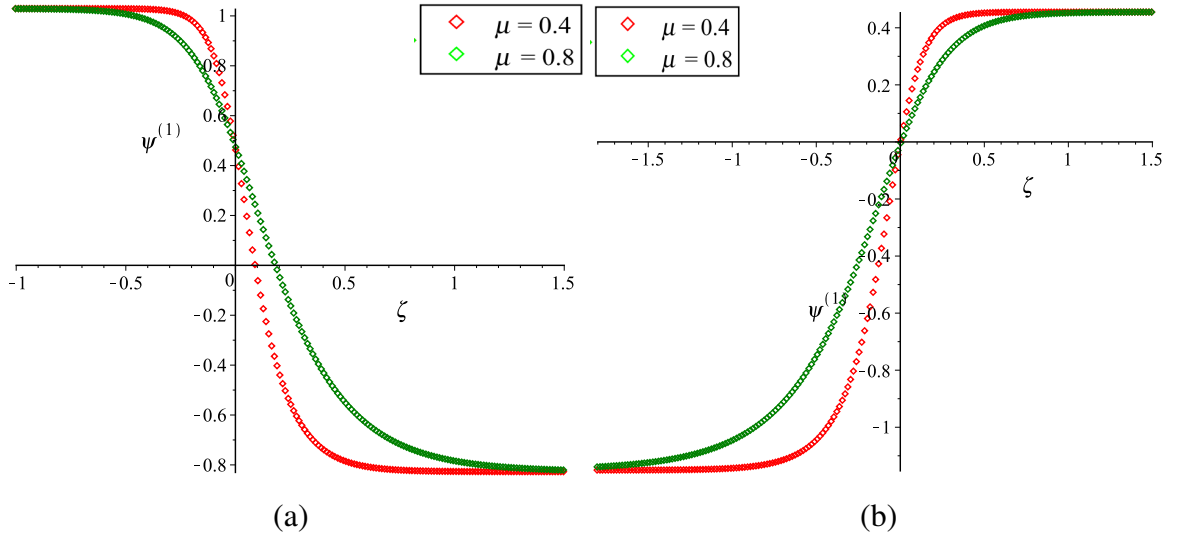


Fig. 3.12 Electrostatic potential HIASWs described by mmBE around $C_v = 0.5895531755$, that is, (a) $r_1 = 0.75$ ($\Lambda = 1$) and (b) $r_1 = 0.45$ ($\Lambda = -1$) in SC plasma ($\Gamma = 3.6$) having superthermality ($\alpha = 0$, $q = 0.4$) for $\mu = 0.4$ (red) and $\mu = 0.8$ (Green) with $r_2 = 0.1$, $\sigma_{ei} = 0.01$, $\gamma_h = 3$, and $\lambda_0 = 0.3$.

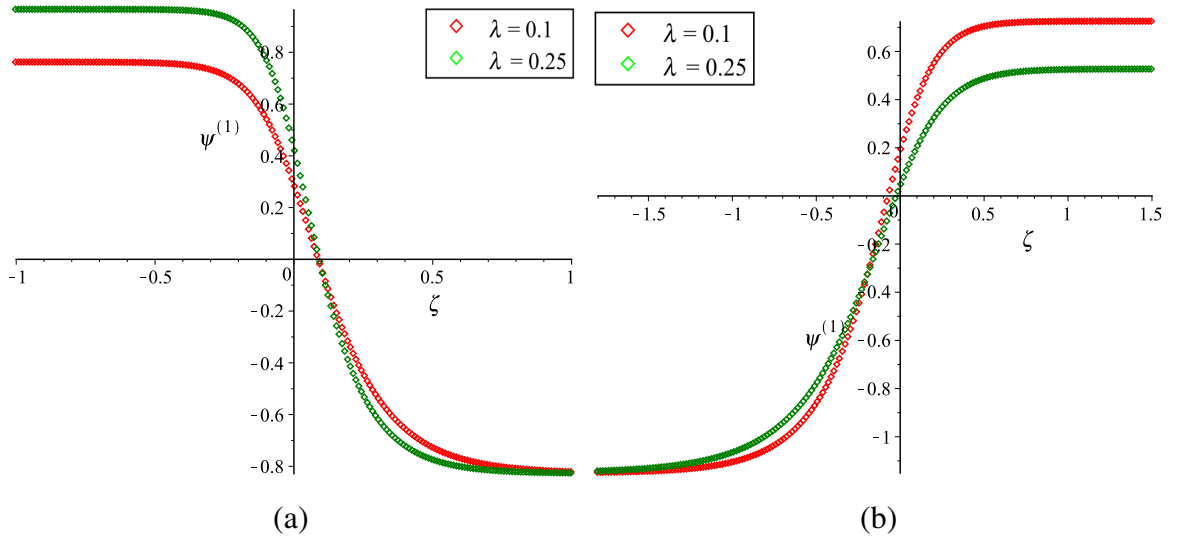


Fig. 3.13 Electrostatic potential ($\psi^{(1)}$) of HIASWs described by mmBE around $C_v = 0.5895531755$, that is, (a) $r_1 = 0.75$ ($\Lambda = 1$) and (b) $r_1 = 0.45$ ($\Lambda = -1$) in SC plasma ($\Gamma = 3.6$) having superthermality ($\alpha = 0, q = 0.4$) for $\lambda_0 = 0.1$ (red) and $\lambda_0 = 0.25$ (Green) with $r_2 = 0.3$, $\sigma_{ei} = 0.01$, $\gamma_h = 3$, and $\mu = 0.8$.

by considering $U(\Gamma) \equiv -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81$ for SCR. Whereas, Figure 3.2 shows the effect of coupling parameter (Γ) along with the constant values of the remaining plasma parameters on nonlinear electrostatic potential HIASW ($\psi^{(1)}$) by considering $U(\Gamma) \equiv -(\sqrt{3}/2)\Gamma^{3/2}$ for WCR with the constant values of the remaining plasma parameters. It is observed from Fig. 3.1 and 3.2 that the amplitude of shocks is remarkably decreased (slightly increased) with the increase of coupling parameter in SCR (WCR). The influence of nonextensivity q ($\alpha = 0$) on the electrostatic potential HIASWs for both cases of SCR and WCR is displayed Figure 3.3(a) and 3.3(b), respectively. It is clearly found from Fig. 3.3 that the growth rate of the amplitude of HIASWs is comparatively higher in SCR rather than WCR, where the magnitude of HIASWs amplitude decreases due to the increase of q . In addition, the amplitude of shocks is comparatively becomes large due to influence of superthermal rather than subthermal electrons in both of SCR and WCR. This means that the strength of nonextensivity is strongly controlled the nonlinearity coefficient A_1 in which A_1 is increased with increased q , as a result, the amplitude of HIASWs is decreased. Figs. 3.4(a) and 3.4(b) display the effect of the population of nonthermality by ignoring nonextensivity ($q = 1$) for both cases

of SCR and WCR, respectively. It is shown from Fig. 3.4 that the amplitude of HIASWs minimized due to the increase of population of nonthermal electrons in both of SCR and WCR. By considering Maxwellian electrons, that is isothermal electrons ($q = 1$, $\alpha = 0$), the effects of r_1 and r_2 on HIASWs are displayed in Figs. 3.5(a) and 3.5(b), whereas the effect of μ on electrostatic HIASWs and widths of electrostatic HIASWs are displayed in Figs. 3.6(a) and 3.6(b), respectively. It is found from Fig. 3.6 that the magnitude of HISWs amplitude decreases with the growing of r_1 , but very slightly decreases with the remarkable growing of r_2 . From the physical point of view, the driving force for the HIASWs is actually provided by HIs inertia in the considered plasmas. With the increase of HI density, that is, depopulation of electrons, the driving force decreases. As a result, the energy of HIASWs losses. Besides, Figs. 3.6(a) clearly indicates that the monotonically HISWs occur with the influence of viscosity coefficient of HIs in which the magnitude of HISWs amplitude unchanged but width of HISWs increased with increased in μ . From the physical point of view, the dissipative media is significantly dominant and leading to monotonically shocks in which the width of shocks increases due to the increase of only the viscosity coefficient of HIs in the plasmas. It is noted that both compressive rarefactive shocks described by BE are supported in the SC plasmas when the SC plasma not only exists with the isothermal, superthermal and subthermal but also nonthermal electrons in space and astrophysical environments.

Figures 3.7 shows the effects of r_1 ($\mu = 0.4$) and μ (around C_v) on the electrostatic HIASWs described by mBE in SCR. It is obversely found from Fig. 3.7(a) that mBE has not only supported HIASWs around C_v but also for $r_1 > C_v$ in which the shock amplitude decreases with the increase of r_1 . Figure 3.8 also displays the effect of μ on the electrostatic HIASWs around by considering four different CVs. It is investigated from Figs. 3.7(b) and 3.8 that the viscosity coefficient of HIs strongly plays a vital role to the formation of monotonically SWEs around the critical values. Also, the compressive SWEs are only supported around CVs. It is noted that the amplitudes SWEs described by mBE are strongly dependent on the CVs based on the related plasma parameters except μ because ψ_{mB} is independent of μ . Whereas, the width (ψ_{mBW}) of SWEs described by mBE is only dependent of μ and λ_0 . Fig. 3.9 shows the variation of width with regards to μ and λ_0 , which is shown that the width of SWEs are remarkably increased (decreased) with the

increase of μ (λ_0).

Figure 3.10 (a) and 3.10(b) illustrate the variation of amplitude and width of electrostatic HIASWs described by mmBE at the neighboring of CVs with regards to μ and λ_0 , which is clearly shown that the width of SWEs decreased, but amplitude remains unchanged with the increase of μ . This means that the viscosity coefficient of plasma particles are only responsible to the formation of SWEs not only around the CVs but at CVs in the plasmas. Whereas, both of the amplitude and width of SWEs around CVs or very closed to CVs increased with the increased of λ_0 . Based on the values given in Table 3.2, Fig. 3.11 illustrates the structure of HIASWs ($\psi^{(1)}$). Finally, Figs. 3.12 shows the electrostatic potential HIASWs described by mmBE around C_v in SC plasma having superthermality. While, Figs. 3.13 shows the electrostatic potential HIASWs described by mmBE around C_v in WC plasma having the strength of nonextensivity (superthermality). It is obviously observed from the above figures that the monotonically SWEs are only appeared with influence of viscosity coefficient of HI fluid in the considered plasmas. Figs. 3.11, 3.12, and 3.13 also indicated that the positive and negative potential SWEs are not only supported but also SWEs in both regimes of space is supported in the plasmas due to the correct solution of mmBE. It is also found that SWEs are propagating left to right and right to left by depending on the neighboring values of CVs. When $\psi_{mmBW} < 0$ and $\psi_{mmB} < 0$, $\psi_{mmBW} < 0$, then the solution as in Eq. (3.37) of mmBE is represented the positive and negative DL structures in the considered plasmas. It is remarkable to be noted that Figures 3.11, 3.12, and 3.13 are displayed by considering all cases of nonextensivity and nonthermality. These figures are clearly shown that the SWEs (DLs) are supported when $r_1 > C_v$ ($r_1 < C_v$) for the case of isothermality, superthermality and nonthermality, whereas SWEs are only supported for case of subthermality.

It can be emphasized here that the outcomes from this work should be helpful for better understanding the process of SWEs in the plasmas having strongly coupled pair-ions, nonextensivity as well nonthermality electrons in many SAEs (especially in stellar polytropes, hadronic matter and quark-gluon plasma, black halos, etc. [60],[62], [129],[130]. To conclude, one may perform to do an experiment in plasma laboratory by considering the predicted theoretical results in this article in such an SC plasma with the effect of adiabaticity and nonextensivity or nonthermality.

3.7 SUMMARY

In this chapter, the nonlinear propagation of HISW in an unmagnetized SC plasma consisting of HI fluids, MLIs and electrons (electrons following (r, q) -distribution) has been investigated. RPT has been used to reduce the governing fluid equations to the nonlinear equations, namely, BE, mBE and mmBE for studying the SWEs, which provides only SWEs in the plasmas. By directly integrating, the appropriate shock wave solutions of mBE and mmBE have been formulated. The effect of plasma parameters has been described by using the useful solutions of the nonlinear equations. It is found that BE is supported both of positive and negative electrostatic SWEs, whereas mBE is only supported the positive SWE around the CVs. Also, BE and mBE are not applicable to study the basic features (viz. polarity, amplitude, width, etc.) of SWEs at CVs. Whereas, mmBE is not useful for studying the nature of SWEs at the neighbouring values of CVs as well as CVs but also the existence of DLs. It is found that not only SWEs but also DLs are supported at the neighboring values of CVs, which reports for the first time via the appropriate shock wave solutions. The results are general and applicable to SAEs or PL, where SWEs are formed.

Chapter 4: EFFECTS OF POLARIZATION FORCE AND COUPLING PARAMETER ON DUST ACOUSTIC SHOCKS AROUND THE CRITICAL AND SUPERCRITICAL VALUES IN A STRONGLY COUPLED DUSTY PLASMA

4.1 INTRODUCTION

Over the past two decades, the dust-acoustic (DA) wave has been studied rigorously for different dusty plasma systems [2],[85],[86],[131]. It is a low frequency electrostatic wave similar to the ion acoustic wave, with the inertia arising from the mass of the dust particles and the restoring force is provided by the thermal pressure of electrons and ions. Previous studies on dusty plasma have primarily focused on negatively charged dust grains. However, dust grains can also acquire positive charges by different mechanisms, such as, photoemission in the presence of a flux of ultra-violet photons, thermionic emission induced by the radiative heating and secondary emission of the electrons from the surface of the dust grains [132]. Lee et. al. [133] have also experimentally investigated the characterization of strongly coupled dusty plasma by producing in argon supercritical fluids. Encouraged by these theoretical predictions and experimental observations, numerous authors [12], [36], [67],[85], [86], [89],[131],[134–136] have considered a dusty plasma system with dust of opposite potential, and investigated the linear and nonlinear DA wave propagation characteristics. Most earlier works have been confined to weakly coupled dusty plasmas, valid for $\Gamma < 1$, where $\Gamma = (Z_d^2/I_D T_d) \exp(-I_d/\lambda_d)$ is the coupling parameter (Z_d , I_d , T_d and λ_d stands for dust grain charge, inter grain distance, temperature of dust and dusty plasma Debye radius). Even in the strongly coupled regime, many researchers [92–95] have overlooked the impact of Γ on the acoustic wave phenomena in various plasma environments due to cumbersome mathematical formulation.

On the other hand, a remarkable development has been observed in the study of the propagation characteristics of DA waves in the presence of polarization force. This

force arises as a consequence of the interaction between the thermal ions and highly negatively charged dust grains. Such force is generated due to the interaction between the energetic electrons and charged dust grains in the dusty plasmas. Hamaguchi et al.[137] explored the importance of the polarization force. Later, Khrapak et al. [138] investigated the effect of polarization force on linear propagation of DA waves by considering the dusty plasma environment. It is found that Khrapak et al. [138] defined the polarization force, mathematically, as $f_p = -(Q^2 \nabla \lambda_d / 2\lambda_d^2)$, where Q is the gained charge, $\lambda_d = \lambda_{de} / \sqrt{1 + \lambda_{de}^2 / \lambda_{di}^2} = \lambda_{de} / \sqrt{(1 + N_i T_e / N_e T_i)}$, $\lambda_{de(di)} = \sqrt{T_{e(i)} / 4\pi e^2 N_{e(i)}}$, $T_{e(i)}$ is the electron (ion) temperature in energy unit, $N_{e(i)}$ is the electron (ion) number density and e is the charge of electron. By considering $T_e \nabla N_e = e N_e \nabla \psi$ (ψ is the electrostatic potential) and $Q = Z_d e$ (Z_d indicates the electron number residing on the dust grain surface), f_p can be simplified as $f_p = Z_d e P_f \sqrt{(N_e / N_{e0})} \nabla \psi$, where $P_f = Z_d e^2 / 4 T_e \lambda_{de0}$ is measuring the impact of polarization force, N_{e0} is the electron density at equilibrium position and $\lambda_{de0} = \sqrt{T_e / 4\pi N_{e0} e^2}$. In order to formulate f_p , the relation $T_i N_e \ll T_e N_i$ is considered which is in good agreement with the charged dust grains for any dusty plasma environments. Due to highly energetic electrons rather than other charged particle and comparatively low energetic ions rather than electrons, one can express the electron and ion density functions as

$$N_e = N_{e0} \left[1 + (q-1) \frac{e\psi}{T_e} \right]^{\frac{(q+1)}{2(q-1)}} \times \left[1 + \Omega_1 \left(\frac{e\psi}{T_e} \right) + \Omega_2 \left(\frac{e\psi}{T_e} \right)^2 \right],$$

and

$$N_i = N_{i0} \exp \left(-\frac{e\psi}{T_i} \right),$$

where

$$\Omega_1 = -16q\alpha / (3 - 14q + 15q^2 + 12\alpha), \Omega_2 = -\Omega_1 (2q - 1).$$

by taking (α, q) -velocity distribution function for electrons [72],[117] and Maxwellian velocity distribution function for ions, respectively. Since, the (α, q) -velocity distribution function of electrons are applicable to describe all cases of electrons energy. However, the interacting charged particle potential energies considerably exceed their thermal kinetic energies due to a comparatively high charge density. As a result, the particles (charged

dust) exhibit collective behavior similar to a liquid and the plasma system becomes strongly coupled. Many researchers [94],[95],[140],[141] have already reported the propagation characteristics of acoustic waves by considering various plasma assumptions in SC plasma environments. Asaduzzaman et al. [140] investigated the impact of polarization force on the linear propagation of DA waves in the dusty plasmas with superthermal electrons. Asaduzzaman and Mamun [141] described the impact of polarization force with the presence of nonthermal electrons and negatively charged dust grains. Zobaer and Mamun [95] observed the effects of polarization force on the nonlinear propagation characteristics of DA waves in strongly coupled dusty plasmas with suprathermal electrons. Ema et al. [94] examined nonlinear ion modes in a strongly coupled plasma in the presence of nonthermal fluids and polarization force.

In most investigations of strongly coupled plasmas, researchers have studied the nonlinear propagation of solitary and shock waves by deriving either Korteweg-de Vries (KdV) and Burgers equations or the KdV–Burgers equation from the model equations considered. However, they were unable (i) to describe what happens in the soliton and shock wave propagation due to the impact of Γ , (ii) to examine the shock wave propagation around the critical values (CVs) of any particular plasma parameter, (iii) to examine the shock wave propagation around the super-CVs (SCVs) of any particular plasma parameter and (iv) to analyze the impact of polarization force together with coupling parameter on the shock wave excitations in the strongly coupled dusty plasmas. Therefore, further studies on strongly coupled dusty plasmas are essential. The main contribution of this chapter is summarized below.

- (i) An unmagnetized strongly coupled dusty plasma is considered by the mixture of (α, q) -velocity distributed electrons, Boltzmann distributed ions and negatively charged dust grains.
- (ii) The nonlinear propagation of DA shock wave excitations in the considered plasma system is explored.
- (iii) The shock wave excitations are examined not around the CVs but also around SCVs of some particular plasma parameters.

It is worth noting that this study contributes to our understanding of the propagation of DA waves in the presence of both polarization force and coupling parameter, not only

around the CVs but also around SCVs of specific parameters. The existence of CVs and SCVs of any specific parameter and the effect of plasma parameter on electrostatic DA shock waves are also reported.

4.2 THEORETICAL MODEL EQUATIONS

Let us consider an unmagnetized coupled dusty plasma system having (α, q) -velocity distributed electrons, Boltzmann distributed ions and negatively charged dust grains. Since the ions and electrons are assumed to be weakly coupled due to ions and electrons having higher temperature and smaller electric charge and the dust grains are strongly coupled because of their lower temperature and larger electric charge. Based on the above assumption, the dimensionless continuity and momentum equations are formulated in the following forms:

$$\frac{\partial N_d}{\partial t} + \frac{\partial}{\partial z} (N_d U_d) = 0, \quad (4.1)$$

$$\left(1 + \tau_r \frac{\partial}{\partial t}\right) \left[N_d \left\{ \left(\frac{\partial}{\partial t} + U_d \frac{\partial}{\partial z} \right) U_d \right\} - N_d \frac{\partial \psi}{\partial z} + N_d P_f \sqrt{N'_e} \frac{\partial \psi}{\partial z} + \mu_d T_{di} \frac{\partial N_d}{\partial z} \right] = \mu \frac{\partial^2 U_d}{\partial z^2}, \quad (4.2)$$

where τ_r , μ_d and μ are the viscoelastic relaxation time, compressibility and longitudinal viscosity coefficient, respectively. τ_r and μ_d are obtained [95],[140] as

$$\tau_r = \mu \frac{T_e}{T_d} \left[1 - \mu_d + \frac{5}{14} U(\Gamma) \right]^{-1}, \mu_d = \left[1 + \frac{1}{3} U(\Gamma) + \frac{\Gamma}{9} \frac{\partial U(\Gamma)}{\partial \Gamma} \right], T_{di} = \frac{T_d}{T_i}. \quad (4.3)$$

The plasma system is closed based on the equilibrium condition by the following:

$$\frac{\partial^2 \psi}{\partial z^2} - N'_e + N'_i - N_d = 0, \quad (4.4)$$

where

$$N'_e = [1 + (q-1) \sigma \psi]^{\frac{(q+1)}{2(q-1)}} \times \left[1 + \Omega_1 (\sigma \psi) + \Omega_2 (\sigma \psi)^2 \right], N'_i = \rho_{ie} e^{-\psi},$$

$$\Omega_1 = -\frac{16q\alpha}{3 - 14q + 15q^2 + 12\alpha}, \Omega_2 = \frac{16q(2q-1)\alpha}{3 - 14q + 15q^2 + 12\alpha}. \quad (4.5)$$

It is noted that Eq. (4.5) can be expanded as

$$N_e' = 1 + \left(\frac{q+1}{2} + \Omega_1 \right) \sigma \psi + \left(\frac{(q+1)(3-q)}{8} + \frac{q+1}{2} \Omega_1 + \Omega_2 \right) \sigma^2 \psi^2 +$$

$$\left(\frac{(q+1)(3-q)(5-3q)}{48} + \frac{(q+1)(3-q)}{8} \Omega_1 + \frac{q+1}{2} \Omega_2 \right) \sigma^3 \psi^3 + \dots,$$

$$N_i' = \rho_{ie} \left[1 - \psi + \frac{\psi^2}{2} + \frac{\psi^3}{6} + \dots \right].$$

In the above equations, N_d , U_d and ψ are normalized by N_{d0} (unperturbed dust particle number density), $C_d = (Z_d T_d / m_d)^{1/2}$ (dust acoustic speed) and T_i / e , respectively. $U(\Gamma)$ is applicable to measure the excess internal energy, τ_r (viscoelastic relaxation time) is normalized $\omega_{dp}^{-1} = (m_d / 4\pi N_{d0} Z_d e^2)^{1/2}$ (dust plasma period),

$$\mu = \left(1 / m_d N_{d0} \omega_{dp} \lambda_{dp}^2 \right) [\mu_b + (4/3) \zeta_b]$$

(normalized viscosity coefficient), q is a parameter measuring the nonextensivity of electrons, and α is a parameter indicating the number of fast (nonthermal) electrons, where T_d , T_i , e , Z_d , m_d , λ_{dp} , and μ_b (ζ_b) are the dust temperature, ions temperature, electron charge, charged number of dust particles, mass of dust particles, dust Debye length and transport coefficients of shear (bulk) viscosity, respectively. Due to the normalization of Eqs. (4.1), (4.2) and (4.4), and maintaining of charge neutrality condition, $\sigma = T_i / T_e$ (T_e is the electrons temperature) and $\rho_{ie} = N_{i0} / N_{e0}$ (N_{e0} and N_{i0} are the unperturbed densities of electrons and ions, respectively) are obtained. It is noted that the term $N_d P_f \sqrt{N_e} \frac{\partial \psi}{\partial z}$ in Eq. (4.2) is the polarization force term, where $P_f = Z_d e^2 / 4 T_e \lambda_{de0}$ is a parameter measuring the influence of polarization force. This force mainly arises from the interaction between thermal ions and highly negatively charged dust grains. Additionally, the term $\mu_d T_{di} \frac{\partial N_d}{\partial z}$ as in Eq. (4.2) occurs due to the thermal pressure of dust. The proposed model equations also show good agreement with earlier investigation [95], with the absence of nonextensivity. It is noted that the nomenclature of the physical variables and parameters is given in Table 4.1.

Table 4.1 List of Physical Variables/Parameters along with usual Meanings

Variable/Parameter	Meaning
N_d	Number density of dust particles
U_d	Dust fluid velocity
ψ	Electrostatic wave potential
N_{j0}	Unperturbed density, where $j = d, i$ and e for dust, ions and electrons
T_j	Temperature for j species
P_f	Measuring the impact of polarization force (PF)
Γ	coupling parameter (CP)
$q(\alpha)$	Strength of electron nonextensivity (population of nonthermal electrons)
μ	Normalized viscosity coefficient
$T_{di}(\sigma)$	Dust to ion temperature ratio's (ion to electron temperature ratio's)
ρ_{ie}	Ion to electron unperturbed density ratio's
$P_s(s_0)$	Linear phase speed (constant speed of reference frame)

4.3 BURGERS EQUATION WITH QUADRATIC NONLINEARITY

Let us begin by scaling out for z and t (see details in [117]) as

$$\zeta = \varepsilon^r(z - p_s t), \tau = \varepsilon^{2r} t, \quad (4.6)$$

and the expansions of perturbed quantities are defined as

$$\begin{bmatrix} N_d \\ U_d \\ \psi \end{bmatrix} = \begin{bmatrix} \rho_{de} \\ 0 \\ 0 \end{bmatrix} + \sum_i^\infty \varepsilon^i \begin{bmatrix} N_d^{(i)} \\ U_d^{(i)} \\ \psi^{(i)} \end{bmatrix}, \quad (4.7)$$

where $\rho_{de} = Z_d N_{d0}/N_{e0}$, p_s is the linear phase speed and ε is a proper fraction measuring the weakness of dissipation. Employing Eqs. (4.5) and (4.7) into Eqs. (4.1), (4.2), and (4.4), along with $r = 1$, several order of ε , that is, $O(\varepsilon^i)$, $i = 2, 3, 4, \dots$ -equations are

obtained. From the $O(\varepsilon^2)$ -equations

$$\begin{aligned}
& -p_s \left(\frac{\partial}{\partial \xi} N_d^{(1)} \right) + p_{de} \left(\frac{\partial}{\partial \xi} U_d^{(1)} \right) = 0, \\
& \mu_d \tau_r \left(\frac{\partial}{\partial \xi} N_d^{(1)} \right) + p_{de} p_s \left(\frac{\partial}{\partial \xi} U_d^{(1)} \right) + (p_{de} - 1) P_f \left(\frac{\partial}{\partial \xi} \psi^{(1)} \right) = 0, \\
& 0 = -(H_1 \sigma + \rho_{di}) \psi^{(1)} - \frac{\partial}{\partial \xi} N_d^{(1)},
\end{aligned}$$

one can yield:

$$N_d^{(1)} = - \left(\frac{\rho_{de} (P_f - 1)}{T_d \mu_d - p_s^2} \right) \psi^{(1)}, U_d^{(1)} = - \left(\frac{p_s (P_f - 1)}{T_d \mu_d - p_s^2} \right) \psi^{(1)}, \quad (4.8)$$

and

$$p_s = \sqrt{T_d \mu_d + \frac{\rho_{de} (-P_f + 1)}{\sigma H_1 + \rho_{ie}}}, \quad (4.9)$$

where

$$H_1 = \left[\frac{q+1}{2} - \frac{16q\alpha}{3-14q+15q^2+12\alpha} \right] \sigma. \quad (4.10)$$

For $O(\varepsilon^3)$, yields

$$\frac{2p_{de} (P_f - 1)^2 p_s}{(T_{di} \mu_d - p_s^2)^2} \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} - p_s \frac{\partial N_d^{(2)}}{\partial \xi} + p_{de} \frac{\partial U_d^{(2)}}{\partial \xi} - \frac{p_{de} (P_f - 1)}{T_{di} \mu_d - p_s^2} \frac{\partial \psi^{(1)}}{\partial \tau} = 0, \quad (4.11)$$

$$\begin{aligned}
& \frac{\tau_r p_s \mu_d T_{di} p_{de} (P_f - 1)}{T_{di} \mu_d - p_s^2} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} - \tau_r p_s p_{de} P_f \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} - \frac{\tau_r p_s^3 (P_f - 1) p_{de} T_{di} \mu_d - p_s^2}{\frac{\partial^2 \psi^{(1)}}{\partial \xi^2}} \\
& + p_{de} P_f a_1 \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} + \tau_r p_s p_{de} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} + \mu_d T_{di} \frac{\partial N_d^{(2)}}{\partial \xi} - \frac{p_{de} (P_f - 1) P_f}{T_{di} \mu_d - p_s^2} \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} \\
& + p_{de} P_f \frac{\partial \psi^{(2)}}{\partial \xi} - p_{de} p_s \frac{\partial U_d^{(2)}}{\partial \xi} + \frac{\mu p_s (P_f - 1)}{T_{di} \mu_d - p_s^2} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} + \frac{p_{de} (P_f - 1)}{T_{di} \mu_d - p_s^2} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} \\
& - p_{de} \frac{\partial \psi^{(2)}}{\partial \xi} - \frac{p_{de} p_s (P_f - 1)}{T_{di} \mu_d - p_s^2} \frac{\partial \psi^{(1)}}{\partial \tau} = 0,
\end{aligned} \quad (4.12)$$

$$0 = -(H_1 \sigma + \rho_{di}) \frac{\partial \psi^{(2)}}{\partial \xi} - 2(H_2 - \rho_{di}) \sigma^2 \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} - \frac{\partial N_d^{(2)}}{\partial \xi}, \quad (4.13)$$

where

$$H_2 = \left[\frac{(q+1)(3-q)}{8} - \frac{8q(q+1)\alpha}{3-14q+15q^2+12\alpha} + \frac{16q(2q-1)\alpha}{3-14q+15q^2+12\alpha} \right] \sigma^2, a_1 = \frac{H_1}{2}. \quad (4.14)$$

Eliminating $N_d^{(2)}$, $U_d^{(2)}$, and $\psi^{(2)}$ from Eqs. (4.11)–(4.13), the following Burgers equation with quadratic nonlinearity is derived:

$$\frac{\partial \psi^{(1)}}{\partial \tau} + A \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} = C \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \quad (4.15)$$

where

$$A = -\frac{\rho_{de} T_d \mu_d - p_s^2}{+} \frac{2\rho_{de} p_s^2 P_f^2}{(T_d \mu_d - p_s^2)^2} - \frac{4\rho_{de} p_s^2 P_f}{(T_d \mu_d - p_s^2)^2} + 2T_d \mu_d \rho_{ie} + 2p_s^2 H_2 \sigma^2 - 2p_s^2 \rho_{ie} \\ - 2T_d \mu_d H_2 \sigma^2 + \rho_{de} P_f a_1 + \frac{2\rho_{de} P_f}{T_d \mu_d - p_s^2} - \frac{\rho_{de} P_f^2}{T_d \mu_d - p_s^2} + \frac{2\rho_{de} p_s^2}{(T_d \mu_d - p_s^2)^2}, \\ C = \frac{\mu}{2}. \quad (4.16)$$

To determine the analytical solution of Burgers with quadratic nonlinearity, one can consider the traveling wave transform as

$$\psi^{(1)}(\xi, \tau) \sim \psi^{(1)}(\zeta), \zeta = \xi - s_0 \tau \quad (4.17)$$

with

$$\psi^{(1)} \rightarrow 0, \frac{d\psi^{(1)}}{d\zeta} \rightarrow 0, \frac{d^2\psi^{(1)}}{d\zeta^2} \rightarrow 0, as \zeta \rightarrow \pm\infty, \quad (4.18)$$

where s_0 is the reference speed. Eq. (4.15) is then converted to the following nonlinear ordinary differential equation (ODE):

$$-s_0 \psi^{(1)} + \frac{A}{2} \left(\psi^{(1)} \right)^2 = C \frac{d\psi^{(1)}}{d\zeta}. \quad (4.19)$$

By directly integrating Eq. (4.19), the well established shock wave solution of Eq. (4.15) is defined as

$$\psi^{(1)} = A_0 \left\{ 1 - \tanh \left(\frac{\zeta}{W_0} \right) \right\}, \quad (4.20)$$

where $A_0 = (s_0/A)$ and $W_0 = (2C/s_0)$ are respectively the amplitude and width of DA waves. Eq. (4.15) yields positive and negative shocks only if $A > 0$ and $A < 0$, respectively. However, Eq. (4.15) is not applicable to describe the nature of shock wave phenomena if $A = 0$. This situation may arise for some particular values of any related parameter, so-called the CVs. One can determine the CVs within the proper ranges by setting $A = 0$ with the aid of computational software, like Maple 18. To overcome such difficulty, one can apply the reductive perturbation technique by taking higher-order powers of the scaling variables into account.

4.4 BURGERS EQUATION WITH CUBIC NONLINEARITY

Let us now change the scale for z and t by taking $r = 2$ for reporting the shock wave phenomena around CVs. As a result, several order of ε , that is, $O(\varepsilon^j)$, $i=3, 4, 5$, ...-equations are obtained. The $O(\varepsilon^3)$ -equations provide the same relations as in Eqs. (4.8), (4.9), and (4.10). From the $O(\varepsilon^4)$ -equations

$$\begin{aligned} & \frac{2p_{de}(P_f - 1)^2 p_s}{(T_{di}\mu_d - p_s^2)^2} \psi^{(1)} \left(\frac{\partial}{\partial \xi} \psi^{(1)} \right) - p_s \left(\frac{\partial}{\partial \xi} N_d^{(2)} \right) + p_{de} \left(\frac{\partial}{\partial \xi} U_d^{(2)} \right) = 0, \\ & \frac{\tau_r p_s \mu_d T_{di} p_{de} (P_f - 1)}{T_{di}\mu_d - p_s^2} \left(\frac{\partial^2}{\partial \xi^2} \psi^{(1)} \right) - \tau_r p_s \left(\frac{\partial^2}{\partial \xi^2} \psi^{(1)} \right) p_{de} P_f + \tau_r p_s \left(\frac{\partial^2}{\partial \xi^2} \psi^{(1)} \right) p_{de} \\ & - \frac{\tau_r p_s^3 (P_f - 1) p_{de}}{T_{di}\mu_d - p_s^2} \left(\frac{\partial^2}{\partial \xi^2} \psi^{(1)} \right) + p_{de} P_f a_1 \psi^{(1)} \left(\frac{\partial}{\partial \xi} \psi^{(1)} \right) - p_{de} p_s \left(\frac{\partial}{\partial \xi} U_d^{(2)} \right) \\ & + \mu_d T_{di} \left(\frac{\partial}{\partial \xi} N_d^{(2)} \right) - \frac{p_{de} (P_f - 1) P_f}{T_{di}\mu_d - p_s^2} \psi^{(1)} \left(\frac{\partial}{\partial \xi} \psi^{(1)} \right) + p_{de} P_f \left(\frac{\partial}{\partial \xi} \psi^{(2)} \right) \\ & + \frac{\mu p_s (P_f - 1)}{T_{di}\mu_d - p_s^2} \left(\frac{\partial^2}{\partial \xi^2} \psi^{(1)} \right) + \frac{p_{de} (P_f - 1)}{T_{di}\mu_d - p_s^2} \left(\frac{\partial^2}{\partial \xi^2} \psi^{(1)} \right) - p_{de} \left(\frac{\partial}{\partial \xi} \psi^{(2)} \right) = 0, \\ & 0 = -(H_1 \sigma + \rho_{di}) \left(\frac{\partial}{\partial \xi} \psi^{(2)} \right) - 2(H_2 - \rho_{di}) \sigma^2 \psi^{(1)} \left(\frac{\partial}{\partial \xi} \psi^{(1)} \right) - \frac{\partial}{\partial \xi} N_d^{(2)}, \end{aligned}$$

one obtains:

$$N_d^{(2)} = \frac{\left(p_s (P_f - 1)^2 \left\{ \psi^{(1)} \right\}^2 + U_d^{(2)} (T_d \mu_d - p_s^2)^2 \right) \rho_{de}}{(T_d \mu_d - p_s^2)^2 p_s}, U_d^{(2)} = h_{11} \psi^{(2)} + h_{12} \left\{ \psi^{(1)} \right\}^2 \quad (4.21)$$

and

$$\frac{1}{2(T_d \mu_d - p_s^2)} \times A \times \left\{ \psi^{(1)} \right\}^2 = 0, \quad (4.22)$$

where

$$\begin{aligned} h_{11} &= -\frac{p_s P_f T_d^2 a_1 \mu_d^2}{2(T_d \mu_d - p_s^2)^3} + \frac{p_s^3 P_f T_d a_1 \mu_d}{(T_d \mu_d - p_s^2)^3} - \frac{p_s^5 P_f a_1}{2(T_d \mu_d - p_s^2)^3} - \frac{p_s P_f^2 T_d \mu_d}{2(T_d \mu_d - p_s^2)^3} \\ &\quad - \frac{p_s^3 P_f^2}{2(T_d \mu_d - p_s^2)^3} + \frac{p_s P_f T_d \mu_d}{(T_d \mu_d - p_s^2)^3} + \frac{p_s^3 P_f}{(T_d \mu_d - p_s^2)^3} - \frac{p_s T_d \mu_d}{2(T_d \mu_d - p_s^2)^3} - \frac{p_s^3}{2(T_d \mu_d - p_s^2)^3}, \\ h_{12} &= -\frac{p_s P_f T_d^2 \mu_d^2}{(T_d \mu_d - p_s^2)^3} + 2 \frac{p_s^3 P_f T_d \mu_d}{(T_d \mu_d - p_s^2)^3} - \frac{p_s^5 P_f}{(T_d \mu_d - p_s^2)^3} + \frac{p_s T_d^2 \mu_d^2}{(T_d \mu_d - p_s^2)^3} \\ &\quad - 2 \frac{p_s^3 T_d \mu_d}{(T_d \mu_d - p_s^2)^3} + \frac{p_s^5}{(T_d \mu_d - p_s^2)^3}. \end{aligned}$$

It is noted that Eq. (4.22) holds only if $A = 0$. As a result, one can determine the so-called CVs of any one specific parameter by setting $A = 0$ while keeping the other remaining parameters constant. If $A = 0$, then one can derive the Burgers equation with cubic nonlinearity. Finally, from $O(\varepsilon^5)$ equations following are determined:

$$N_d^{(1)} \frac{\partial U_d^{(2)}}{\partial \xi} + \rho_{de} \frac{\partial U_d^{(3)}}{\partial \xi} + N_d^{(2)} \frac{\partial U_d^{(1)}}{\partial \xi} - p_s \frac{\partial N_d^{(3)}}{\partial \xi} + U_d^{(1)} \frac{\partial N_d^{(2)}}{\partial \xi} U_d^{(2)} \frac{\partial N_d^{(1)}}{\partial \xi} + \frac{\partial N_d^{(1)}}{\partial \tau} = 0, \quad (4.23)$$

$$\begin{aligned} &\rho_{de} P_f a_2 \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} + \rho_{de} P_f a_1 \psi^{(2)} \frac{\partial \psi^{(1)}}{\partial \xi} + \rho_{de} P_f a_1 \psi^{(1)} \frac{\partial \psi^{(2)}}{\partial \xi} + N_d^{(1)} P_f a_1 \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} \\ &\quad + \tau_r p_s^2 \rho_{de} \frac{\partial^2 U_d^{(1)}}{\partial \xi^2} - \tau_r p_s \mu_d T_{di} \frac{\partial^2 N_d^{(1)}}{\partial \xi^2} - \tau_r p_s P_f \rho_{de} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} + \tau_r p_s \rho_{de} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} \\ &\quad + N_d^{(1)} P_f \frac{\partial \psi^{(2)}}{\partial \xi} + \rho_{de} P_f \frac{\partial \psi^{(3)}}{\partial \xi} + N_d^{(2)} P_f \frac{\partial \psi^{(1)}}{\partial \xi} + N_d^{(1)} U_d^{(1)} \frac{\partial U_d^{(1)}}{\partial \xi} - N_d^{(1)} p_s \frac{\partial U_d^{(2)}}{\partial \xi} \\ &\quad - p_s N_d^{(2)} \frac{\partial U_d^{(1)}}{\partial \xi} + \rho_{de} U_d^{(2)} \frac{\partial U_d^{(1)}}{\partial \xi} - \rho_{de} p_s \frac{\partial U_d^{(3)}}{\partial \xi} + \rho_{de} U_d^{(1)} \frac{\partial U_d^{(2)}}{\partial \xi} + \mu_d T_{di} \frac{\partial N_d^{(3)}}{\partial \xi} \\ &\quad - N_d^{(1)} \frac{\partial \psi^{(2)}}{\partial \xi} - \rho_{de} \frac{\partial \psi^{(3)}}{\partial \xi} - N_d^{(2)} \frac{\partial \psi^{(1)}}{\partial \xi} - \mu \frac{\partial^2 U_d^{(1)}}{\partial \xi^2} + \rho_{de} \frac{\partial U_d^{(1)}}{\partial \tau} = 0, \end{aligned} \quad (4.24)$$

$$0 = -(H_1\sigma + \rho_{ie}) \frac{\partial \psi^{(3)}}{\partial \xi} - 3(H_3\sigma^3 + \rho_{ie}) \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} - \frac{\partial N_d^{(3)}}{\partial \xi} - 2(H_2\sigma^2 - \rho_{ie}) \frac{\partial}{\partial \xi} \left(\psi^{(1)} \psi^{(2)} \right), \quad (4.25)$$

where,

$$H_3 = \left[\frac{(q+1)(3-q)(5-3q)}{48} - \frac{2q(q+1)(3-q)\alpha}{3-14q+15q^2+12\alpha} + \frac{8q(q+1)(2q-1)\alpha}{3-14q+15q^2+12\alpha} \right] \sigma^3, \\ a_2 = \frac{H_2}{2} - \frac{H_1^2}{8}. \quad (4.26)$$

Eliminating $N_d^{(3)}$, $U_d^{(3)}$, and $\psi^{(3)}$ from Eqs. (4.23)-(4.25) with the help of Eq. (4.8) and (4.21), the following Burgers equation with cubic nonlinearity is formulated:

$$\frac{\partial \psi^{(1)}}{\partial \tau} + B \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} = C \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \quad (4.27)$$

where

$$B = \frac{\rho_{de} P_f T_d^2 a_1 \mu_d^2}{2(T_d \mu_d - p_s^2)^3} - \frac{\rho_{de} P_f^2 T_d^2 a_1 \mu_d^2}{2(T_d \mu_d - p_s^2)^3} - \frac{9\rho_{de} p_s^2 P_f^2 T_d \mu_d}{(T_d \mu_d - p_s^2)^4} + \frac{9\rho_{de} p_s^2 P_f T_d \mu_d}{(T_d \mu_d - p_s^2)^4} \\ + \frac{3\rho_{de} p_s^2 P_f^3 T_d \mu_d}{(T_d \mu_d - p_s^2)^4} + \frac{3\rho_{de} p_s^6 P_f^2 a_1}{(T_d \mu_d - p_s^2)^4} - \frac{3\rho_{de} p_s^2 \mu_d T_d}{(T_d \mu_d - p_s^2)^4} - \frac{3\rho_{de} p_s^6 P_f a_1}{(T_d \mu_d - p_s^2)^4} + 3p_s^2 \rho_{de} \\ - \frac{\rho_{de}}{(T_d \mu_d - p_s^2)^2} - \frac{3\rho_{de} P_f^2}{(T_d \mu_d - p_s^2)^2} - \frac{3\rho_{de} p_s^4}{(T_d \mu_d - p_s^2)^4} + \frac{3\rho_{de} P_f}{(T_d \mu_d - p_s^2)^2} + 3p_s^2 H_3 \sigma^3 \\ + \frac{\rho_{de} P_f^3}{(T_d \mu_d - p_s^2)^2} - 3\mu_d T_d \rho_{ie} + \rho_{de} P_f a_2 + \frac{\rho_{de} p_s^4 P_f a_1}{2(T_d \mu_d - p_s^2)^3} - \frac{\rho_{de} p_s^4 P_f^2 a_1}{2(T_d \mu_d - p_s^2)^3} \\ + \frac{3\rho_{de} P_f^2 T_d \mu_d}{2(T_d \mu_d - p_s^2)^3} - \frac{3\rho_{de} P_f \mu_d T_d}{2(T_d \mu_d - p_s^2)^3} - \frac{\rho_{de} P_f^3 T_d \mu_d}{2(T_d \mu_d - p_s^2)^3} + \frac{7\rho_{de} p_s^2}{2(T_d \mu_d - p_s^2)^3} \\ - \frac{\rho_{de} P_f^2 a_1}{T_d \mu_d - p_s^2} + \frac{\rho_{de} P_f a_1}{T_d \mu_d - p_s^2} - \frac{9\rho_{de} p_s^4 P_f^2}{(T_d \mu_d - p_s^2)^4} + \frac{9\rho_{de} p_s^4 P_f}{(T_d \mu_d - p_s^2)^4} - 3\mu_d T_d H_3 \sigma^3 \\ - \frac{21\rho_{de} p_s^2 P_f}{2(T_d \mu_d - p_s^2)^3} + \frac{\rho_{de} \mu_d T_d}{2(T_d \mu_d - p_s^2)^3} - \frac{7\rho_{de} p_s^2 P_f^3}{2(T_d \mu_d - p_s^2)^3} + \frac{21\rho_{de} p_s^2 P_f^2}{2(T_d \mu_d - p_s^2)^3} \\ + \frac{\rho_{de} p_s^2 P_f^2 T_d a_1 \mu_d}{(T_d \mu_d - p_s^2)^3} - \frac{3\rho_{de} p_s^2 P_f T_d^2 a_1 \mu_d^2}{(T_d \mu_d - p_s^2)^4} - \frac{6\rho_{de} p_s^4 P_f^2 T_d a_1 \mu_d}{(T_d \mu_d - p_s^2)^4} \\ + \frac{3\rho_{de} p_s^4 P_f^3}{(T_d \mu_d - p_s^2)^4} - \frac{\rho_{de} p_s^2 P_f T_d a_1 \mu_d}{(T_d \mu_d - p_s^2)^3} + \frac{6\rho_{de} p_s^4 P_f T_d a_1 \mu_d}{(T_d \mu_d - p_s^2)^4} + \frac{3\rho_{de} p_s^2 P_f^2 T_d^2 a_1 \mu_d^2}{(T_d \mu_d - p_s^2)^4} \quad (4.28)$$

By using Eqs. (4.17) and (4.18) into Eq. (4.27), one can convert Eq. (4.27) to a nonlinear ODE (ignored for simplicity). Integrating the obtained ODE, the appropriate shock wave solution of Eq. (4.27) is determined [117] as:

$$\psi^{(1)} = \left[A_{01} \left\{ 1 - \tanh \left(\frac{\zeta}{W_{01}} \right) \right\} \right]^{1/2}, \quad (4.29)$$

where $A_{01} = (3s_0/2B)$ and $W_{01} = (C/s_0)$ are respectively the amplitude and width of DA waves around CVs.

4.5 BURGERS EQUATION WITH QUARTIC NONLINEARITY

To investigate shock wave phenomena around SCVs, one can change the scale for z and t by taking $r = 3$. As a result, several order of ε , that is, $O(\varepsilon^j), i = 4, 5, 6, 7, \dots$ -equations are obtained. The $O(\varepsilon^4)$ -equations provide the same relations as in Eqs. (4.8), (4.9), and (4.10). Again, the $O(\varepsilon^5)$ -equations yield the same relations as in Eqs. (4.21) and (4.22). Now, the $O(\varepsilon^6)$ -equations (almost similar to Eqs. (4.23), (4.24) and (4.25) except the time derivative terms) give the mathematical expressions for third order perturb quantities as:

$$\begin{aligned} N_d^{(3)} &= \frac{\rho_{de}}{p_s} U_d^{(3)} - h_{13} \left[\left\{ \psi^{(1)} \right\}^3 - \left\{ \psi^{(1)} \psi^{(2)} \right\} \right], \\ U_d^{(3)} &= -h_{14} \left\{ \psi^{(1)} \right\}^3 - \frac{p_s (P_f - 1)}{(T_d \mu_d - p_s^2)} \psi^{(3)} + h_{15} \left\{ \psi^{(1)} \psi^{(2)} \right\} \end{aligned} \quad (4.30)$$

and the relation as

$$2A \left\{ \psi^{(1)} \psi^{(2)} \right\} + A_{sc} \left\{ \psi^{(1)} \right\}^3 = 0, \quad (4.31)$$

where

$$\begin{aligned}
A_{sc} = & \frac{P_f a_2 \rho_{de}}{3(T_d \mu_d - p_s^2)} - \frac{2T_d^2 \mu_d^2 \rho_{de}}{(T_d \mu_d - p_s^2)^5} - \rho_{de} - H_3 \sigma^3 - \frac{5\rho_{de}}{2(T_d \mu_d - p_s^2)^3} \\
& - \frac{3P_f^2 \rho_{de} a_1}{2(T_d \mu_d - p_s^2)^2} + \frac{3\rho_{de} P_f a_1}{2(T_d \mu_d - p_s^2)^2} + \frac{13\rho_{de} T_d \mu_d}{3(T_d \mu_d - p_s^2)^4} + \frac{5P_f^3 \rho_{de}}{2(T_d \mu_d - p_s^2)^3} \\
& - \frac{15P_f^2 \rho_{de}}{2(T_d \mu_d - p_s^2)^3} + \frac{15P_f \rho_{de}}{2(T_d \mu_d - p_s^2)^3} - \frac{P_f \rho_{de} T_d \mu_d a_1}{(T_d \mu_d - p_s^2)^3} + \frac{P_f^2 \rho_{de} T_d \mu_d a_1}{(T_d \mu_d - p_s^2)^3} \\
& - \frac{6T_d^2 \mu_d^2 P_f^2 \rho_{de}}{(T_d \mu_d - p_s^2)^5} + 6 \frac{T_d^2 \mu_d^2 P_f \rho_{de}}{(T_d \mu_d - p_s^2)^5} - \frac{13P_f^3 \rho_{de} T_d \mu_d}{3(T_d \mu_d - p_s^2)^4} + 13 \frac{P_f^2 \rho_{de} T_d \mu_d}{(T_d \mu_d - p_s^2)^4} \\
& - 13 \frac{P_f \rho_{de} T_d \mu_d}{(T_d \mu_d - p_s^2)^4} + 2 \frac{T_d^2 \mu_d^2 P_f^3 \rho_{de}}{(T_d \mu_d - p_s^2)^5}, \tag{4.32}
\end{aligned}$$

$$\begin{aligned}
h_{13} = & 2 \frac{\rho_{de} P_f^3}{(T_d \mu_d - p_s^2)^3} - 6 \frac{\rho_{de} P_f^2}{(T_d \mu_d - p_s^2)^3} + 6 \frac{\rho_{de} P_f}{(T_d \mu_d - p_s^2)^3} - \frac{P_f^2 a_1 \rho_{de}}{(T_d \mu_d - p_s^2)^2} \\
& - 2 \frac{\rho_{de} T_d \mu_d P_f^3}{(T_d \mu_d - p_s^2)^4} + 6 \frac{\rho_{de} T_d \mu_d P_f^2}{(T_d \mu_d - p_s^2)^4} - 6 \frac{\rho_{de} T_d \mu_d P_f}{(T_d \mu_d - p_s^2)^4} - 2 \frac{\rho_{de}}{(T_d \mu_d - p_s^2)^3} \\
& + \frac{P_f a_1 \rho_{de}}{(T_d \mu_d - p_s^2)^2} + 2 \frac{\rho_{de} T_d \mu_d}{(T_d \mu_d - p_s^2)^4},
\end{aligned}$$

$$\begin{aligned}
h_{14} = & - \frac{p_s P_f^2 a_1}{2(T_d \mu_d - p_s^2)^2} + \frac{p_s P_f a_1}{2(T_d \mu_d - p_s^2)^2} + \frac{P_f^3 p_s}{2(T_d \mu_d - p_s^2)^3} - \frac{3p_s P_f^2}{2(T_d \mu_d - p_s^2)^3} \\
& + \frac{3p_s P_f}{2(T_d \mu_d - p_s^2)^3} - \frac{T_d \mu_d A_7 p_s}{\rho_{de} (T_d \mu_d - p_s^2)} + \frac{P_f a_2 p_s}{3(T_d \mu_d - p_s^2)} - \frac{p_s}{2(T_d \mu_d - p_s^2)^3} \\
& - \frac{P_f^3 p_s T_d \mu_d}{3(T_d \mu_d - p_s^2)^4} + \frac{p_s P_f^2 T_d \mu_d}{(T_d \mu_d - p_s^2)^4} - \frac{p_s P_f T_d \mu_d}{(T_d \mu_d - p_s^2)^4} + \frac{p_s T_d \mu_d}{3(T_d \mu_d - p_s^2)^4},
\end{aligned}$$

$$\begin{aligned}
h_{15} = & \frac{p_s P_f^2}{(T_d \mu_d - p_s^2)^2} - 2 \frac{p_s P_f}{(T_d \mu_d - p_s^2)^2} + \frac{T_d^3 \mu_d^3 p_s P_f^2 a_1}{(T_d \mu_d - p_s^2)^5} - 2 \frac{T_d^2 \mu_d^2 p_s^3 P_f^2 a_1}{(T_d \mu_d - p_s^2)^5} \\
& + \frac{T_d \mu_d p_s^5 P_f^2 a_1}{(T_d \mu_d - p_s^2)^5} - \frac{T_d^3 \mu_d^3 p_s P_f a_1}{(T_d \mu_d - p_s^2)^5} + 2 \frac{T_d^2 \mu_d^2 p_s^3 P_f a_1}{(T_d \mu_d - p_s^2)^5} - \frac{T_d \mu_d p_s^5 P_f a_1}{(T_d \mu_d - p_s^2)^5} \\
& + 2 \frac{T_d \mu_d p_s^3 P_f^3}{(T_d \mu_d - p_s^2)^5} - 6 \frac{T_d \mu_d p_s^3 P_f^2}{(T_d \mu_d - p_s^2)^5} + 6 \frac{T_d \mu_d p_s^3 P_f}{(T_d \mu_d - p_s^2)^5} - 2 \frac{T_d \mu_d p_s^3}{(T_d \mu_d - p_s^2)^5} \\
& - \frac{p_s P_f a_1}{T_d \mu_d - p_s^2} + \frac{p_s}{(T_d \mu_d - p_s^2)^2}.
\end{aligned}$$

It is noted that Eq. (4.31) holds only if $A_{sc} = 0$ and $A = 0$. As a result, one can determine the so-called SCVs of any one specific parameter by satisfying both $A_{sc} = 0$ and $A = 0$ while keeping the other remaining parameters constant. If $A_{sc} = 0$ and $A = 0$, then one can derive the Burgers equation with quartic nonlinearity. This means that one can determine CVs, say K_c as $K_c = K = f(P_f, \rho_{de}, \rho_{ei}, T_{di}, \mu_d, q, \alpha)$ if $A = 0$, whereas one can determine SCVs, say K_{sc} as $K_{sc} = K = f(P_f, \rho_{de}, \rho_{ei}, T_{di}, \mu_d, q, \alpha)$ if both $A = 0$ and $A_{sc} = 0$. Finally, $O(\varepsilon^7)$ equations provide following forms:

$$\begin{aligned} N_d^{(1)} \frac{\partial U_d^{(3)}}{\partial \xi} + \rho_{de} \frac{\partial U_d^{(4)}}{\partial \xi} + N_d^{(2)} \frac{\partial U_d^{(2)}}{\partial \xi} + N_d^{(3)} \frac{\partial U_d^{(1)}}{\partial \xi} + U_d^{(2)} \frac{\partial N_d^{(2)}}{\partial \xi} \\ + U_d^{(3)} \frac{\partial N_d^{(1)}}{\partial \xi} + U_d^{(3)} \frac{\partial N_d^{(1)}}{\partial \xi} - p_s \frac{\partial N_d^{(4)}}{\partial \xi} + \frac{\partial N_d^{(1)}}{\partial \tau} = 0, \end{aligned} \quad (4.33)$$

$$\begin{aligned} \rho_{de} P_f a_3 \left\{ \psi^{(1)} \right\}^3 \frac{\partial \psi^{(1)}}{\partial \xi} + P_f a_1 N_d^{(1)} \psi^{(2)} \frac{\partial \psi^{(1)}}{\partial \xi} + P_f a_1 N_d^{(2)} \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} \\ + \rho_{de} P_f a_1 \psi^{(2)} \frac{\partial \psi^{(2)}}{\partial \xi} + \rho_{de} P_f a_1 \psi^{(1)} \frac{\partial \psi^{(3)}}{\partial \xi} + P_f a_2 N_d^{(1)} \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} \\ + P_f a_1 N_d^{(1)} \psi^{(1)} \frac{\partial \psi^{(2)}}{\partial \xi} + \mu_d T_{di} \frac{\partial^2 \psi^{(3)}}{\partial \xi^2} + P_f N_d^{(1)} \frac{\partial \psi^{(3)}}{\partial \xi} + N_d^{(1)} \frac{\partial}{\partial \xi} \left(U_d^{(1)} U_d^{(2)} \right) \\ + \rho_{de} P_f a_1 \psi^{(1)} \frac{\partial \psi^{(3)}}{\partial \xi} - p_s N_d^{(1)} \frac{\partial U_d^{(3)}}{\partial \xi} + P_f N_d^{(2)} \frac{\partial U_d^{(3)}}{\partial \xi} + N_d^{(2)} U_d^{(1)} \frac{\partial U_d^{(1)}}{\partial \xi} - p_s N_d^{(2)} \frac{\partial U_d^{(2)}}{\partial \xi} \\ - p_s N_d^{(3)} \frac{\partial U_d^{(1)}}{\partial \xi} + P_f N_d^{(3)} \frac{\partial \psi^{(1)}}{\partial \xi} + \rho_{de} P_f \frac{\partial \psi^{(4)}}{\partial \xi} + \rho_{de} \frac{\partial}{\partial \xi} \left(U_d^{(1)} U_d^{(3)} \right) \\ + \rho_{de} U_d^{(2)} \frac{\partial U_d^{(2)}}{\partial \xi} - p_s \rho_{de} \frac{\partial U_d^{(4)}}{\partial \xi} - \frac{\partial}{\partial \xi} \left(N_d^{(1)} \psi^{(3)} \right) - N_d^{(2)} \frac{\partial \psi^{(2)}}{\partial \xi} - \rho_{de} \frac{\partial \psi^{(4)}}{\partial \xi} \\ + \rho_{de} \frac{\partial \psi^{(1)}}{\partial \tau} - \tau_r p_s \mu_d T_{di} \frac{\partial^2 N_d^{(1)}}{\partial \xi^2} - \tau_r p_s P_f \rho_{de} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} + \rho_{de} P_f a_1 \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(2)}}{\partial \xi} \\ + \tau_r p_s \rho_{de} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} + \tau_r p_s^2 \rho_{de} \frac{\partial^2 U_d^{(1)}}{\partial \xi^2} + 2 P_f a_2 \rho_{de} \psi^{(1)} \psi^{(2)} \frac{\partial \psi^{(1)}}{\partial \xi} - \mu \frac{\partial^2 U_d^{(1)}}{\partial \xi^2} = 0, \end{aligned} \quad (4.34)$$

$$\begin{aligned}
0 = & -(H_1\sigma + \rho_{ie}) \frac{\partial \psi^{(4)}}{\partial \xi} - 3(H_3\sigma^3 + \rho_{ie}) \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(2)}}{\partial \xi} \\
& - 2(H_2\sigma^2 - \rho_{ie}) \frac{\partial}{\partial \xi} \left[\psi^{(1)} \psi^{(3)} \right] - 2(H_2\sigma^2 - \rho_{ie}) \psi^{(2)} \frac{\partial \psi^{(2)}}{\partial \xi} - \frac{\partial}{\partial \xi} N_d^{(4)} \\
& - 6(H_3\sigma^3 + \rho_{ie}) \psi^{(1)} \psi^{(2)} \frac{\partial \psi^{(1)}}{\partial \xi} = 0.
\end{aligned} \tag{4.35}$$

By eliminating $N_d^{(4)}$, $U_d^{(4)}$, and $\psi^{(4)}$ from Eqs. (4.33)-(4.35), we formulate the following Burgers equation with quartic nonlinearity :

$$\frac{\partial \psi^{(1)}}{\partial \tau} + P \left\{ \psi^{(1)} \right\}^3 \frac{\partial \psi^{(1)}}{\partial \xi} = C \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \tag{4.36}$$

where

$$\begin{aligned}
P = & \frac{15\rho_{de}p_s^2}{(T_d\mu_d - p_s^2)^4} + \frac{15\rho_{de}p_s^2P_f^4}{(T_d\mu_d - p_s^2)^4} - \frac{60\rho_{de}p_s^2P_f^3}{(T_d\mu_d - p_s^2)^4} + \frac{90\rho_{de}p_s^2P_f^2}{(T_d\mu_d - p_s^2)^4} \\
& - \frac{60\rho_{de}p_s^2P_f}{(T_d\mu_d - p_s^2)^4} + \frac{3\rho_{de}P_f^3a_1}{(T_d\mu_d - p_s^2)^2} - \frac{5P_f^4\rho_{de}}{2(T_d\mu_d - p_s^2)^3} + P_f a_3 \rho_{de} + \frac{4P_f a_2 \rho_{de}}{3(T_d\mu_d - p_s^2)} \\
& - \frac{2T_d^2\mu_d^2\rho_{de}}{(T_d\mu_d - p_s^2)^5} - \frac{8p_s^2P_f a_2 \rho_{de}}{3(T_d\mu_d - p_s^2)^2} + \frac{8p_s^2P_f^2 a_2 \rho_{de}}{3(T_d\mu_d - p_s^2)^2} + \frac{\rho_{de}p_s^2P_f^2 a_1^2}{(T_d\mu_d - p_s^2)^2} \\
& - \frac{12\rho_{de}p_s^2P_f^3 a_1}{(T_d\mu_d - p_s^2)^3} + \frac{24\rho_{de}p_s^2P_f^2 a_1}{(T_d\mu_d - p_s^2)^3} - \frac{12\rho_{de}p_s^2P_f a_1}{(T_d\mu_d - p_s^2)^3} - \frac{104\rho_{de}p_s^2T_d\mu_d}{3(T_d\mu_d - p_s^2)^5} \\
& + \frac{20\rho_{de}p_s^2T_d^2\mu_d^2}{(T_d\mu_d - p_s^2)^6} - \frac{P_f^2 a_1^2 \rho_{de}}{2(T_d\mu_d - p_s^2)} - \frac{5\rho_{de}}{2(T_d\mu_d - p_s^2)^3} - \frac{6\rho_{de}P_f^2 a_1}{(T_d\mu_d - p_s^2)^2} \\
& + \frac{3P_f a_1 \rho_{de}}{(T_d\mu_d - p_s^2)^2} + \frac{13\rho_{de}T_d\mu_d}{3(T_d\mu_d - p_s^2)^4} + \frac{10\rho_{de}P_f^3}{(T_d\mu_d - p_s^2)^3} - \frac{15\rho_{de}P_f^2}{(T_d\mu_d - p_s^2)^3} \\
& + \frac{10P_f \rho_{de}}{(T_d\mu_d - p_s^2)^3} - \frac{4P_f^2 a_2 \rho_{de}}{3(T_d\mu_d - p_s^2)} + \frac{12\rho_{de}p_s^2P_f^3 a_1 T_d\mu_d}{(T_d\mu_d - p_s^2)^4} - \frac{24\rho_{de}p_s^2P_f^2 a_1 T_d\mu_d}{(T_d\mu_d - p_s^2)^4} \\
& + 12 \frac{\rho_{de}p_s^2P_f a_1 T_d\mu_d}{(T_d\mu_d - p_s^2)^4} - 2 \frac{\rho_{de}P_f^3 a_1 T_d\mu_d}{(T_d\mu_d - p_s^2)^3} - 2 \frac{\rho_{de}T_d\mu_d P_f a_1}{(T_d\mu_d - p_s^2)^3} + 4 \frac{\rho_{de}T_d\mu_d P_f^2 a_1}{(T_d\mu_d - p_s^2)^3} \\
& - \frac{2P_f^4 T_d^2 \mu_d^2 \rho_{de}}{(T_d\mu_d - p_s^2)^5} + \frac{13P_f^4 \rho_{de} T_d\mu_d}{3(T_d\mu_d - p_s^2)^4} - \frac{104\rho_{de}p_s^2P_f^4 T_d\mu_d}{3(T_d\mu_d - p_s^2)^5} + \frac{416\rho_{de}p_s^2P_f^3 T_d\mu_d}{3(T_d\mu_d - p_s^2)^5} \\
& - \frac{208\rho_{de}p_s^2P_f^2 T_d\mu_d}{(T_d\mu_d - p_s^2)^5} + \frac{416\rho_{de}p_s^2P_f T_d\mu_d}{3(T_d\mu_d - p_s^2)^5} + \frac{20\rho_{de}p_s^2T_d^2\mu_d^2 P_f^4}{(T_d\mu_d - p_s^2)^6} - \frac{80\rho_{de}p_s^2T_d^2\mu_d^2 P_f^3}{(T_d\mu_d - p_s^2)^6} \\
& + \frac{120\rho_{de}p_s^2T_d^2\mu_d^2 P_f^2}{(T_d\mu_d - p_s^2)^6} - \frac{80\rho_{de}p_s^2T_d^2\mu_d^2 P_f}{(T_d\mu_d - p_s^2)^6} + \frac{8T_d^2\mu_d^2 P_f^3 \rho_{de}}{(T_d\mu_d - p_s^2)^5} - \frac{12T_d^2\mu_d^2 P_f^2 \rho_{de}}{(T_d\mu_d - p_s^2)^5} \\
& + \frac{8T_d^2\mu_d^2 P_f \rho_{de}}{(T_d\mu_d - p_s^2)^5} - \frac{52\rho_{de}P_f^3 T_d\mu_d}{3(T_d\mu_d - p_s^2)^4} + \frac{26\rho_{de}T_d\mu_d P_f^2}{(T_d\mu_d - p_s^2)^4} - \frac{52\rho_{de}T_d\mu_d P_f}{3(T_d\mu_d - p_s^2)^4}. \tag{4.37}
\end{aligned}$$

By implementing Eqs. (4.17) and (4.18) into Eq. (4.36), one can convert Eq. (4.36) into a nonlinear ODE, which is ignored for simplicity. Integrating the obtained ODE, the useful shock wave solution of Eq. (4.36) is defined [89] as:

$$\psi^{(1)} = \left[A_{02} \left\{ 1 - \tanh \left(\frac{\zeta}{W_{02}} \right) \right\} \right]^{\frac{1}{3}}, \tag{4.38}$$

where $A_{02} = (2s_0/P)$ and $W_{02} = (3C/2s_0)$ are respectively the amplitude and width of DA waves around SCVs.

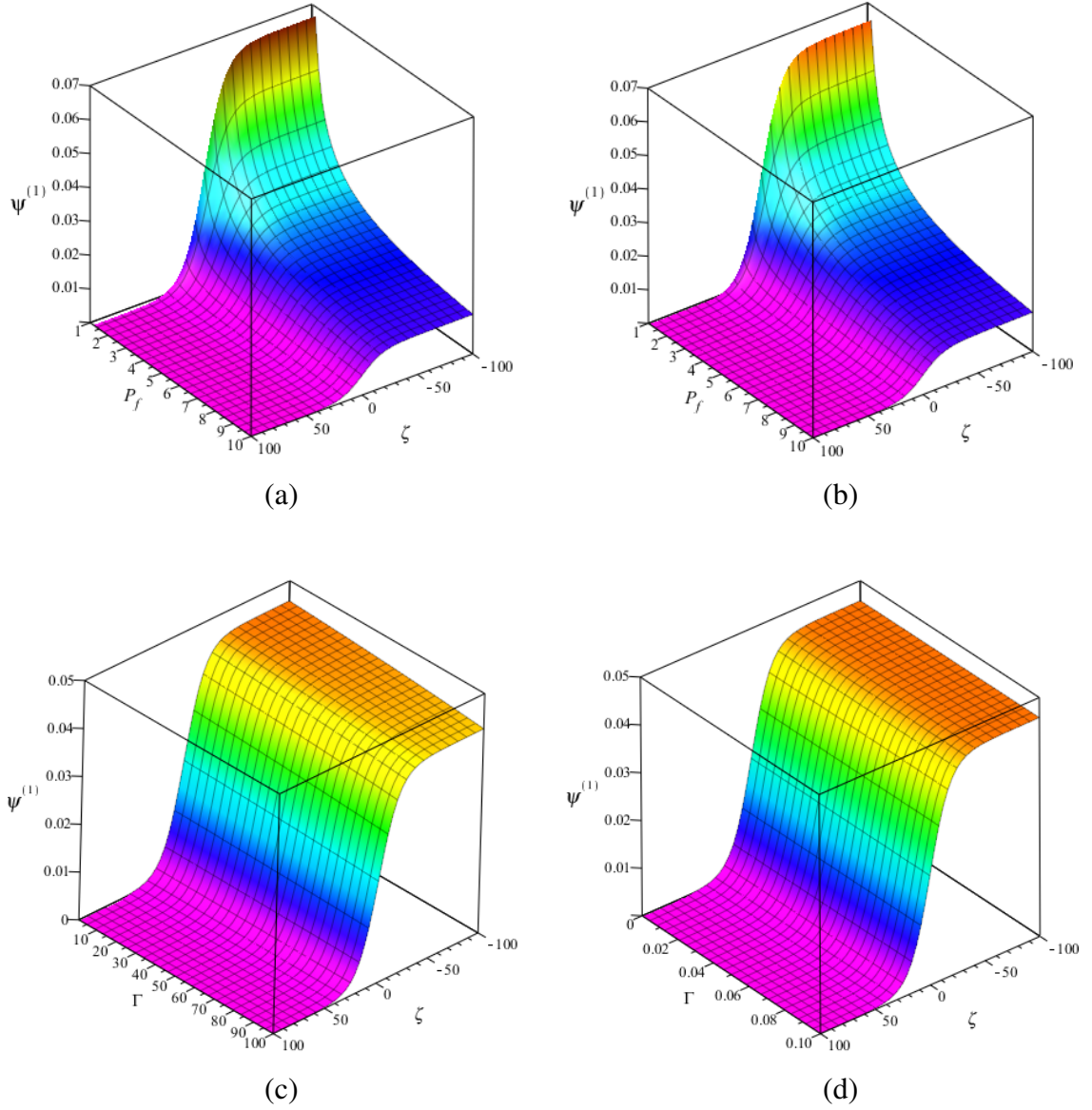


Fig. 4.1 Effect of P_f [Γ] by considering (a) [(c)] strongly coupled regime ($\Gamma = 5.5$) and (b) [(d)] weakly coupled regime ($\Gamma = 0.01$) with $\sigma = 0.6$, $T_{di} = 0.01$, $\rho_{ie} = 0.2$, $\mu = 0.56$, and $s_0 = 0.03$ on the electrostatic shocks described by Eq. (4.15) in the presence of nonthermality (i.e. $q = 1$ $\alpha = 0.35$).

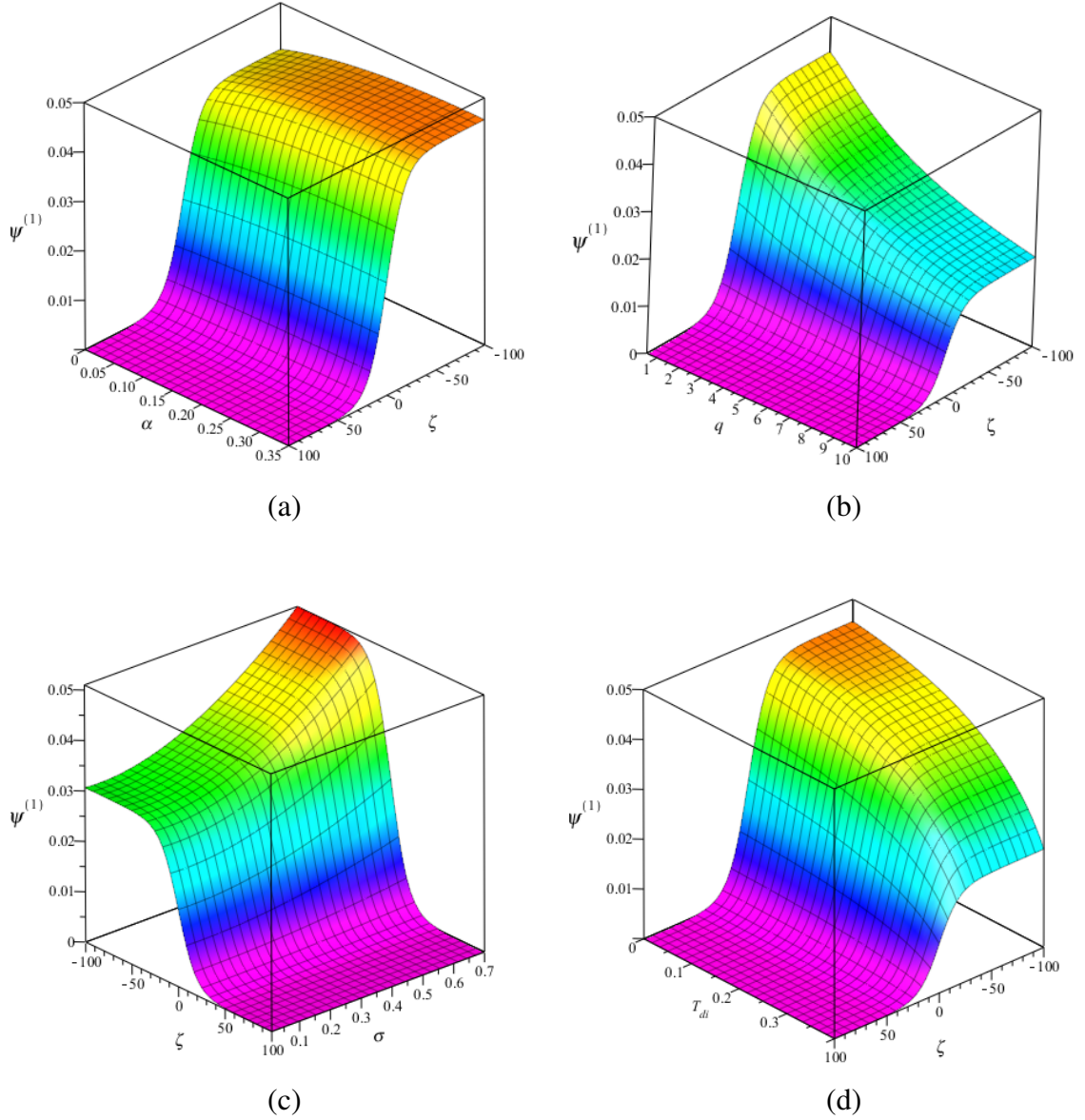


Fig. 4.2 Effect of (a) α ($q = 1$, $\sigma = 0.6$, $T_{di} = 0.01$), (b) q ($\alpha = 0$, $\sigma = 0.6$, $T_{di} = 0.01$), (c) σ ($\alpha = 0.25$, $q = 1$, $T_{di} = 0.01$) and (d) T_{di} ($\alpha = 0.25$, $\sigma = 0.6$, $q = 1$) by considering the strongly coupled regime ($\Gamma = 10$) with $P_f = 1.5$, $\rho_{ie} = 0.2$, $\mu = 0.56$, and $s_0 = 0.03$ on the electrostatic shocks described by Eq. (4.15).

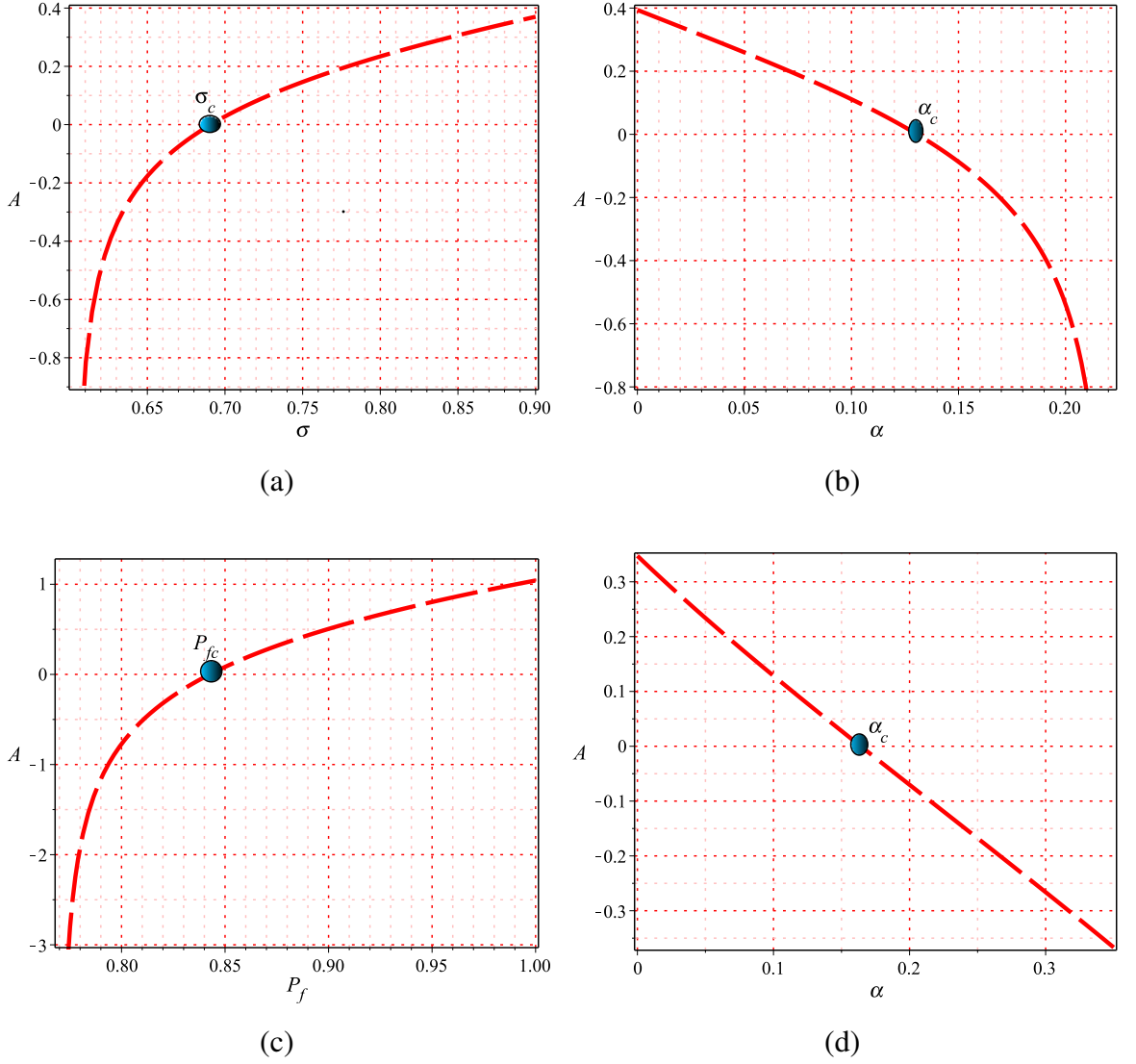


Fig. 4.3 Existence of critical points $\sigma_c = 0.6907750218$, $\alpha_c = 0.1306612739$, $P_{fc} = 0.8425731011$ and $\alpha_c = 0.1641464060$ by plotting the nonlinear coefficient (A) of Eq. (4.15) with regards to (a) σ with $\alpha = 0$, $q = 0.45$, $T_{di} = 0.3$, $\rho_{ie} = 0.5$, $P_f = 0.90$, $\Gamma = 2.5$; (b) α with $\sigma = 0.75$, $q = 1$, $T_{di} = 0.3$, $\rho_{ie} = 0.5$, $P_f = 0.90$, $\Gamma = 2.5$; (c) P_f with $\sigma = 0.7$, $q = 0.45$, $T_{di} = 0.3$, $\rho_{ie} = 0.7$, $\alpha = 0$, $\Gamma = 2.5$; and (d) α with $\sigma = 0.6$, $q = 1$, $T_{di} = 0.1$, $\rho_{ie} = 0.5$, $P_f = 0.90$, $\Gamma = 0.001$, respectively.

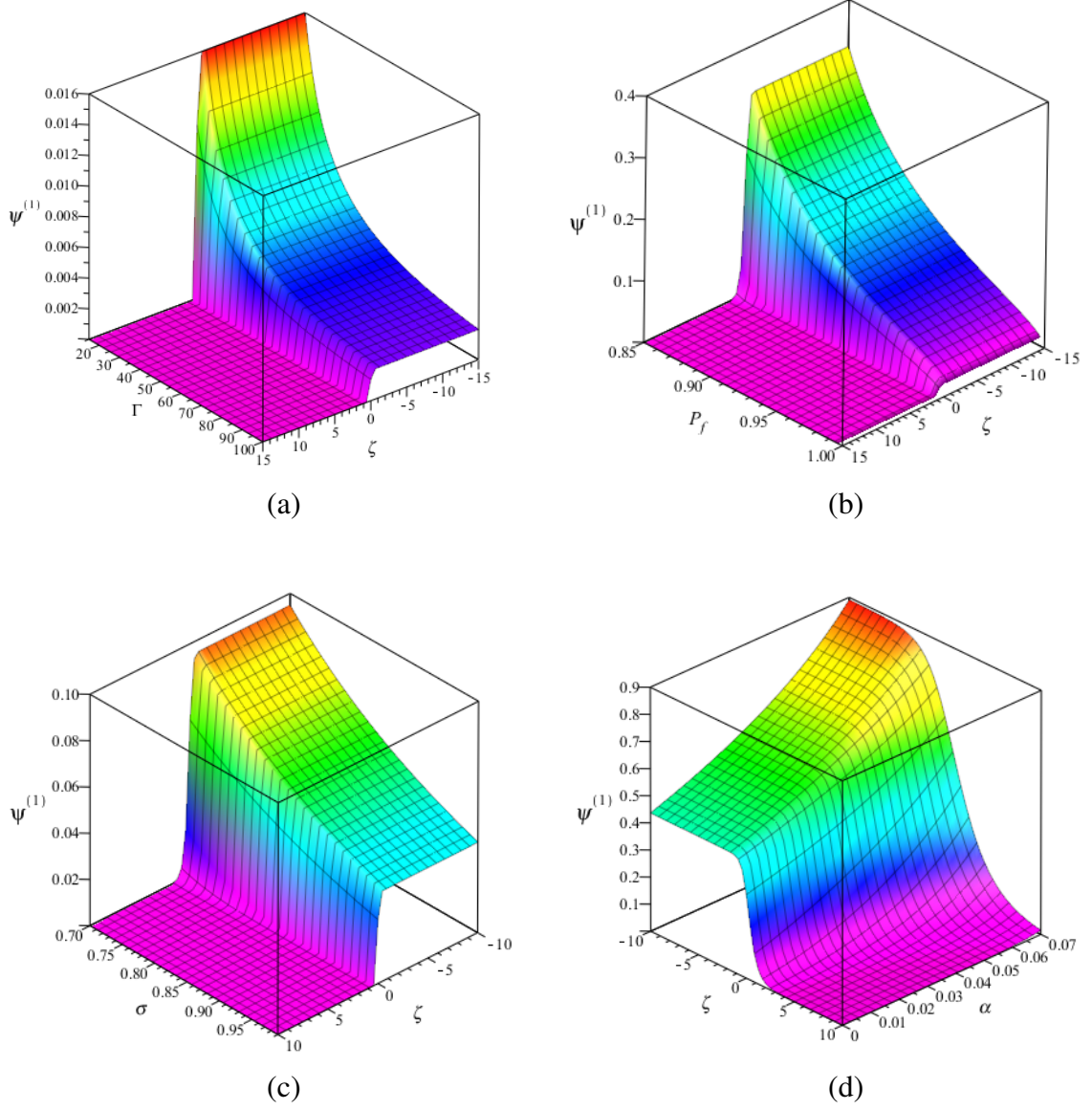


Fig. 4.4 Effect of (a) Γ ($\alpha = 0.2, q = 1, \sigma = 0.75, T_{di} = 0.3, \rho_{ie} = 0.5, \mu = 0.56, P_f = 0.90$ and $s_0 = 0.03$), (b) P_f ($\sigma = 0.7, \alpha = 0, q = 0.45, T_{di} = 0.3, \rho_{ie} = 0.7, \mu = 0.56, \Gamma = 2.5$ and $s_0 = 0.03$), (c) σ ($\alpha = 0, q = 0.45, T_{di} = 0.3, \rho_{ie} = 0.5, P_f = 0.90, \mu = 0.56, \Gamma = 2.5$, and $s_0 = 0.03$) and (d) α ($\Gamma = 0.001, q = 1, \sigma = 0.6, T_{di} = 0.1, \rho_{ie} = 0.5, \mu = 0.56, P_f = 0.90$ and $s_0 = 0.03$) on the electrostatic shocks described by Eq. (4.27).

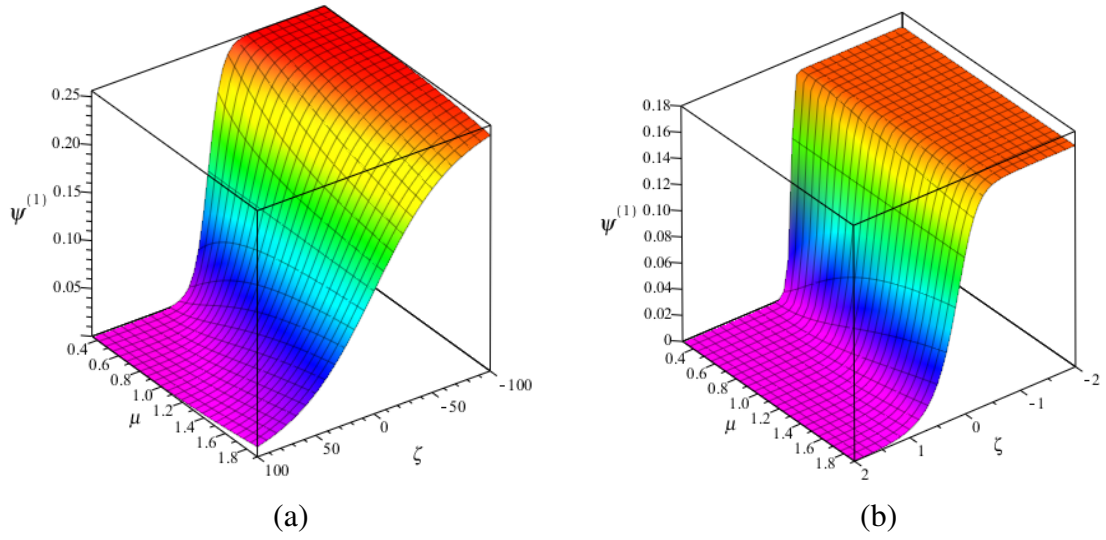


Fig. 4.5 Effect of (a) μ ($\Gamma = 2.5$, $\alpha = 0$, $q = 0.45$, $\sigma = 0.8$, $T_{di} = 0.3$, $\rho_{ie} = 0.5$, $P_f = 0.90$ and $s_0 = 0.03$) on the electrostatic shocks described by BE with quadratic nonlinearity and (b) μ ($\Gamma = 3.8$, $\alpha = 0$, $q = 0.45$, $\sigma = 0.6907750218$, $T_{di} = 0.3$, $\rho_{ie} = 0.5$, $P_f = 0.90$ and $s_0 = 0.03$) on the electrostatic shocks described by Eq. (4.15) and Eq. (4.27), respectively.

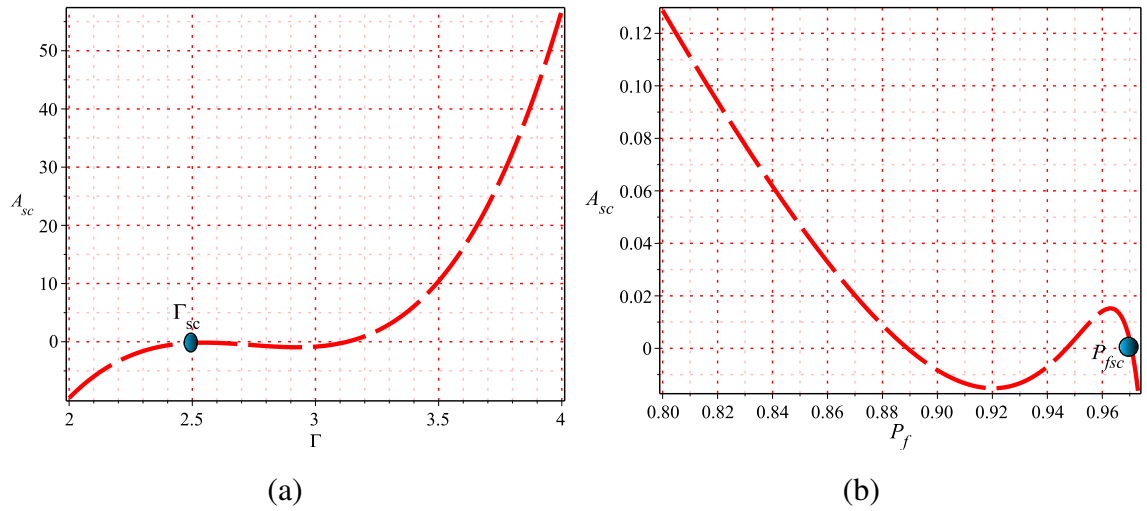


Fig. 4.6 Existence of super critical points by plotting A_{sc} as in Eq. (4.32) with regards to (a) Γ with $\alpha = 0$, $q = 0.45$, $T_{di} = 0.3$, $\rho_{ie} = 0.5$, $P_f = 0.90$, $\sigma = 0.6907750218$, and (b) P_f with $\alpha = 0.1$, $q = 0.35$, $T_{di} = 0.3$, $\rho_{ie} = 0.3$, $\sigma = 0.7$, $\Gamma = 2.5$, respectively.

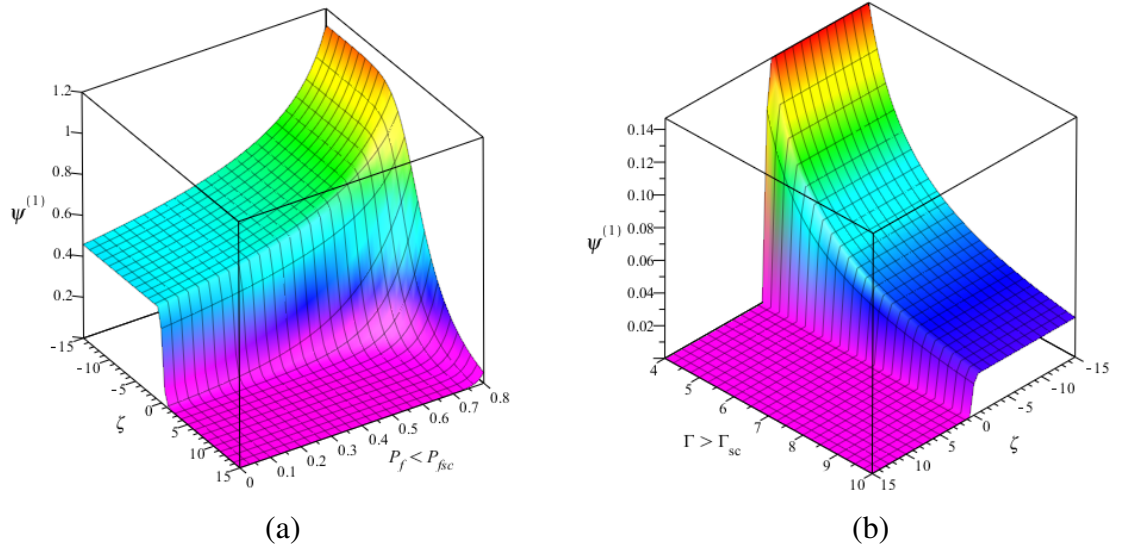


Fig. 4.7 Effect of (a) P_f with $\alpha = 0.1$, $q = 0.35$, $T_{di} = 0.3$, $\rho_{ie} = 0.3$, $\sigma = 0.7$, $\Gamma = 2.5$, and (b) Γ with $\alpha = 0.1$, $q = 0.35$, $T_{di} = 0.3$, $\rho_{ie} = 0.3$, $\sigma = 0.7$, $\Gamma = 2.5$, on the electrostatic DA shocks around the super critical points described by Eq. (4.36), respectively.

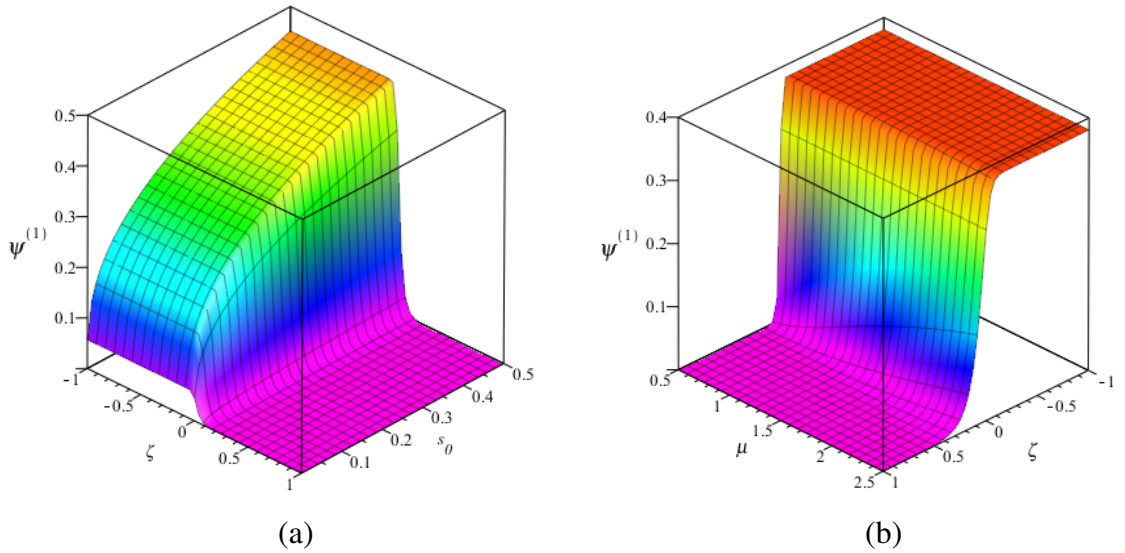


Fig. 4.8 Effect of (a) s_0 with $\alpha = 0$, $q = 0.45$, $T_{di} = 0.3$, $\rho_{ie} = 0.5$, $\sigma = 0.6907750218$, $\Gamma = 3.8$, $\mu = 0.96$, $P_f = 0.9$; and (b) μ with $\alpha = 0$, $q = 0.45$, $T_{di} = 0.3$, $\rho_{ie} = 0.5$, $\sigma = 0.6907750218$, $\Gamma = 3.8$, $s_0 = 0.3$, $P_f = 0.9$ on the electrostatic DA shocks around the super critical points described by Eq. (4.36), respectively.

4.6 RESULTS AND DISCUSSION

This section explores the effects of Γ , P_f , q , α , ρ_{ie} , σ , T_{di} , and μ on the electrostatic DA shocks in both of strongly and weakly coupled regimes. In the presented investigation, the parameter ranges are considered based on Ref. [95], which correspond to the experimental conditions of Ref. [142]. That is, the ranges of density and temperature ratios are considered as $N_{i0} = 7 \times 10^7 \text{ cm}^{-3}$, $N_{e0} = 2.3 \times 10^7 \text{ cm}^{-3}$, $T_e = 1 - 8 \text{ eV}$ and $T_i = 0.1 - 0.9 \text{ eV}$.

Figures 4.1(a)-4.1(d) show the effects of P_f and Γ in the presence of electron nonthermality with $q = 1$ and $\alpha = 0.35$, on the nonlinear propagation of electrostatic DA shocks. This is considered under the conditions of $U(\Gamma) \equiv -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81$ for strongly coupled regime, and $U(\Gamma) \equiv -(\sqrt{3}/2)\Gamma^{3/2}$ for weakly coupled regime, while keeping the remaining parameters constant. It is observed from Fig. 4.1 that (i) P_f effectively contributes to minimizing the amplitudes of DA shocks with its increase, and (ii) Γ contributes less effectively than P_f to minimizing the amplitudes of DA shocks with its increase in both regimes. Accordingly, Γ exhibits a similar impact on the amplitudes of DA shocks because $U(\Gamma)$ is slightly increased with the increase of Γ . The effects of other parameters, that is α , q , σ and T_{di} on the nonlinear propagation of electrostatic DA shocks are displayed in Figures 4.2(a)-4.2(d), respectively. Figures 4.2(a) and 4.2(b) clearly demonstrate the significant role played by electron nonthermality (nonextensivity) in increasing (decreasing) the amplitude of DA shocks with the increase of α (q). It is noted that Figure 4.1 and 4.2 are displayed using the solution of Eq. (4.15). Eq. (4.15) is only applicable if $A > 0$ and $A < 0$. When $A = 0$, the amplitude of shocks becomes infinity. Therefore, one can determine CVs of any particular parameter within the proper ranges by setting $A = 0$. It is also found that the CVs of any particular parameter is independent of μ and s_0 . Figures 4.3(a)-(d) demonstrate the existence of CVs of some particular parameters. It is found from Fig. 4.3 that:

- (i) The negative (positive) electrostatic shocks support for $0.5 < \sigma < 0.69$ ($\sigma > 0.69$). The amplitude of electrostatic shocks becomes infinity at $\sigma = 0.6907750218$ and the electrostatic shocks are not supported due to complexity for $0.5 < \sigma$ with the choice of other parameters as $\alpha = 0$, $q = 0.45$, $T_{di} = 0.3$, $\rho_{ie} = 0.5$, $P_f = 0.90$, $\Gamma = 2.5$ in strongly coupled regime.

- (ii) The negative (positive) electrostatic shocks support for $0.1306612739 < \alpha < 0.21$ ($0 \leq \alpha < 0.1306612739$). The amplitude of electrostatic shocks becomes infinity at $\alpha = 0.1306612739$ and the electrostatic shocks are not supported due to complexity for $\alpha < 0.21$ with the choice of other parameters as $\sigma = 0.75$, $q = 1$, $T_{di} = 0.3$, $\rho_{ie} = 0.5$, $P_f = 0.90$, $\Gamma = 2.5$ in strongly coupled regime.
- (iii) The negative (positive) electrostatic shocks support for $0.78 < P_f < 0.8425731011$ ($0.8425731011 < P_f$). The amplitude of electrostatic shocks becomes infinity at $\alpha = 0.8425731011$ and the electrostatic shocks are not supported due to complexity for $P_f < 0.78$ with the choice of other parameters as $\sigma = 0.7$, $q = 0.45$, $T_{di} = 0.3$, $\rho_{ie} = 0.7$, $\alpha = 0$, $\Gamma = 2.5$ in strongly coupled regime.
- (iv) The negative (positive) electrostatic shocks support for $0.1641464060 < \alpha$ ($0 \leq \alpha < 0.1641464060$). The amplitude of electrostatic shocks becomes infinity at $\alpha = 0.1641464060$ with the choice of other parameters as $\sigma = 0.6$, $q = 1$, $T_{di} = 0.1$, $\rho_{ie} = 0.5$, $P_f = 0.90$, $\Gamma = 0.001$ in weakly coupled regime.

To address the aforementioned difficulty and report the propagation of DA shocks around the CVs, Eq. (4.27) is derived. Figures 4.4(a)-(d) display the impact of Γ , P_f , σ and α on the propagation of electrostatic DA shocks around CVs with regards to ζ by the solution of Eq. (4.27). It is evident from Figs. 4.4 that the amplitude of shocks around CVs decreases with the increase of Γ , P_f and σ but increases with the increase of faster electrons. Additionally, it is found from Figs. 4.4 that the compressive electrostatic DA shocks are supported only if $\Gamma_c > 2.5$, $P_{fc} > 0.78$, $\sigma_c > 0.6907750218$ and $\alpha_c < 0.1306612739$, respectively. Otherwise, the electrostatic DA shocks are not supported due to the complex numeric value of the nonlinear coefficient of Eq. (4.27). Furthermore, the impact of μ on the nonlinear propagation of electrostatic DA shocks is displayed in Figs. 4.5(a) and 4.5(b) by the solutions of Eq. (4.15) and Eq. (4.27), respectively. It is observed from Fig. 4.5 that the DA shocks change monotonically, with the width of shocks monotonically decreases with the increase of μ . Additionally, Eq. (4.15) and Eq. (4.27) do not support the shock wave excitation around CVs, whereas Eq. (4.27) supports the excitation of shock waves around CVs based on the conditions of Fig. 4.3.

Additionally, Eq. (4.27) is not applicable for describing the nature of DA shock at the CVs. Consequently, one can derive another evolution equation involving more higher-order

nonlinearity by incorporating more higher-order corrections into the implemented method. To derive this equation, the solvability condition becomes $A_{sc} = 0$ and $A = 0$. This implies that one can determine the SCVs of any particular parameter in which both A_{sc} and A are equal to zero. Figures 4.6(a) and 4.6(b) are displayed with regards to Γ and P_f for predicting the SCVs of Γ and P_f such that $A_{sc} = 0$ and $A = 0$. It is interesting to note from Fig. 4.6 that both A_{sc} and A become zero for $\Gamma = 2.5$ with the choice of $\alpha = 0$, $q = 0.45$, $T_{di} = 0.3$, $\rho_{ie} = 0.5$, $P_f = 0.90$, $\sigma = 0.6907750218$ and $P_f \sim 0.97$ with the choice of $\alpha = 0.1$, $q = 0.35$, $T_{di} = 0.3$, $\rho_{ie} = 0.3$, $\sigma = 0.7$, $\Gamma = 2.5$, respectively. Consequently, the amplitudes of DA shocks are approaching to infinity. Furthermore, Eq. (4.27) is unable to analyze the nature of the propagation of shocks around such types of SCVs. That is why one cannot ignore studying whether or not the shock wave phenomena exist around the SCVs. Thus, Eq. (4.36) is derived by correcting the reductive perturbation method. The existence of electrostatic DA shocks around some SCVs is displayed in Figs. 4.7 and 4.8. It is observed from Fig. 4.7 that DA shocks support around the SCVs only if the numeric value of P_f is less than its P_{fsc} (SCV) and the numeric value of Γ is greater than its Γ_{sc} (SCV). Also, the amplitude of shocks around SCVs increases (decreases) with the increase of P_f (Γ). It is provided that the parameters do not change their role for minimizing or growing the amplitude of shocks with the increase or decrease of their parametric values except for P_f . It is also found that the amplitude of shocks decreases with the increase of P_f described in Eq. (4.15) and Eq. (4.27), whereas the amplitude of shocks increases with the increase of P_f described in Eq. (4.36). Hence, P_f significantly contributed by changing its role for examining the nature of shocks around SCVs in the considered plasma environment. From Fig. 4.8, it is observed that the amplitude and width of shocks increase around SCV with the increase of s_0 , as expected. Accordingly, the width of shocks slightly increase monotonically due to the increase of μ . Physically, the dissipative media are significantly influenced by the presence of viscosity coefficient of negatively charged dust fluid, leading to monotonically shocks with changes in the viscosity coefficient. It is also predicted that the viscosity coefficient is solely responsible for generating shock wave excitations in the considered plasma, based on the solutions of the obtained equations, which is in good agreement with earlier investigations [94],[119], [143]. In the absence of viscosity coefficient, one may study the DA soliton propagation in the considered plasma

environment. In this article, the considered ranges of plasma parameters are relevant to some astrophysical compact objects and many cosmic environments, especially dark-matter halos, stellar polytropes, hadronic matter and quark-gluon plasma, protoneutron stars, etc. [9], [144], [145]. Thus, the obtained results may be helpful in understanding the dissipative process of localized electrostatic disturbances in such astrophysical compact object and cosmic environments.

4.7 CONCLUSIONS

The electrostatic DA shock wave propagation in an unmagnetized strongly coupled dusty plasma consisting of negatively charged dust grains fluid, Maxwellian ions and (α, q) -distributed electrons, where ions and electrons are assumed to be weakly coupled has been investigated. The reductive perturbation technique has been implemented to derive Burgers equation with quadratic, cubic and quartic nonlinearity from the proposed theoretical model equations. It is found that DA waves described by Burgers equation with quadratic nonlinearity support both positive and negative electrostatic potential in the presence of not only superthermal but also nonthermal electrons. Accordingly, the amplitude of shocks approaches to infinity for some CVs of any specific plasma parameter. In such case, Burgers with quadratic nonlinearity is not applicable to describe the electrostatic shocks in the considered plasmas. As a result, Burgers equation with cubic nonlinearity is an arena to overcome such difficulty and supports the compressive electrostatic DA shocks only around the CVs with some limitation. In addition, DA shock waves described by this equation support the positive electrostatic potential around the CVs in the presence of not only superthermal but also nonthermal electrons. However, the considered plasma environments support SCVs of some specific plasma parameter. Progress in reporting the propagation of DA waves not only around CVs but also SCVs in the considered strongly coupled complex plasmas, Burgers equation with quartic nonlinearity has been derived. Based on the useful solution of it, DA acoustic shock waves with only positive polarity have found not only at the CVs but also around SCVs. Some other most important finding are:

- (i) The height of electrostatic shocks decreased due to the effects of polarization force, coupling parameter, strength of nonextensivity and ion temperature but increased due to

the effects of charged dust grains and population of faster electrons (nonthermal).

(ii) The height of shocks around CVs also decreased with the increase in the values of polarization force, coupling parameter, strength of nonextensivity and ion temperature but increased with the increase of faster electrons.

(iii) DA shocks were supported and changed monotonically due to the impact of viscosity coefficient.

(iv) The polarization force significantly contributed by changing its role for examining the nature of shocks around SCVs.

Thus, the presented results should be very useful in understanding the basic features of the localized electrostatic DA shock wave propagation in cosmic environments and some compact astrophysical objects, as well as laboratory dusty plasmas.

Chapter 5: HEAVY ION-ACOUSTIC SOLITON AND DRESSED SOLITON IN AN UNMAGNETIZED WEAKLY AND STRONGLY COUPLED PLASMA

5.1 INTRODUCTION

It is well confirmed that the plasma consists of pair-ion species in addition to electrons in not only most of the astrophysical and space environments (ASEs) but also in laboratory plasmas [19], [22], [52], [103], [120], [146–148]. In refs. [19], [22], [52], [103], [120], [146–148], researchers have provided evidence of such plasmas that are existed in both of ASEs and laboratory plasmas. Besides, a new evolution of the universe was observed by the existence of the heavy elements which led to the prestellar stage due to all matter of the universe being compressed to extreme densities and possessed corresponding high temperatures [103]. As a result, the multi-species plasma having several charged particles belongs to the regime of strongly coupled plasmas (SCPs). Any plasma species can enter in strongly coupled regime (SCR) only if its comparatively extensive charge quantity and very low temperature. Apart from the existence of SCPs, it is not only produced in exotic environments (e.g., stller polytropes [60], dark matter halos [149], hardonic matters and quark-gluon plasmas [62], neutral stars and earth terrestrial planets around other stars [10],[11], etc.) but also generated in laboratory for processing and tools at very low temperature [122],[150] as well as plasma crystals [151]. In ref. [150], researchers have already generated experimentally SCP by the laser heated solids. They have clearly mentioned that the laser-produced plasmas are actually generated from solids along with many initial lattice structures by depending on the low temperature of ions. Besides, the ions become strongly coupled via their mutual Coulomb interaction because of their high charge density. Such interaction strength is described by the coupling parameter $\Gamma = z_i^2 q_i^2 / a_i k_B T_i$ where z_i is the charge state, q_i is the charge of ion, a_i is the inter ion spacing, k_B is the Baltzmann constant and T_i is the ion temperature. It is noted that the plasma system can be identified as strongly (weakly) connected by depending on $\Gamma > 1$ ($\Gamma < 1$). It is, therefore, the essential requirement for describing the basic features of SCPs

with the influence of Γ .

A few researchers [96],[127] have already reported the linear and nonlinear features of heavy ion acoustic (HIA) planar and nonplanar shocks and solitons in SCPs having nonextensive distributed electrons without considering the influence of Γ . In their studies, they ignored the influence of the coupling parameter without mentioning it. Their outcomes may lead to truncation errors due to the improperly defined nonlinear coefficient of the nonlinear evolution equations. In addition, the electrostatic wave structures have existed in the magnetosphere with density depressions, which is confirmed by the Viking spacecraft [63] and Freja satellite [64]. In such situation, one cannot ignore the influence of the population of nonthermal electrons on the propagation HIA coherent structures in SCPs. Very recently, Tamang and Saha [152] have reported a phase plane analysis of the dust-acoustic waves for the Burgers equation in a strongly coupled dusty plasma with the influence of Γ . Based on the bifurcation analysis, [153], the Burgers-like equations support the shock wave excitation, whereas the Korteweg-de Vries (KdV)-like equations form bell-shaped (soliton) structures in the plasmas. Later, Islam et al. [117] have studied only the HIA shock wave excitation in such plasmas by considering heavy ions (HIs) fluid, Maxwellian lighter ions (MLIs) and the hybrid Tsallis—nonthermal, that is (α, q) velocity distribution [72] electrons with the useful ranges of the nonextensive and nonthermal parameters [75],[76]. They have clearly shown that this distribution function is very useful to study the influence of not only strength of nonextensivity and nonthermality but also concurrently acting nonextensivity and nonthermality on the HIA shocks. Due to the limitations of these previous studies [96],[117],[127] a further research work on the nonlinear propagation HIA soliton in both of SCR and weakly coupled regime (WCR) are still now required by considering all the cases of electron energies. In addition, the nonlinear propagation of HIA dressed soliton is not previously reported in such plasmas by considering both WCR and SCR due to the tedious mathematical formation. Therefore, this chapter explores the HIA soliton and dressed soliton propagation in an unmagnetized plasma having HIs, MLIs and (α, q) distributed electrons in both WCR and SCR with the detailed mathematical formulation, where HIs are strongly and weakly correlated to MLIs. The influence of plasma parameters on the HIA soliton and dressed soliton propagation are also examined with the description.

5.2 MODEL EQUATIONS

By considering an unmagnetized collisionless strongly and weakly coupled plasma having inertial positively charged HIs fluid, MLIs and inertialess (α, q) -distributed electrons, the following normalized hydrodynamic fluid equations are determined [117]:

$$\frac{\partial n_h}{\partial t} + \frac{\partial(n_h u_h)}{\partial x} = 0, \quad (5.1)$$

$$\left(1 + \tau_R \frac{\partial}{\partial t}\right) \left[n_h \left\{ \left(\frac{\partial}{\partial t} + u_h \frac{\partial}{\partial x} \right) u_h + \frac{\partial \phi}{\partial x} \right\} + k_f \frac{\partial n_h}{\partial x} \right] = \eta \frac{\partial^2 u_h}{\partial x^2}, \quad (5.2)$$

$$\begin{aligned} \frac{\partial^2 \phi}{\partial x^2} = & -\rho_2 \exp(-\sigma_{ei} \phi) - n_h + \rho_1 [1 + (q-1)\phi]^{\frac{(q+1)}{2(q-1)}} \\ & \times \left[1 + \left(\frac{-16q\alpha}{3-14q+15q^2+12\alpha} \right) \phi + \left(\frac{16q(2q-1)\alpha}{3-14q+15q^2+12\alpha} \right) \phi^2 \right], \end{aligned} \quad (5.3)$$

where $\eta = (\omega_f/m_h n_{h0} \lambda_D^2)[\eta_t + (4/3)\eta_b]$, $k_f = \eta_h \gamma_h$, $\rho_1 = n_{e0}/z_h n_{h0}$, $\rho_2 = z_i n_{i0}/z_h n_{h0}$, $\sigma = T_e/T_i$ and $\eta_t(\eta_b)$ is transport coefficients of shear (bulk) viscosity. τ_R is defined [125] by

$$\tau_R = \eta \frac{T_e}{T_h} \left\{ 1 - \eta_h + \frac{4}{15} U(\Gamma) \right\}^{-1}, \quad (5.4)$$

with the compressibility [125] condition as

$$\eta_h = \frac{1}{k_B T_h} \partial_{n_h} P_h = \frac{1}{k_B T_h} \left\{ 1 + \frac{1}{3} U(\Gamma) + \frac{\Gamma}{9} \frac{\partial U(\Gamma)}{\partial \Gamma} \right\}. \quad (5.5)$$

Eqs. (5.1)–(5.3) are normalized by introducing $n_h \rightarrow n_h/n_{h0}$, $u_h \rightarrow u_h/C_{hs}$ ($C_{hs} = (z_h k_B T_e/m_h)^{1/2}$), $\phi \rightarrow e\phi/k_B T_e$, $x \rightarrow x\lambda_{dh}$ ($\lambda_{dh} = (k_B T_e/4z_h^2 \phi n_{h0} e^2)^{1/2}$), $t \rightarrow t\omega_{hf}$ ($\omega_{hf} = (4z_h^2 \phi n_{h0} e^2/m_h)^{1/2}$) by considering the charge neutrality condition $z_i n_{i0} + z_h n_{h0} = n_{e0}$ into account, where n_h (u_h), ϕ and x (t) are the normalized HI density (fluid velocity), electrostatic potential and length (time), m_h is the HI mass, n_{h0} and n_{i0} (n_{e0}) are the HIs and lighter ions (electrons) unperturbed densities, $z_h(z_i)$ is charge number of HIs (lighter ions), T_e (T_i) is the electrons (lighter ions) temperature, e is the electric charge, k_B is the Boltzmann constant, respectively. Besides, $U(\Gamma)$ plays an important role by measuring the excess internal energy. For $\Gamma \ll 1$, one can consider $U(\Gamma) \equiv -(\sqrt{3}/2)\Gamma^{3/2}$. One can

also express by neglecting small correction term due to finite number of charged particles as $U(\Gamma) \equiv -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81$ for $1 < \Gamma < 100$ [126]. It is noted that the relation between η_t and Γ is not possible to represent in the analytical form due to complexity. At these stage, one can find out the tabulated/graphical results of their functional behavior by deriving the molecular dynamic simulations based on the various statistical schemes [125].

5.3 FORMATION OF KDV EQUATION AND ITS STATIONARY WAVE SOLUTION

Let us start with the new coordinates system based on reductive perturbation technique (RPT) [67], [154] as

$$\zeta = \varepsilon^{1/2}(x - \lambda_p t), \tau = \varepsilon^{3/2}t \quad (5.6)$$

and the perturbed quantities can expand [117] as

$$\begin{bmatrix} n_h \\ u_h \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \sum_i \varepsilon^i \begin{bmatrix} n_h^{(i)} \\ u_h^{(i)} \\ \phi^{(i)} \end{bmatrix}, \quad (5.7)$$

where $\lambda_p(\varepsilon)$ is the linear phase speed (a proper fraction quantity measuring the weakness of dispersion). Inserting Eq. (5.6) together with Eq. (5.7) into Eqs. (5.1)–(5.3) in consequently, the set of equations are formulated (ignored for simplicity) by depending on the several order of ε , that is $O(\varepsilon^j)$, $j = 3/2, 5/2, 7/2, \dots$ Solving $O(\varepsilon^{3/2})$ order equations, $n_h^{(1)}$, $u_h^{(1)}$ and λ_p are formulated as

$$n_h^{(1)} = \frac{1}{\lambda_p^2 - k_f} \phi^{(1)}, u_h^{(1)} = \frac{\lambda_p}{\lambda_p^2 - k_f} \phi^{(1)}, \quad (5.8)$$

$$\lambda_p = \sqrt{k_f + \frac{1}{\rho_1 \left\{ \frac{q+1}{2} - \frac{16q\alpha}{3-14q+15q^2+12\alpha} \right\} + \rho_2 \sigma}}. \quad (5.9)$$

Now, the following $O(\varepsilon^{5/2})$ order equations from Eq. (5.1)–(5.3) are formulated:

$$\frac{\partial n_h^{(1)}}{\partial \tau} - \lambda_p \frac{\partial n_h^{(2)}}{\partial \xi} + \frac{\partial u_h^{(2)}}{\partial \xi} + \frac{\partial}{\partial \xi} [n_h^{(1)} u_h^{(1)}] = 0, \quad (5.10)$$

$$\frac{\partial u_h^{(1)}}{\partial \tau} - \lambda_p \frac{\partial u_h^{(2)}}{\partial \xi} - \lambda_p n_h^{(1)} \frac{\partial u_h^{(1)}}{\partial \xi} + u_h^{(1)} \frac{\partial u_h^{(1)}}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} + k_f \frac{\partial n_h^{(2)}}{\partial \xi} + n_h^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} = 0, \quad (5.11)$$

$$\frac{\partial^3 \phi^{(1)}}{\partial \xi^2} = K_1 \frac{\partial \phi^{(2)}}{\partial \xi} + 2K_2 \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} - \frac{\partial n_h^{(2)}}{\partial \xi}, \quad (5.12)$$

where,

$$K_1 = \rho_1 \left\{ \frac{q+1}{2} - \frac{16q\alpha}{3-14q+15q^2+12\alpha} \right\} + \rho_2 \sigma,$$

$$K_2 = \rho_1 \left\{ \frac{(q+1)(3-q)}{8} - \frac{8q(q+1)\alpha}{3-14q+15q^2+12\alpha} + \frac{16q(2q-1)\alpha}{3-14q+15q^2+12\alpha} \right\} - \frac{\rho_2 \sigma^2}{2}.$$

Simplifying Eqs. (5.10)–(5.12), the following KdV equation is obtained:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + P \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + Q \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \quad (5.13)$$

where

$$P = \frac{\lambda_p^2 - k_f}{2\lambda_p} \left[\frac{3\lambda_p^2 - k_f}{(\lambda_p^2 - k_f)^2} - 2 \frac{K_2}{K_1} \right], Q = \frac{(\lambda_p^2 - k_f)^2}{2\lambda_p}. \quad (5.14)$$

The well-established stationary wave solution of KdV equation is defined as

$$\phi^{(1)} = \phi_A \operatorname{sech}^2 \left(\frac{\zeta}{\phi_W} \right). \quad (5.15)$$

Here $\zeta = \xi - V\tau$ (V is the constant speed), $\phi_A = (3V/P)$ is the amplitude of HIA solitons and $\phi_W = \sqrt{4Q/V}$ is the width of HIA solitons, respectively. Eq. (5.15) clearly indicates the nonlinear propagation of electrostatic potential is independent of viscosity coefficient of HIs. This means that the viscosity coefficient of HIs are only response to the formation of shocks in the consider plasmas.

5.4 FORMATION OF INHOMOGENEOUS KDV LIKE EQUATION AND ITS STATIONARY WAVE SOLUTION

To study the effect of plasma parameters on the new humped type electrostatic soliton, so called the dressed soliton, one needs to consider further higher order of ε equations. To do it, one can determine the following equations by taking more higher order of ε , that is, $O(\varepsilon^{7/2})$:

$$\frac{\partial n_h^{(2)}}{\partial \tau} - \lambda_p \frac{\partial n_h^{(3)}}{\partial \xi} + \frac{\partial u_h^{(3)}}{\partial \xi} + \frac{\partial}{\partial \xi} \left[n_h^{(1)} u_h^{(2)} + n_h^{(2)} u_h^{(1)} \right] = 0, \quad (5.16)$$

$$\begin{aligned} & \frac{\partial u_h^{(2)}}{\partial \tau} - \lambda_p \frac{\partial u_h^{(3)}}{\partial \xi} + k_f \frac{\partial n_h^{(3)}}{\partial \xi} + \frac{\partial \phi^{(3)}}{\partial \xi} + n_h^{(1)} \frac{\partial u_h^{(1)}}{\partial \tau} - \lambda_p n_h^{(2)} \frac{\partial u_h^{(1)}}{\partial \xi} - \lambda_p n_h^{(1)} \frac{\partial u_h^{(2)}}{\partial \xi} \\ & + \frac{\partial}{\partial \xi} \left[u_h^{(1)} u_h^{(2)} \right] + n_h^{(1)} u_h^{(1)} \frac{\partial u_h^{(1)}}{\partial \xi} + n_h^{(1)} \frac{\partial \phi^{(2)}}{\partial \xi} + n_h^{(2)} \frac{\partial \phi^{(1)}}{\partial \xi} = 0, \end{aligned} \quad (5.17)$$

$$\frac{\partial^3 \phi^{(2)}}{\partial \xi^2} = K_1 \frac{\partial \phi^{(3)}}{\partial \xi} + 2K_2 \frac{\partial}{\partial \xi} \left[\phi^{(1)} \phi^{(2)} \right] + 3K_3 \left(\phi^{(1)} \right)^2 \frac{\partial \phi^{(1)}}{\partial \xi} - \frac{\partial n_h^{(3)}}{\partial \xi}, \quad (5.18)$$

where

$$K_3 = \rho_1 \left[\frac{(q+1)(3-q)(5-q)}{48} - \frac{2q\alpha(q+1)(3-q)}{3-14q+15q^2+12} + \frac{8q(2q-1)\alpha(q+1)}{3-14q+15q^2+12} \right] + \frac{\rho_2 \sigma^3}{6}.$$

Solving Eqs. (5.10)–(5.12) with the aid of Eq. (5.13), one obtains

$$\begin{aligned} n_h^{(2)} &= \frac{1}{\lambda_p^2 - k_f} \phi^{(2)} + K_2 \left(\phi^{(1)} \right)^2 - \frac{\partial^2 \phi^{(1)}}{\partial \xi^2}, u_h^{(2)} = \frac{\lambda_p}{\lambda_p^2 - k_f} \phi^{(2)} + \lambda_p \times \\ & \left\{ K_2 - \frac{1}{(\lambda_p^2 - k_f)^2} + \frac{P}{2\lambda_p (\lambda_p^2 - k_f)} \right\} \left(\phi^{(1)} \right)^2 - \left(\lambda_p - \frac{Q}{\lambda_p^2 - k_f} \right) \frac{\partial^2 \phi^{(1)}}{\partial \xi^2}. \end{aligned} \quad (5.19)$$

Now, eliminating $n_h^{(3)}$, $u_h^{(3)}$ and $\phi^{(3)}$ from Eqs. (5.16)–(5.18) which gives

$$\begin{aligned} & \frac{\partial \phi^{(2)}}{\partial \tau} + \frac{\lambda_p^2 - k_f}{2\lambda_p} \left[\frac{3\lambda_p^2 - k_f}{(\lambda_p^2 - k_f)^2} - 2\frac{K_2}{K_1} \right] \frac{\partial}{\partial \xi} \left[\phi^{(1)} \phi^{(2)} \right] + \frac{(\lambda_p^2 - k_f)^2}{2\lambda_p} \frac{\partial^3 \phi^{(2)}}{\partial \xi^3} = \\ & R_1 \left\{ \phi^{(1)} \right\}^2 \frac{\partial \phi^{(1)}}{\partial \xi} + R_2 \phi^{(1)} \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} + R_3 \frac{\partial}{\partial \xi} \left(\frac{\partial \phi^{(1)}}{\partial \xi} \right)^2 + R_4 \frac{\partial^5 \phi^{(1)}}{\partial \xi^5}, \end{aligned} \quad (5.20)$$

where

$$\begin{aligned}
R_1 &= \frac{2K_2P}{K_1} - \frac{3P}{2K_1(\lambda_p^2 - k_f)^2} + \frac{P^2}{2\lambda_p K_1(\lambda_p^2 - k_f)} - \frac{3\lambda_p K_2}{K_1(\lambda_p^2 - k_f)} \\
&+ \frac{3\lambda_p}{2K_1(\lambda_p^2 - k_f)^3} - \frac{K_2}{2\lambda_p K_1} + \frac{3K_3}{2\lambda_p K_1^2}, \\
R_2 &= \frac{2K_2Q}{K_1} - \frac{Q}{K_1(\lambda_p^2 - k_f)^2} + \frac{QP}{\lambda_p K_1(\lambda_p^2 - k_f)} - \frac{P}{K_1} + \frac{\lambda_p}{K_1(\lambda_p^2 - k_f)}, \\
R_3 &= \frac{1 - 2P\lambda_p}{4\lambda_p K_1} + \frac{PQ}{4\lambda_p K_1(\lambda_p^2 - k_f)} + \frac{\lambda_p}{2K_1(\lambda_p^2 - k_f)} - \frac{Q}{2K_1(\lambda_p^2 - k_f)^2}, \\
R_4 &= \frac{Q^2}{2\lambda_p K_1(\lambda_p^2 - k_f)} - \frac{Q}{K_1}.
\end{aligned}$$

Eq. (5.20) can then be converted to

$$\frac{\partial \phi^{(2)}}{\partial \tau} + P \frac{\partial}{\partial \xi} [\phi^{(1)} \phi^{(2)}] + Q \frac{\partial^3 \phi^{(2)}}{\partial \xi^3} = F(\phi^{(1)}). \quad (5.21)$$

It is noted that Eq. (5.21) is known as the so called inhomogeneous KdV like equation, which divulges only the dressed soliton in the plasmas. Now, one may determine the solution of Eq.(5.21) for investigating the features of such soliton via the well established renormalization method [155],[156]. Based on this method, one can minimize the resonant term in $F(\phi^{(1)})$ by adding an extra term $\delta N \frac{\partial \phi^{(1)}}{\partial \xi}$. As a result, Eqs. (5.13) and (5.21) can be converted respectively to

$$\frac{\partial \phi^{(1)}}{\partial \tau} + P \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + Q \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} + \delta V \frac{\partial \phi^{(1)}}{\partial \xi} = 0, \quad (5.22)$$

and

$$\frac{\partial \phi^{(2)}}{\partial \tau} + P \frac{\partial}{\partial \xi} [\phi^{(1)} \phi^{(2)}] + Q \frac{\partial^3 \phi^{(2)}}{\partial \xi^3} + \delta V \frac{\partial \phi^{(2)}}{\partial \xi} = F(\phi^{(1)}) + \delta V \frac{\partial \phi^{(1)}}{\partial \xi}. \quad (5.23)$$

Introducing new reference frame as $\zeta = \xi - (V + \delta V)\tau$, the stationary solution of Eq. (5.22) is formulated as

$$\phi^{(1)} = \phi_A \operatorname{sech}^2(\phi_W^{-1} \zeta), \quad (5.24)$$

where $\phi_A = 3V/P$ and $\phi_W = \sqrt{4Q/V}$. Using the stationary frame and Eq.(5.24) consequently, Eq. (5.23) reduces to

$$\begin{aligned} [-V + P\phi_A \sec h^2(\phi_W^{-1}\zeta)] \phi^{(2)} + Q \frac{d^2 \phi^{(2)}}{d\zeta^2} = \left(\delta V \phi_A + \frac{16\phi_A R_4}{\phi_W^4} \right) \sec h^2(\phi_W^{-1}\zeta) \\ + R_{11} \sec h^4(\phi_W^{-1}\zeta) + R_{12} \sec h^6(\phi_W^{-1}\zeta), \end{aligned} \quad (5.25)$$

where

$$\begin{aligned} R_{11} &= \frac{2\phi_A^2}{\phi_W^2} R_2 + \frac{4\phi_A^2}{\phi_W^2} \left(R_3 - \frac{R_2}{2} \right) - \frac{120\phi_A}{\phi_W^4} R_4 \\ R_{12} &= \frac{\phi_A^3}{3} R_1 - \frac{6\phi_A^2}{\phi_W^2} R_2 - \frac{4\phi_A^2}{\phi_W^2} \left(R_3 - \frac{R_2}{2} \right) + \frac{120\phi_A}{\phi_W^4} R_4. \end{aligned} \quad (5.26)$$

Inserting $\left(\delta V \phi_A + \frac{16\phi_A R_4}{\phi_W^4} \right) = 0$, one can remove the secular term and yields

$$\delta V = -\frac{16R_4}{\phi_W^4} = -\frac{R_4 V^2}{Q^2}. \quad (5.27)$$

As a result, Eq. (5.25) can rewrite as

$$Q \frac{d^2 \phi^{(2)}}{d\zeta^2} + V [3 \sec h^2(\phi_W^{-1}\zeta) - 1] \phi^{(2)} = R_{11} \sec h^4(\phi_W^{-1}\zeta) + R_{12} \sec h^6(\phi_W^{-1}\zeta), \quad (5.28)$$

In the previous studies, researchers [156] have clearly provided that the particular integral $(\phi_p^{(2)})$ of Eq. (5.28) is only contributed to $\phi^{(2)}$ for the formation of acoustic waves in the plasmas. Hence, the useful solution of Eq. (5.28) can be written by using the series solution method [156] as

$$\phi^{(2)} = \phi_p^{(2)} = \phi_{A1} \sec h^2(\phi_{W'}^{-1}\zeta) + \phi_{A2} \sec h^4(\phi_{W'}^{-1}\zeta), \quad (5.29)$$

where $\phi_{A1} = (1/V)[(2/3)R_{11} + R_{12}]$ and $\phi_{A2} = (-1/2V)R_{12}$. Finally, the total stationary solution (ϕ_T) up to the 2nd order of V is defined by

$$\phi_T = \phi^{(1)} + \phi^{(2)} = \phi_{A0} \sec h^2(\phi_{W'}^{-1}\zeta) + \phi_{A1} \sec h^2(\phi_{W'}^{-1}\zeta) + \phi_{A2} \sec h^4(\phi_{W'}^{-1}\zeta), \quad (5.30)$$

which represents the dressed soliton. It is noted that the amplitude of soliton and dressed soliton due to the 2nd order contribution is obtained as $\phi_{A'} = 3(V + \delta V)/P$ and $\phi_{A''} = 3(V + \delta V)/P + \phi_{A1} + \phi_{A2}$, respectively, where as the width of both soliton are same and yields $\phi_{W'} = \sqrt{4Q/(V + \delta V)}$.

5.5 RESULTS AND DISCUSSION

This section have explored the effect of coupling and other related plasma parameters on the electrostatic HIA soliton and dressed soliton in the considered plasma for both of WCR and SCR. In addition, the conditions for the existence of dressed soliton in both regimes have incorporated. In the presented analysis, the ranges of the plasma parameters are considered based on the previous refs. [64], [96],[127], which are revelent to ASEs, especially, hadronic matter and stellar polytropes, quark-gluon plasma, black halos, etc.

In Figures 5.1(a) and 5.1(b), the effect of only strength of nonextensivity ($\alpha = 0$) and nonthermality ($q = 1$) on the propagation of HIA soliton are displayed, respectively by considering WCR ($\Gamma = 0.01$) and the constant values of the other parameters. In Figs. 5.2(a) and (b), the effect of $\rho_1 (= n_{e0}/z_h n_{h0})$ and $\rho_2 (= z_i n_{i0}/z_h n_{h0})$ on the propagation of HIA soliton are displayed respectively by considering WCR ($\Gamma = 0.01$) and the constant values of the other parameters. In Fig. 5.3, the effect of Γ on the propagation of HIA soliton is displayed by considering $U(\Gamma) \equiv -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81$ with the presence nonthermal nonextensive as well as only the population of nonthermal electrons and the constant values of the other parameters. The above figures clearly indicates that the electrostatic potentials are produced with the presence of not only isothermal ($q \rightarrow 1$ and $\alpha = 0$), superthermal ($\alpha = 0, 0.33 < q < 1$), subthermal ($\alpha = 0, q > 1$), nonthermal ($q \rightarrow 1, 0 < \alpha < 0.25$) but also nonextensive nonthermal electrons, except some limitations for the coupling parameter. Because, the nonlinear propagation of HIA solitons are supported within the ranges $\Gamma < 100$ of Γ with the influence of both α and q only. Besides, it is exited not only with the influence of nonextensivity but also nonthermality of electrons for the range $\Gamma < 10$ of Γ by depending on the other parametric values of the related parameters. Figs. 5.1–5.3 also demonstrate that the amplitudes and widths of solitons are considerably decreased (increased) with the increase of q , ρ_1 and Γ (α) but very slightly increase with the increase of ρ_2 . In addition, the maximum soliton energies are produced

when HIs are weakly connected to MLIs because the growth rate of the amplitude of HIA soliton is increased in weakly correlated plasmas rather than strongly correlated plasmas. Besides, the nonlinearity coefficient P is strongly affected by q in which P is significantly increased with the increase q . Due to this, the HIA soliton amplitude is significantly decreased. Besides, the HIA soliton amplitude are significantly increased with the presence of population of nonthermal electrons increased. It is also observed that the driving force are only decreased only when the HIs density increased or the depopulation of electrons. As a result, the energies of HIA soliton losses, which is in good agreement with our finding as displayed in Fig. 5.2. Both compressive and rarefactive HIA solitons are also supported in considered plasmas with the presence of strongly (weakly) correlated ions.

To obtain the physical structures of HIA dressed solitons, one needs to follow some criteria of the perturbation theories. Based on such theories, one might be considered the higher order contribution only for the condition $|\phi_{A12} = \phi_{A1} + \phi_{A2}| < |\phi_A|$. In Ref.[156], authors have clearly mentioned that the dressed soliton solution is not physically meaningful if $|\phi_{A1} + \phi_{A2}| \geq |\phi_A|$. It is provided that one can study the nature of dressed solitons only if $|\phi_{A12}| < |\phi_A|$. To get the clear idea whether or not the dressed solitons exists, the amplitudes of soliton, wave propagation due to second order contribution and dressed soliton along with their widths against the population of nonthermal electrons, strength of nonextensivity for electrons and nonextensivity with the population of nonthermal electrons are displayed, respectively in Figs.5.4(a)–(f). The figure 5.4 has obviously shown that the dressed solitons are produced in both of WCR and SCR with the variation of the α , q except when $q > 4$ ($\alpha = 0$) in WCR and some specific values of q in the SCR based on the considered values of the other parameters. Based on $|\phi_{A12}| < |\phi_A|$, the structures of some HIA deressed soliton along with HIA wave propagation due to the second order contribution and HIA soliton for the case of nonthermality of electrons, superthermality of electrons, isothermality of electrons and subthermality of electrons are displayed in Figs. 5.5(a)–(d). The variation of dressed soliton against q is also displayed in Fig. 5.6. Finally, Fig. 5.7 display the nature of normalized electric field due to the HIA soliton and dressed soliton for different values of τ with $\alpha = 0.1$, $q = 1$, $\rho_1 = 0.5$, $\rho_2 = 0.1$, $V = 0.05$ and $\Gamma = 0.01$. From the above discussion, the finding with our mathematical validations presented in this article may be very useful for realizing the nature of low frequency wave

dynamics due to the propagation of not only HIA soliton but also HIA dressed soliton in many ASEs, where the plasma consists of strongly (weakly) correlated pair-ions with the presence of not only nonextensivity butl also nonthermality electrons. The theoretical finding are also useful to perform an experimental verification in future laboratory studies.

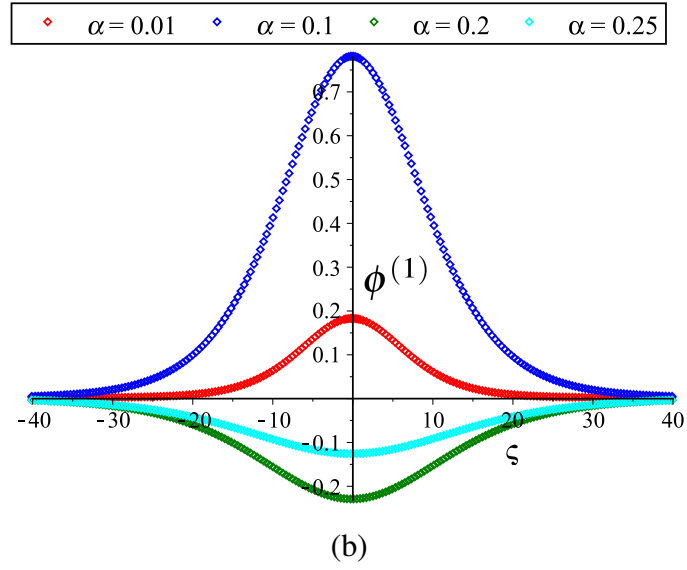
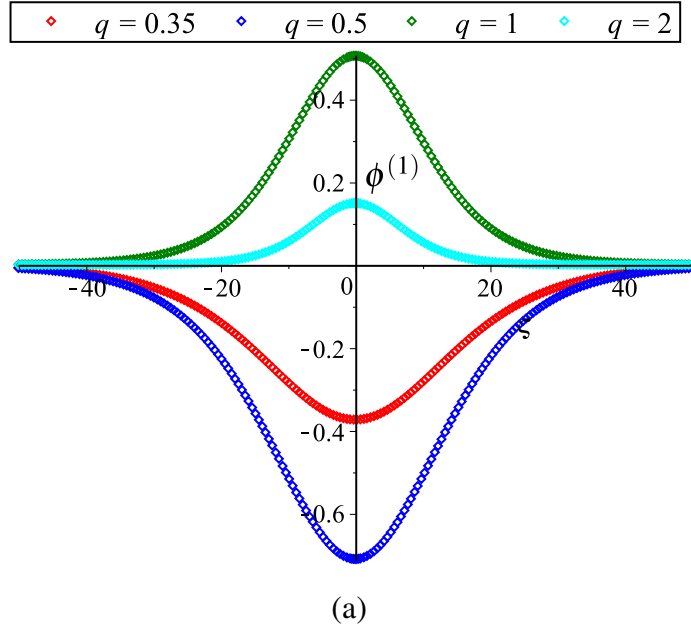


Fig. 5.1 Effect of (a) strength of electrons nonextensivity, that is, q ($\alpha = 0$) and (b) electrons nonthermality index, that is, α ($q = 1$) on the HIA soliton ($\phi^{(1)}$) by considering the WCR ($\Gamma = 0.01$) with $\sigma = 0.01$, $\rho_1 = 0.3$, $\rho_2 = 0.1$ and $V = 0.05$.

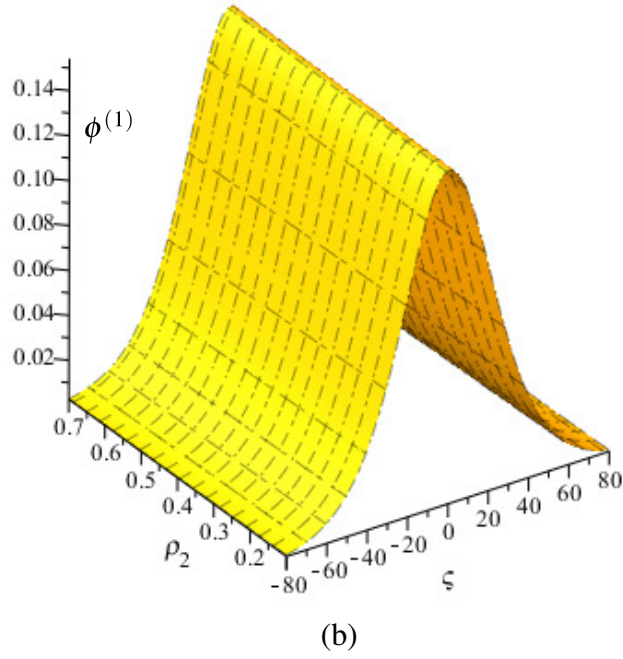
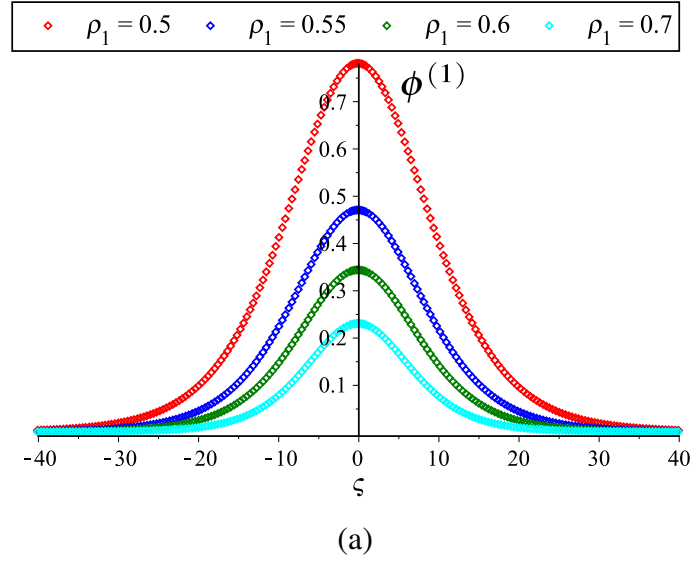


Fig. 5.2 Effect of (a) ρ_1 ($\rho_2 = 0.1$, $\sigma = 0.01$, $\alpha = 0.1$, $q = 1$ and $V = 0.05$) and (b) ρ_2 ($\rho_1 = 0.6$, $\sigma = 0.01$, $\alpha = 0.1$, $q = 0.35$ and $V = 0.05$) on the HIA soliton ($\phi^{(1)}$) by considering the WCR ($\Gamma = 0.01$).

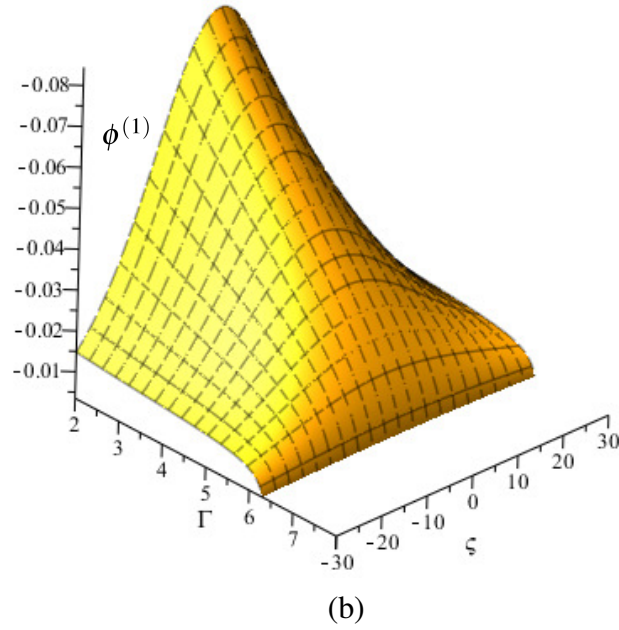
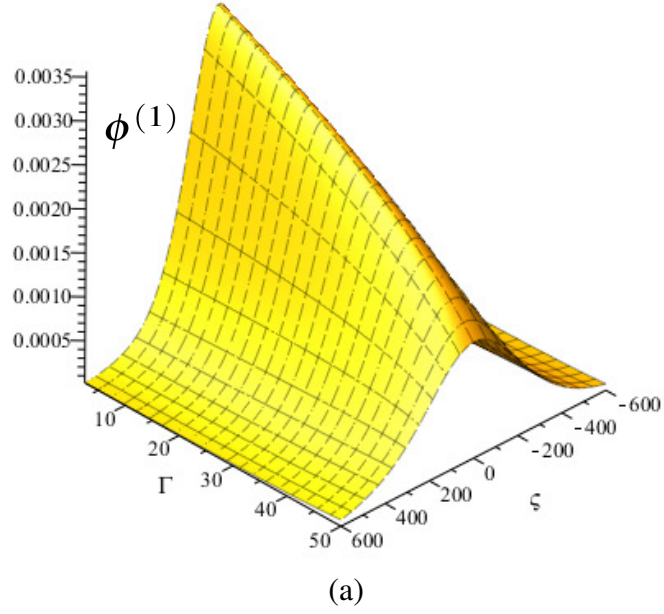


Fig. 5.3 Effect of (a) the coupling parameter Γ with $q = 0.55$, $\alpha = 0.01$, $\rho_1 = 0.3$, $\rho_2 = 0.1$ and $V = 0.05$; and (b) Γ with $q = 1$, $\alpha = 0.01$, $\rho_1 = 0.3$, $\rho_2 = 0.1$ and $V = 0.05$ on the HIA soliton ($\phi^{(1)}$) for the case of SCR.

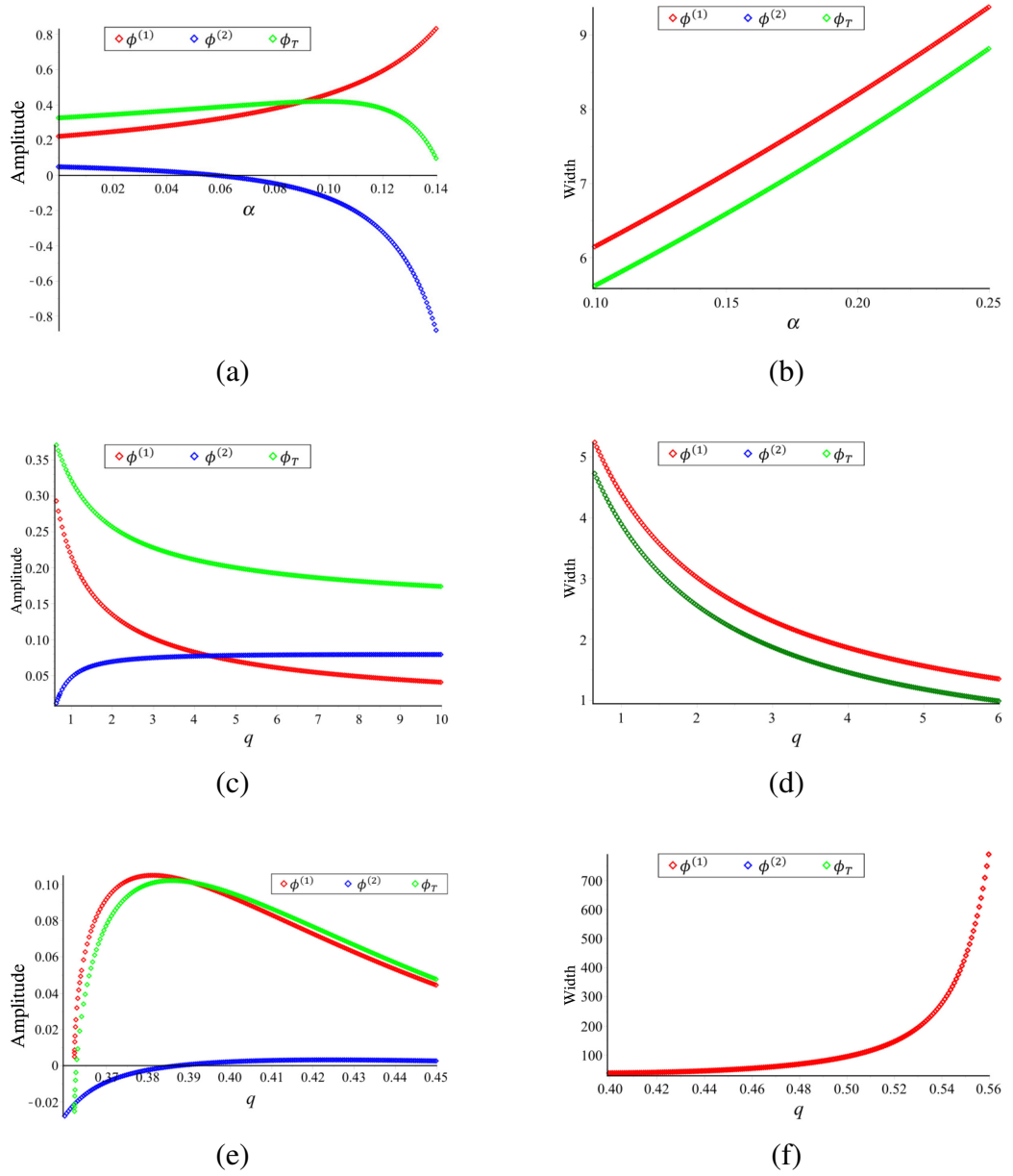


Fig. 5.4 Variation of (a) amplitude of $\phi^{(1)}$ (red color), $\phi^{(2)}$ (blue color) and ϕ_T (green color), and (b) width of soliton and dressed soliton against α ($q = 1$) in WCR ($\Gamma = 0.01$),; (c) amplitude of $\phi^{(1)}$ (red color), $\phi^{(2)}$ (blue color) and ϕ_T (green color), and (d) width of soliton and dressed soliton against q ($\alpha = 0$) in WCR ($\Gamma = 0.01$),; and (e) amplitude of $\phi^{(1)}$ (red color), $\phi^{(2)}$ (blue color) and ϕ_T (green color), and (f) width of soliton and dressed soliton against q ($\alpha = 0.25$) in SCR ($\Gamma = 10$). The remaining parameters are considered as $\rho_1 = 0.7$, $\rho_2 = 0.25$ and $V = 0.1$.

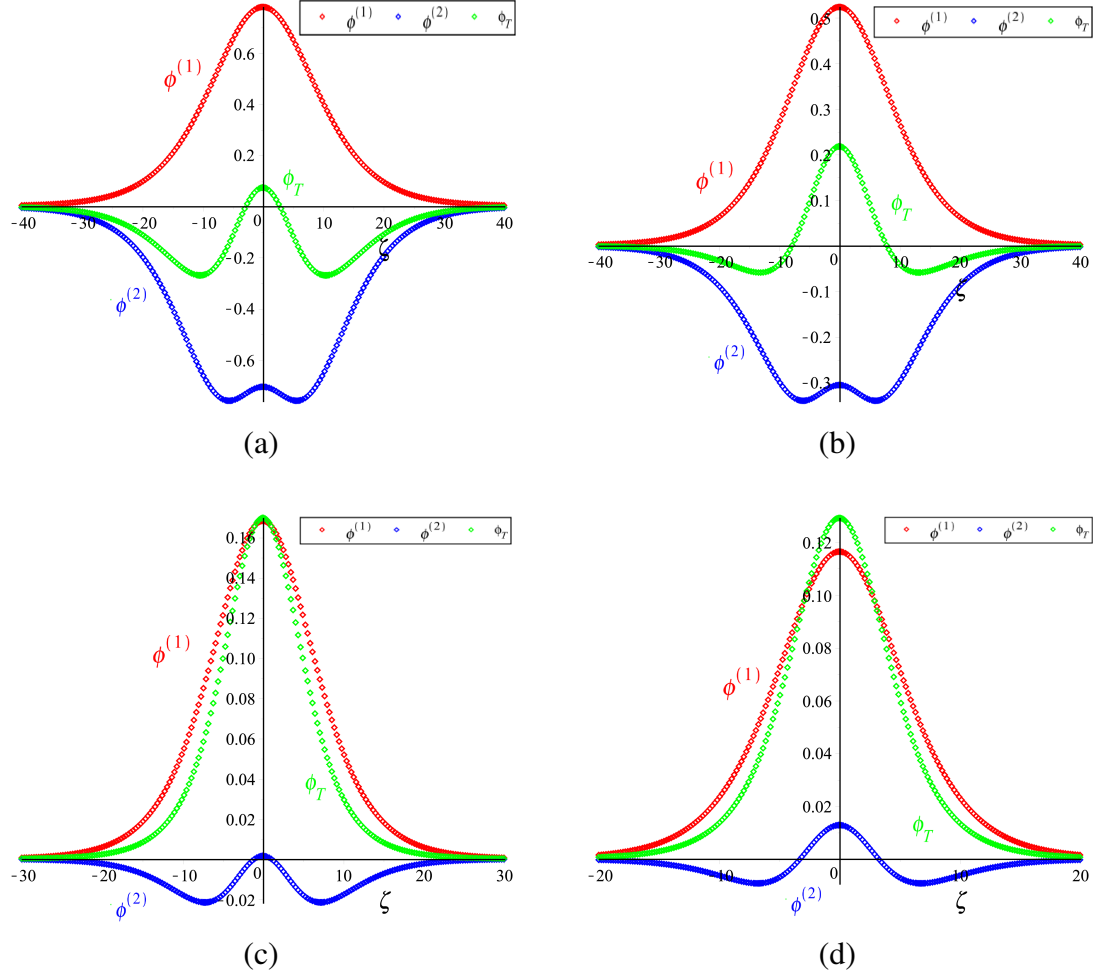


Fig. 5.5 Structures of HIA soliton ($\phi^{(1)}$ (red color), HIA wave propagation due to second order contribution $\phi^{(2)}$ (blue color) and HIA dressed soliton ϕ_T (green color) in WCR ($\Gamma = 0.01$) with the influence of (a) nonthermality of electrons ($\alpha = 0.1, q = 1$), (b) superthermality of electrons ($\alpha = 0, q = 0.4$), (c) isothermality of electrons ($\alpha = 0, q = 1$) and (d) subthermality of electrons ($\alpha = 0, q = 4$), respectively. The remaining parameters are considered as $\rho_1 = 0.5$, $\rho_2 = 0.1$ and $V = 0.05$.

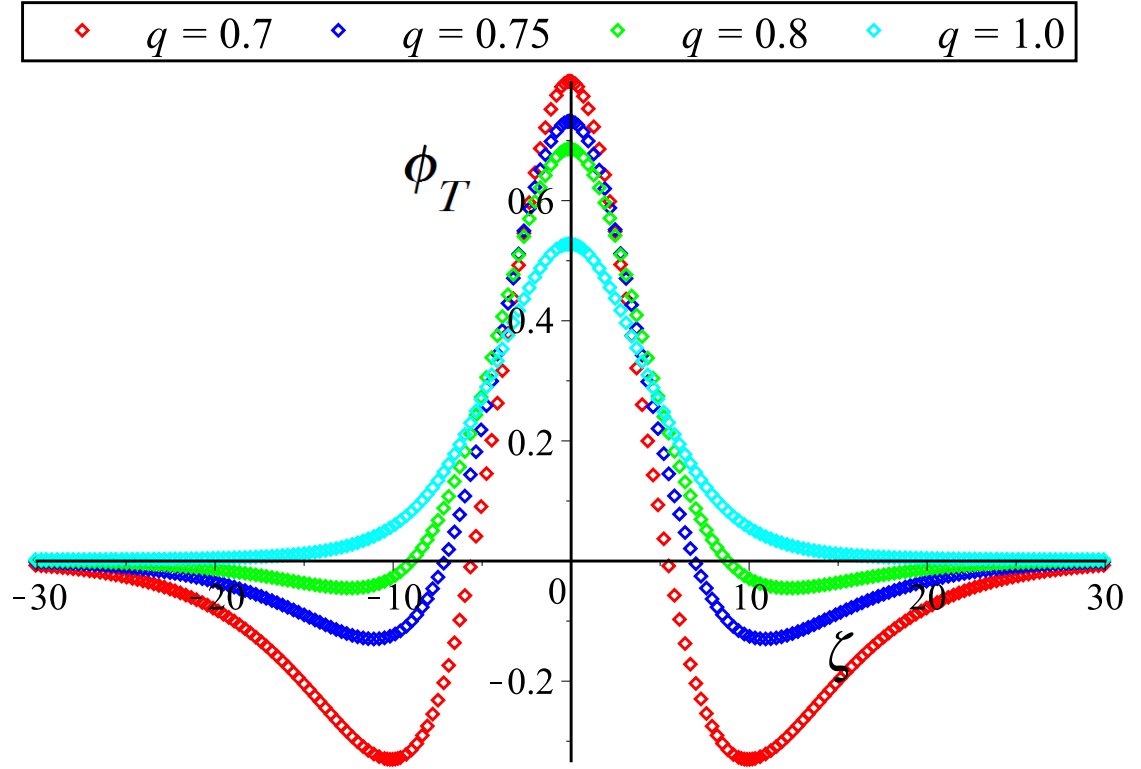


Fig. 5.6 Variation of HIA dressed soliton in SCR ($\Gamma = 3$) with the influence of nonextensivity of electrons q ($\alpha = 0$). The remaining parameters are considered as $\rho_1 = 0.5$, $\rho_2 = 0.1$ and $V = 0.05$.

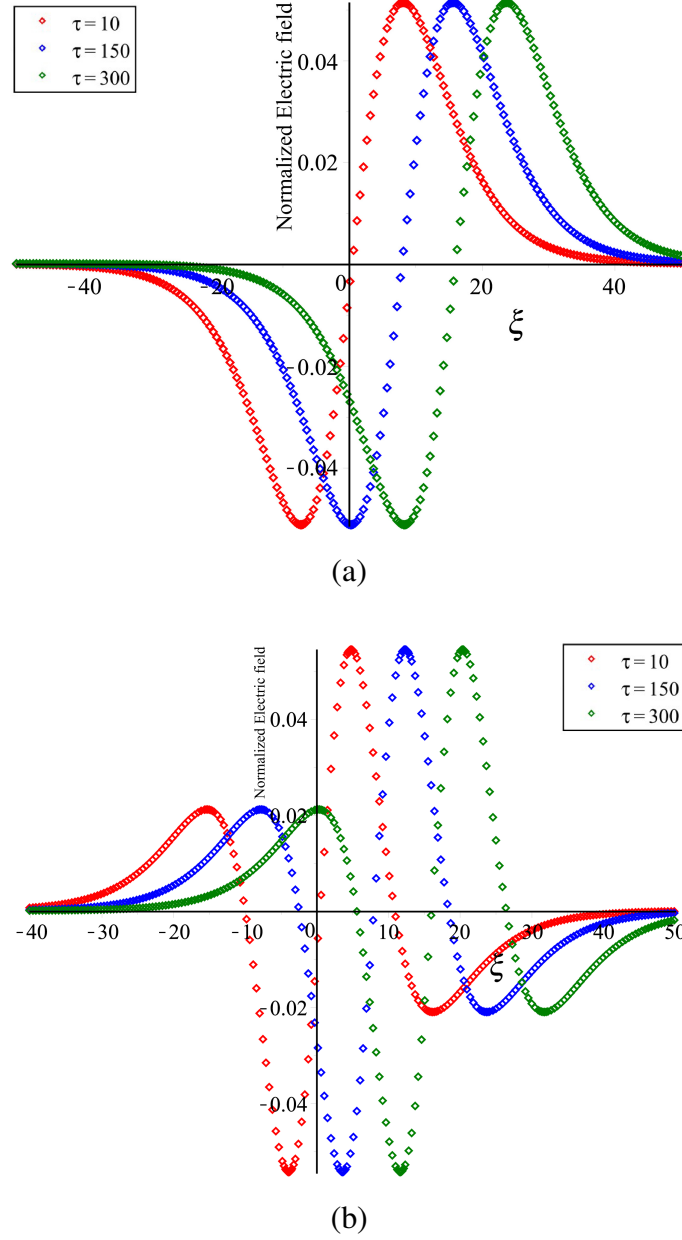


Fig. 5.7 Nature of normalized electric field due to (a) the HIA soliton and (b) the HIA dressed soliton for $\tau = 10$ (red color), $\tau = 150$ (blue color) and $\tau = 300$ (green color) with $\alpha = 0.1$, $q = 1$, $\rho_1 = 0.5$, $\rho_2 = 0.1$, $V = 0.05$ and $\Gamma = 0.01$.

5.6 CONCLUDING REMARKS

In this chapter, the characteristics of the HIA nonlinear soliton and dressed soliton structures in the weakly and strongly coupled plasmas have studied consisting of HIs fluid, MLIs which are strongly and weakly correlated to HIs and (α, q) -distributed electrons. By adopting the RPT, the KdV and inhomogeneous KdV-like equations have obtained from the considered plasma system. The nature of both HIA soliton and dressed soliton due to higher order contribution have been investigated with the influence of different plasma parameters. It is observed that the coupling parameter plays an significant role to the formation of not only HIA soliton but also HIA dressed soliton in which the amplitude and width are significantly decreased in SCR, whereas increased in WCR. The density ratio ρ_1 are also dominated to the formation of such solitons rather than ρ_2 in which the amplitude and the width are significantly decreased with the increase of ρ_1 but vary slightly increased with the increase of ρ_2 . The inclusion of higher order nonlinearity and dispersion corrections yields the evolution of HIA dressed soliton. The contribution of higher order effects leads to the formation of both compressive and rarefactive polarities such dressed solitons. These outcomes would be efficacious to describe various aspects of different kind of nonlinear coherent structures observed in many ASEs.

Chapter 6: BURGERS EQUATION WITH CUBIC AND QUARTIC NONLINEARITY: DYNAMICS OF HEAVY ION-ACOUSTIC COHERENT STRUCTURES IN UNMAGNETIZED STRONGLY COUPLED PLASMA

6.1 INTRODUCTION

Plasmas can be found in various environments, such as Earth's ionosphere, magnetosphere, cometary tails, solar wind, and the bow shock near the magnetopause boundary [157–159]. These plasmas contain both positive and negative ions, along with electrons. Heavy elements are thought to have formed in the early universe during a phase when matter was packed into extremely high densities and temperatures [160]. Many scientists have studied multi-ion plasmas using both theoretical models and experimental research [161–165]. However, strongly coupled plasma (SCP) is a critical area of study because it plays a key role in advancing plasma physics, fusion research, and understanding astrophysical and space phenomena, both experimentally [166],[167] and theoretically [168–170]. In SCP, particles interact intensely, leading to complex behaviors like phase transitions and collective structures, which differ from the weakly coupled plasma (WCP) that are more commonly studied. This state is important for understanding extreme environments, such as stellar interiors, white dwarfs, and fusion reactors, where high-density plasmas exist. Due to the high charge and lower temperature of the dust particles, the dusty plasma enters a strongly coupled state. The ions, which also have high charge densities, interact with each other through their mutual Coulomb forces and become strongly coupled as well. The strength of the interaction can be measured using the coupling parameter, $\Gamma = \frac{Z_i^2 Q_i^2}{I_i k_B T_i}$, where Q_i , I_i , Z_i , and T_i represent the charge, spacing between particles, charge state, and temperature of the ions, respectively. When $\Gamma > 1$, the system is in a strongly coupled regime, and when $\Gamma < 1$, it is in a weakly coupled regime. The experimental evidence from dusty plasmas, high-energy density laboratories, and fusion devices has provided valuable

insights into SCP's properties, aiding advancements in fusion energy, space weather prediction, and materials science. However, the presence of heavy charged particles, prevalent in the early stages of universal evolution under high-density and high-temperature conditions, has also been confirmed [170]. Recent experimental work by Lyon et al. [171] demonstrated the production of SCPs from laser-heated solids, highlighting ion temperature behaviors under various initial lattice structures. These developments provide opportunities for studying fundamental SCP characteristics in multi-ion and electron systems.

In contrast, plasma particle velocity distributions often deviate from the Maxwellian distribution due to long-range interactions like gravitational or Coulomb forces, resulting in non-thermal behaviors, such as subthermality, superthermality, and nonthermality. These deviations have been widely documented in astrophysical contexts [172],[173], with the existence of charged particles in different phase spaces also confirmed [174]. Nonextensive statistics, introduced by Tsallis [175], offer a framework for understanding these equilibrium deviations, particularly in scenarios with power-law distributions observed in astrophysical and cosmological environments. This framework deviates from traditional Boltzmann-Gibbs statistics by introducing a generalized entropy function, which accounts for long-range interactions in complex systems. By incorporating the nonextensive parameter q , it allows for a more accurate description of systems that do not adhere to the classical assumptions of extensive thermodynamics, making it especially relevant in the study of anomalous behaviors in various scientific fields. Furthermore, investigations have explored nonlinear ion-acoustic (IA) waves, assuming nonthermal distributions for electrons, which incorporate long-range interactions and phase space distributions [176]. Tribeche et al. [177] introduced a hybrid Tsallis-nonthermal electron velocity distribution function, demonstrating that such models better fit space observations and influence IA wave structures. They also outlined criteria for nonextensive and nonthermal parameters, crucial for analyzing physical phenomena across different plasma regimes. Recent studies [177–183] have also highlighted the importance of SCPs by theoretically investigating nonlinear IA shock and solitary wave phenomena with the influence of plasma parameters.

Despite significant advancements, most of the earlier studies overlook the nonlinear characteristics of shock and solitary wave excitations, particularly near critical values

(CVs) and at CVs, due to the complex mathematical calculations. A few studies have focused solely on shock excitations around CVs, deriving the Burgers equation (BE) with a cubic nonlinear term, particularly when the BE with a quadratic nonlinear term fails to adequately describe shock propagation in SCPs. For instance, Nazrul et al. [182],[183] investigated the propagation features of heavy ion acoustic (HIA) shock and dressed soliton propagation characteristics by deriving the BE with a quadratic nonlinear term and in-homogeneous KdV equations, respectively in an unmagnetized SCP, integrating heavy ions (HIs) fluids, (α, q) -distributed electrons, and Maxwellian light ions (LIs). They demonstrated that the plasma environment supports both compressive and rarefactive shocks when the nonlinear coefficient of quadratic nonlinear BE is positive or negative, respectively. They also found that the nonlinear coefficient becomes zero for specific plasma parameter values, referred to as the CVs, leading to a situation where shock wave propagation cannot be analyzed using this equation. To address this issue, they derived the BE with a cubic nonlinear term to study shock wave excitations around CVs. This raises an important question: what happens to the propagation of not only for shock waves but also periodic waves, along with their dynamical features, when the nonlinear coefficients of both the quadratic and cubic terms simultaneously approach to zero at a specific plasma parameter value—referred to as the super-critical value (SCV)—in the aforementioned plasma environment?

Given these gaps in existing research [182],[183], there is a clear need to investigate the effects of plasma parameters with the existence of SCVs on shock and periodic waves, along with their dynamic behaviors. This could be achieved by extending the BE to include higher-order nonlinear terms or by combining multiple higher-order nonlinear terms. Therefore, this study aims to investigate the nonlinear characteristics of HIA shock waves, periodic waves, and their dynamic behavior in an unmagnetized SCP [182],[183]. The focus will be on understanding the propagation features, both near and at the CVs and SCVs, through the derivation of new nonlinear evolution equations (NLEEs). Additionally, the research will explore the critical roles of the plasma parameters on the propagation HIA dynamics in the considered plasma environment. Notably, since the integrable NLEEs can be solved analytically, numerical methods are generally not needed. Experimentalists typically prefer comparing their findings with these analytical solutions. Therefore, this

study will also address whether numerical techniques are necessary for investigating acoustic wave phenomena in such plasmas.

6.2 GOVERNING FLUID EQUATIONS

The unmagnetized collisionless SCP consists of inertial positively charged HIs, inertialess electrons with (α, q) -distributed velocity, and LIs with Maxwellian-distributed velocity species, subject to $Z_i N_{i0} + Z_h N_{h0} = N_{e0}$, where the unperturbed density are defined as N_{s0} , $s = h$ (HI), $s = e$ (electron), $s = i$ (LI) [182],[183]. HIs are strongly coupled with LIs due to their heavy mass m_h , large electric charge Z_h , and low temperature T_h . The electron density is determined using the (α, q) -velocity distribution [182], [183]:

$$N_e = N_{e0} \left[1 + (q-1) \frac{e\psi}{T_e} \right]^{\frac{q+1}{2(q-1)}} \times \left[1 + \Sigma_1 \left(\frac{e\psi}{T_e} \right) + \Sigma_2 \left(\frac{e\psi}{T_e} \right)^2 \right], \quad (6.1)$$

with specific terms Σ_1 and Σ_2 given by:

$$\Sigma_1 = -\frac{16q\alpha}{3 - 14q + 15q^2 + 12\alpha}, \quad \Sigma_2 = -\Sigma_1(2q - 1).$$

Here, the strength of nonextensivity is represented by q , while α quantifies the population of nonthermal electrons. It is noted that the (α, q) -velocity distribution can be employed to investigate the physical phenomena observed in many astrophysical environments, provided the appropriate ranges for α and q are considered. Specifically, these include: (i) isothermal ($q \rightarrow 1$ and $\alpha = 0$), (ii) superthermal ($\alpha = 0, 0.33 < q < 1$), (iii) subthermal ($\alpha = 0, q > 1$), and (iv) nonthermal ($q \rightarrow 1$ and $0 < \alpha < 0.35$). The normalized mass and momentum conservation equations for the HIs fluid are [182],[183]:

$$\frac{\partial N_h}{\partial t} + \frac{\partial (N_h U_h)}{\partial z} = 0, \quad (6.2)$$

$$\left(1 + \tau_R \frac{\partial}{\partial t} \right) \left[N_h \left(\left(\frac{\partial}{\partial t} + U_h \frac{\partial}{\partial z} \right) U_h + \frac{\partial \psi}{\partial z} \right) + c_f \frac{\partial N_h}{\partial z} \right] = \eta \frac{\partial^2 U_h}{\partial z^2}, \quad (6.3)$$

where N_h (U_h) is the normalized HIs density (velocity), η is the plasma viscosity coefficient, ψ is normalized electrostatic potential and $c_f = \mu_h k_B T_h \gamma_h$ is the compressibility

factor. The relaxation time τ_R is:

$$\tau_R = \eta \frac{T_e}{T_h} \left[1 - \mu_h + \frac{4}{15} U(\Gamma) \right]^{-1}, \mu_h = \frac{1}{k_B T_h} \left[1 + \frac{1}{3} U(\Gamma) + \frac{\Gamma}{9} \frac{\partial U(\Gamma)}{\partial \Gamma} \right].$$

The system is closed by the Poisson equation for the electrostatic potential [182]:

$$\frac{\partial^2 \psi}{\partial z^2} = r_1 [1 + (q-1)\psi]^{\frac{q+1}{2(q-1)}} \times [1 + \Sigma_1 \psi + \Sigma_2 \psi^2] - r_2 \exp(-\sigma_{ei} \psi) - N_h, \quad (6.4)$$

where $r_1 = N_{e0}/(Z_h N_{h0})$ is the electron to HI density ratio, $r_2 = (Z_i N_{i0})/(Z_h N_{h0})$ is the LI to HI density ratio, and $\sigma_{ei} = T_e/T_i$ is the electron to LI temperature ratio [182]. It is noted that one can be studied the effect of coupling parameter (Γ) on nonlinear electrostatic potential HIA waves by considering $U(\Gamma) \equiv -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81$, $\Gamma > 1$ for SCP [184] and $U(\Gamma) \equiv -(\sqrt{3}/2)\Gamma^{3/2}$, $\Gamma < 1$ for WCP [185]. It is important to mention that the ranges of plasma parameters are used in this investigation are very wide and corresponds to space and laboratory plasma system. Because Rosenberg et al. [163] provided the parameters corresponding to the laboratory experiment as electron density $3.3 \times 10^8 \text{ cm}^{-3}$, ion densities $5 \times 10^8 \text{ cm}^{-3}$, & $7.5 \times 10^7 \text{ cm}^{-3}$, ion temperature 0.05eV & 0.025eV, and electron temperature $T_e = 2.5\text{eV}$ & 3eV. They have also compared their experimental results with the analytical results using the aforementioned data. Later, Mamun and his research group [179],[180] studied not only dust acoustic waves but also HIA waves in SCPs by defining the appropriate ranges of the plasma parameters ratio's. Thus the ranges of related plasma parameters for the considered plasma system are taken as $r_1 = 0.1 - 0.7$, $r_2 = 0.1 - 0.6$, $\eta = 0.4 - 0.8$, and $\sigma_{ei} = 0.01$ based on the references [179],[180],[182] to study HIA shock wave excitations. They have also mentioned that their outcomes should be helpful for understanding the positively charged heavy ion and nonextensive effects in various space plasmas [149, 186–188], particularly in stellar polytropes [186], hadronic matter and quark-gluon plasma [187], dark-matter halos [149], etc., as well as laboratory plasmas [163],[188].

6.3 BE WITH QUARTIC NONLINEAR TERM AND SHOCK WAVE EXCITATIONS

In previous study [182], researchers have defined the new stretching coordinates as $\zeta = \varepsilon^i(z - \lambda_v t)$ and $\tau = \varepsilon^{2i}t$, where $0 < \varepsilon < 1$ and λ_v is the linear phase velocity. By considering $i = 1$ and $i = 2$, they have derived BE with quadratic and cubic nonlinear term, respectively. To derive BE with quartic nonlinear term, one needs to define new coordinates with $i = 3$ as

$$\zeta = \varepsilon^3(z - \lambda_v t), \tau = \varepsilon^6 t, \quad (6.5)$$

and the power series expansion of unknown variables [182] as

$$\begin{bmatrix} N_h \\ U_h \\ \psi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \sum_i \varepsilon^i \begin{bmatrix} N_h^{(i)} \\ U_h^{(i)} \\ \psi^{(i)} \end{bmatrix}, \quad (6.6)$$

Applying Eqs. (6.5) and (6.6) into Eqs. (6.2), (6.3) and (6.4) in a systematic way, Eqs. (6.2), (6.3) and (6.4) are obtained with the involvement of different order of ε , that is $O(\varepsilon^k)$, $k = 4, 5, 6, \dots$. From $O(\varepsilon^4)$ -equations, one obtains

$$N_h^{(1)} = \frac{1}{\lambda_v^2 - c_f} \psi^{(1)}, U_h^{(1)} = \frac{\lambda_v}{\lambda_v^2 - c_f} \psi^{(1)}, \lambda_v = \sqrt{c_f + \frac{1}{\Delta_1}}, \quad (6.7)$$

where

$$\Delta_1 = r_1 \left[\frac{q+1}{2} - \frac{16q\alpha}{3-14q+15q^2+12\alpha} \right] + r_2 \sigma_{ei}.$$

It is observed from Eq. (6.7) that $N_h^{(1)}$ and $U_h^{(1)}$ are the first order perturb quantities and λ_v is the linear dispersion relation of the HIA waves, which are same as we have obtained before in [182],[183] for studying HIA shocks, solitons and dressed solitons propagations.

Again, $O(\varepsilon^5)$ order equations yields the following relations:

$$\begin{aligned} N_h^{(2)} &= \frac{1}{\lambda_v^2 - c_f} \psi^{(2)} + \left[\frac{3}{2(\lambda_v^2 - c_f)^2} + \frac{c_f}{(\lambda_v^2 - c_f)^3} \right] \left\{ \psi^{(1)} \right\}^2, \\ U_h^{(2)} &= \frac{\lambda_v}{\lambda_v^2 - c_f} \psi^{(2)} + \left[\frac{\lambda_v}{2(\lambda_v^2 - c_f)^2} + \frac{\lambda_v c_f}{(\lambda_v^2 - c_f)^3} \right] \left\{ \psi^{(1)} \right\}^2, \end{aligned} \quad (6.8)$$

and

$$-\frac{\lambda_v^2 - c_f}{2\lambda_v} \left[\frac{3\lambda_v^2 - c_f}{(\lambda_v - c_f)^2} - 2\frac{\Delta_2}{\Delta_1} \right] \left\{ \psi^{(1)} \right\}^2 = 0, \quad (6.9)$$

where

$$\Delta_2 = r_1 \left[\frac{(q+1)(3-q)}{8} - \frac{q+1}{2} \times \frac{16q\alpha}{3-14q+15q^2+12\alpha} + \frac{16q(2q-1)\alpha}{3-14q+15q^2+12\alpha} \right] - \frac{r_2 \sigma_{ei}^2}{2}.$$

Eq. (6.8) represents the second order perturb quantities, whereas Eq. (6.9) yields the solvability condition, which are same as we have obtained before in [182] for studying the propagation of HIA shocks around CVs described by BE with cubic nonlinearity. Eq. (6.9) holds only if

$$A_1 = \frac{\lambda_v^2 - c_f}{2\lambda_v} \left[\frac{3\lambda_v^2 - c_f}{(\lambda_v - c_f)^2} - 2\frac{\Delta_2}{\Delta_1} \right] = 0, \quad (6.10)$$

which yields the CVs ($C_v's$) for any specific parameter. It is noted that A_1 is the nonlinear coefficient of BE with quadratic nonlinear term that derived in Ref. [182]. Again, $O(\varepsilon^6)$ order equations (ignored for simplicity) yields the following relations:

$$\begin{aligned} N_h^{(3)} &= \frac{(3/2)\lambda_v^4 + 4c_f\lambda_v^2 + (1/2)c_f}{(\lambda_v^2 - c_f)^5} \left\{ \psi^{(1)} \right\}^3 + \frac{3\lambda_v^2 - c_f}{(\lambda_v^2 - c_f)^3} \left\{ \psi^{(1)} \psi^{(2)} \right\} \\ &\quad - \frac{1}{(\lambda_v^2 - c_f)^4} \psi^{(3)}, \end{aligned} \quad (6.11)$$

$$\begin{aligned} U_h^{(3)} &= \frac{\lambda_v((3/2)\lambda_v^4 + 4c_f\lambda_v^2 + (1/2)c_f)}{(\lambda_v^2 - c_f)^5} \left\{ \psi^{(1)} \right\}^3 + \frac{\lambda_v^2 + c_f}{(\lambda_v^2 - c_f)^3} \left\{ \psi^{(1)} \psi^{(2)} \right\} \\ &\quad - \frac{1}{(\lambda_v^2 - c_f)^4} \psi^{(3)}, \end{aligned} \quad (6.12)$$

and

$$\left[-\Delta_3 + \frac{(3/2)\lambda_v^4 + 4c_f\lambda_v^2 + (1/2)c_f}{(\lambda_v^2 - c_f)^5} + \frac{2\lambda_v^2}{(\lambda_v^2 - c_f)^4} \right] \left\{ \psi^{(1)} \right\}^3 + \frac{A_c}{\lambda_v^2 - c_f} \left\{ \psi^{(1)} \psi^{(2)} \right\} - \Delta_1 (1 - \Delta_1^3) \psi^{(3)} = 0, \quad (6.13)$$

where

$$\Delta_3 = r_1 \left[\frac{(q+1)(3-q)(5-q)}{48} - \frac{2q\alpha(q+1)(3-q)}{3-14q+15q^2+12} + \frac{8q(2q-1)\alpha(q+1)}{3-14q+15q^2+12} \right] + \frac{r_2\sigma_{ei}^3}{6}. \quad (6.14)$$

Eqs. (6.11) and (6.12) represent the second order perturb quantities, whereas Eq. (6.13) yields the solvability condition, which are determined for the first time. Eq. (6.13) is solvable only if

$$A_{sc} = -\Delta_3 + \frac{(3/2)\lambda_v^4 + 4c_f\lambda_v^2 + (1/2)c_f}{(\lambda_v^2 - c_f)^5} + \frac{2\lambda_v^2}{(\lambda_v^2 - c_f)^4} = 0, \quad (6.15)$$

and $\psi^{(3)} = 0$. It is noted that A_{sc} is the nonlinear coefficient of BE with cubic nonlinear term that derived in Ref. [182]. It is clearly found that the condition for the derivation of BE with more higher order nonlinear term is $A_1 = 0$ and $A_{sc} = 0$ at the same time. So, one needs to determine a value of any specific parameter with the constant values of the remaining parameters for which both $A_1 = 0$ and $A_{sc} = 0$, which so-called the SCV (C_{sv}). The existence of C_v and C_{sv} are given in Fig. 1. It is found from this figure that both C_v and C_{sv} are same, that is, $C_v = C_{sv} \sim 0.48$ for which both $A_1 = 0$ and $A_{sc} = 0$.

Finally, $O(\varepsilon^7)$ order equations are determined from Eqs. (6.2)-(6.4) by implementing Eqs. (6.5) and (6.6) in the following forms:

$$\frac{1}{(\lambda_v^2 - c_f)} \frac{\partial \psi^{(1)}}{\partial \tau} - \lambda_v \frac{\partial N_h^{(4)}}{\partial \xi} + \frac{\partial U_h^{(4)}}{\partial \xi} + \frac{\partial}{\partial \xi} \left[N_h^{(2)} U_h^{(2)} \right] + \frac{1}{(\lambda_v^2 - c_f)} \frac{\partial}{\partial \xi} \left[\psi^{(1)} U_h^{(3)} \right] + \frac{\lambda_v}{(\lambda_v^2 - c_f)} \frac{\partial}{\partial \xi} \left[\psi^{(1)} N_h^{(3)} \right] = 0, \quad (6.16)$$

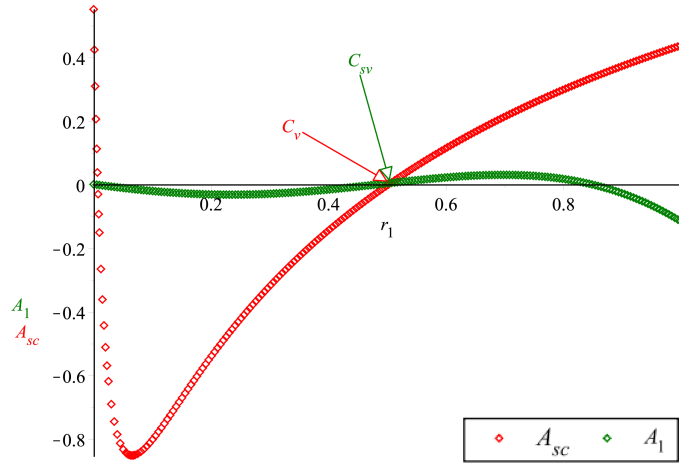


Fig. 6.1 Shape of A_1 and A_{sc} with regards to r_1 with $r_2 = 0.3$, $\Gamma = 8$, $\sigma_{ei} = 0.1$, $\alpha = 0$ and $q = 0.63$.

$$\begin{aligned}
& \frac{\lambda_v}{(\lambda_v^2 - c_f)} \frac{\partial \psi^{(1)}}{\partial \tau} + U_h^{(2)} \frac{\partial U_h^{(2)}}{\partial \xi} - \lambda_v N_h^{(2)} \frac{\partial U_h^{(2)}}{\partial \xi} + N_h^{(3)} \frac{\partial \psi^{(1)}}{\partial \xi} + \frac{\lambda_v^2 \left\{ \psi^{(1)} N_h^{(2)} \right\}}{(\lambda_v^2 - c_f)^2} \frac{\partial \psi^{(1)}}{\partial \xi} \\
& + N_h^{(2)} \frac{\partial \psi^{(2)}}{\partial \xi} + \frac{\lambda_v}{(\lambda_v^2 - c_f)} U_h^{(3)} \frac{\partial \psi^{(1)}}{\partial \xi} + \frac{1}{(\lambda_v^2 - c_f)} \psi^{(1)} \frac{\partial \psi^{(3)}}{\partial \xi} - \frac{\lambda_v^2}{(\lambda_v^2 - c_f)} N_h^{(3)} \frac{\partial \psi^{(1)}}{\partial \xi} \\
& + \frac{\lambda_v}{(\lambda_v^2 - c_f)^2} \left\{ \psi^{(1)} \right\}^2 \frac{\partial U_h^{(2)}}{\partial \xi} + \frac{\lambda_v}{(\lambda_v^2 - c_f)^2} \left\{ \psi^{(1)} U_h^{(2)} \right\} \frac{\partial \psi^{(1)}}{\partial \xi} + \frac{\tau_R \lambda_v^3}{(\lambda_v^2 - c_f)} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} \\
& - \frac{\tau_R \lambda_v c_f}{(\lambda_v^2 - c_f)} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2} - \tau_R c_f \lambda_v \frac{\partial^2 N_h^{(1)}}{\partial \xi^2} - \lambda_v \frac{\partial U_h^{(4)}}{\partial \xi} + c_f \frac{\partial N_h^{(4)}}{\partial \xi} + \frac{\partial \psi^{(4)}}{\partial \xi} \\
& = \frac{\eta \lambda_v}{(\lambda_v^2 - c_f)} \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \tag{6.17}
\end{aligned}$$

$$\begin{aligned}
& -4\Delta_4 \left\{ \psi^{(1)} \right\}^3 \frac{\partial}{\partial \xi} \psi^{(1)} - 3\Delta_3 \frac{\partial}{\partial \tau} \left(\left\{ \psi^{(1)} \right\}^2 \psi^{(2)} \right) - 2\Delta_2 \frac{\partial}{\partial \tau} \left(\psi^{(1)} \psi^{(3)} \right) \\
& - \Delta_2 \frac{\partial}{\partial \xi} \left(\psi^{(2)} \right)^2 - \Delta_1 \frac{\partial}{\partial \xi} \psi^{(4)} + \frac{\partial}{\partial \xi} N_h^{(4)} = 0, \tag{6.18}
\end{aligned}$$

Simplifying Eqs. (6.16)–(6.18), one obtains the following BE with quartic nonlinear term:

$$\frac{\partial \psi^{(1)}}{\partial \tau} + P \left\{ \psi^{(1)} \right\}^3 \frac{\partial \psi^{(1)}}{\partial \xi} = Q \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \tag{6.19}$$

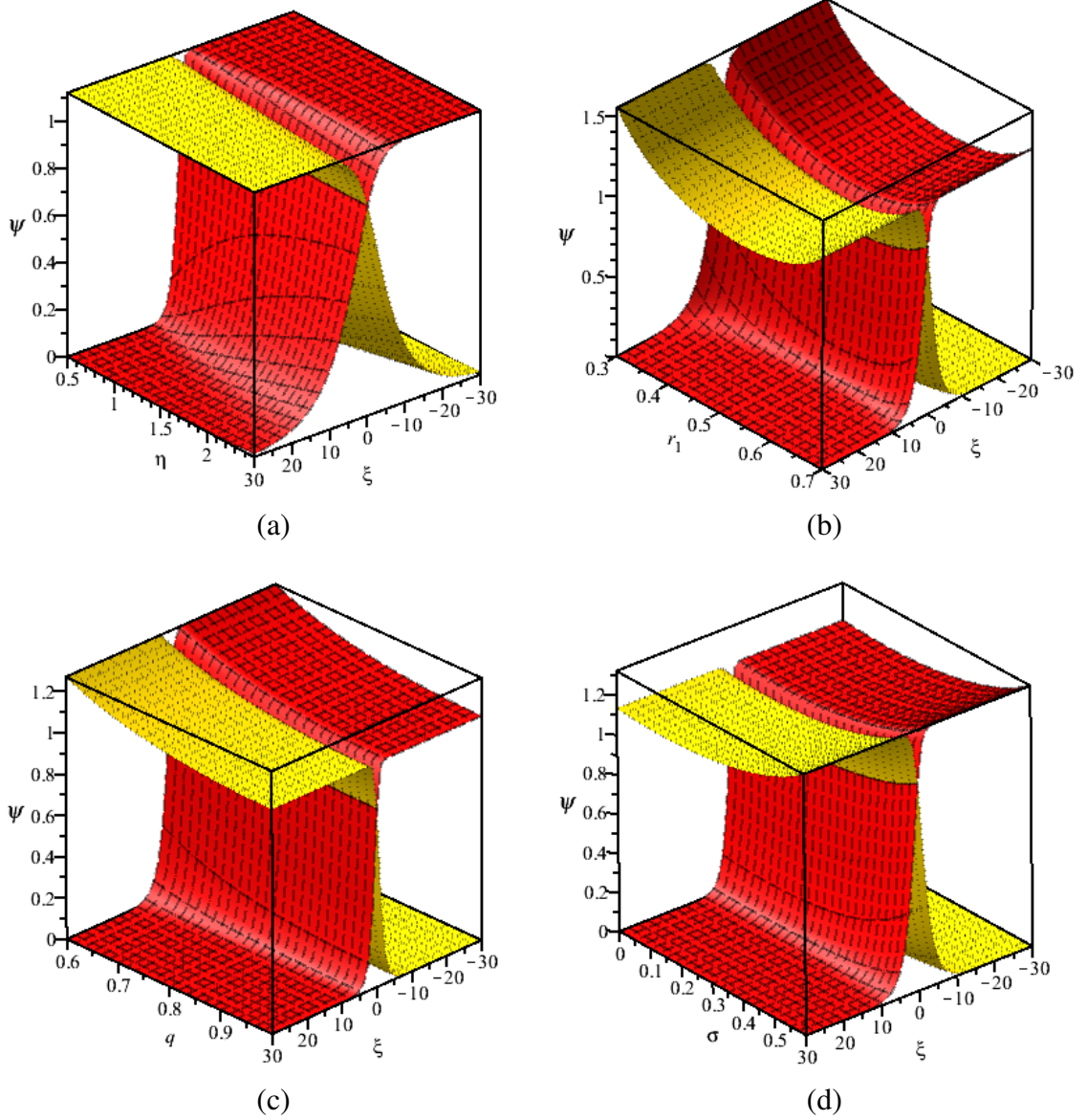


Fig. 6.2 Kink and anti-kink analysis of shock wave based on the solution as in Eq. (21) for (a) η with $\alpha = 0, \sigma = 0.1, q = 0.8, \Gamma = 8, r_2 = 0.3, r_1 = 0.55$; (b) r_1 with $\alpha = 0, \sigma = 0.1, q = 0.8, \Gamma = 8, r_2 = 0.3, \Xi = 1$, and $\eta = 0.5$; (c) q with $\alpha = 0, \Gamma = 8, r_2 = 0.3, r_1 = 0.55$, and $\eta = 0.5$; (d) σ with $\alpha = 0, \Gamma = 8, r_2 = 0.3, r_1 = 0.55$, and $\eta = 0.5$.

where

$$\begin{aligned}
P = & \frac{13\lambda_v^7}{2(-\lambda_v^2 + c_f)^6} + \frac{21\lambda_v^5 c_f}{2(-\lambda_v^2 + c_f)^6} - \frac{21\lambda_v^5}{4(-\lambda_v^2 + c_f)^5} + \frac{7\lambda_v^3 c_f}{4(-\lambda_v^2 + c_f)^5} \\
& - \frac{\lambda_v c_f}{4(-\lambda_v^2 + c_f)^5} + \frac{c_f^2}{4\lambda_v(-\lambda_v^2 + c_f)^5} - 2\lambda_v^3 H_4 + 4\lambda_v H_4 c_f + \frac{3\lambda_v^3}{2(-\lambda_v^2 + c_f)^4} \\
& - \frac{3\lambda_v c_f}{2(-\lambda_v^2 + c_f)^4} - 2\frac{c_f^2 H_4}{\lambda_v} - \frac{33\lambda_v^3 c_f^2}{2(-\lambda_v^2 + c_f)^6} - \frac{\lambda_v c_f^3}{2(-\lambda_v^2 + c_f)^6} + \frac{2\lambda_v^3 c_f}{(-\lambda_v^2 + c_f)^6} \\
& - \frac{2\lambda_v c_f^2}{(-\lambda_v^2 + c_f)^6} + \frac{7\lambda_v c_f^2}{2(-\lambda_v^2 + c_f)^5}, Q = \frac{\eta}{2}.
\end{aligned} \tag{6.20}$$

Eq. (6.19) is a integrable NLEE, where P is the nonlinear coefficient and Q is the dissipative coefficient. So, one can easily determine the analytical solution by directly integrating. The shock and anti-shock wave solution of Eq. (6.19) is given below [189].

$$\psi^{(1)} = \left[\psi_A \left\{ 1 \pm \tanh \left(\frac{\xi}{\psi_W} \right) \right\} \right]^{\frac{1}{3}}, \tag{6.21}$$

where, $\psi_A = 2\lambda_s/P$ is the amplitude and $\psi_W = 3Q/2\lambda_s$ is the width of the shock wave. Based on this solution, the impact of plasma parameters around the SCV on the excitation of HIA shock and anti-shock waves is illustrated in Figures 2(a) to 2(d). From these figures, it is evident that both the shocks and anti-shocks, as described by the quartic nonlinear BE, undergo significant variations when the plasma parameters are altered. Specifically, the presence of a viscosity coefficient leads to a monotonic change in the shock and anti-shock structures, where the width of these waves changes monotonically with an increase in viscosity, while their amplitude remains relatively unaffected. This behavior can be understood as a consequence of increased viscosity, which dampens the wave's spatial gradients, thus affecting the width of the shock or anti-shock front without substantially altering the energy contained in the wave. Furthermore, the degree of nonextensivity, often represented by the ratio of electron to light ion temperatures, plays a crucial role in modifying the characteristics of the shocks and anti-shocks around SCVs. As the nonextensivity increases, the amplitude of compressive shocks and anti-shocks decreases.

This can be attributed to the thermodynamic effects of nonextensive distributions, where the increase in the temperature ratio tends to enhance the non-idealities in the plasma, leading to weaker compression (lower amplitude) in shocks and anti-shocks. However, the most striking changes occur when the density ratio, particularly the electron to heavy ion density ratio, is varied. As the density ratio increases up to approximately 0.5, the amplitude of the compressive shocks and anti-shocks decreases. Beyond this threshold, within the range $0.5 < r_1 < 0.9$, the amplitude of these waves starts to increase. This phenomenon can be explained by the different inertial responses of electrons and heavy ions to perturbations: a higher electron-to-ion density ratio results in enhanced electron dynamics, which tends to modify the shock structures, while a lower ratio allows for more pronounced ion dynamics that can reinforce the shock or anti-shock amplitudes.

6.4 BE WITH BOTH CUBIC AND QUARTIC NONLINEAR TERMS

To study the nature HIA wave phenomena at the not only critical composition but super-critical composition of any specific parameter (say J), one assume $A_{sc} = A_{sc}^0$ for not only very near to C_{sv} but also at C_{sv} as

$$A_{sc}^0 = \Lambda \left(\frac{\partial A_{sc}}{\partial J} \right)_{J=C_{sv}} |J - C_{sv}| = \Xi \epsilon \left(\frac{\partial A_{sc}}{\partial J} \right)_{J=C_{sv}}, \quad (6.22)$$

where $|J - C_{sv}| \equiv \epsilon$ because $|J - C_{sv}|$ is small quantity and $\Xi = 1(-1)$ for $J > C_{sv}(J < C_{sv})$. As a result, one can re-evaluate the Eq. (6.18) by introducing

$$\rho^{(4)} \equiv -\epsilon^7 \frac{1}{3} \Xi \left(\frac{\partial A_{sc}}{\partial J} \right)_{J=C_{sv}} \left\{ \psi^{(1)} \right\}^3$$

in the Eq. (6.4) and yields

$$\begin{aligned} & -4\Delta_4 \left\{ \psi^{(1)} \right\}^3 \frac{\partial \psi^{(1)}}{\partial \xi} - 3\Delta_3 \frac{\partial}{\partial \tau} \left(\left\{ \psi^{(1)} \right\}^2 \psi^{(2)} \right) - 2\Delta_2 \frac{\partial}{\partial \tau} \left(\psi^{(1)} \psi^{(3)} \right) - \\ & \Delta_2 \frac{\partial}{\partial \xi} \left(\psi^{(2)} \right)^2 - \Delta_1 \frac{\partial \psi^{(4)}}{\partial \xi} + \frac{\partial N_h^{(4)}}{\partial \xi} - \Xi \left(\frac{\partial A_{sc}}{\partial J} \right)_{J=C_{sv}} \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} = 0 \end{aligned} \quad (6.23)$$

Simplifying Eqs. (6.16), (6.17) and (6.23), one obtains

$$\frac{\partial \psi^{(1)}}{\partial \tau} + \Xi R \left\{ \psi^{(1)} \right\}^2 \frac{\partial \psi^{(1)}}{\partial \xi} + P \left\{ \psi^{(1)} \right\}^3 \frac{\partial \psi^{(1)}}{\partial \xi} = Q \frac{\partial^2 \psi^{(1)}}{\partial \xi^2}, \quad (6.24)$$

where

$$R = \frac{\lambda_v^2 - c_f}{\lambda_v} \left(\frac{\partial A_{sc}}{\partial J} \right)_{J=C_{sv}}, \quad (6.25)$$

which is the BE involving both cubic and quartic nonlinearity. It is observed that Eq. (6.24) is a non-integrable NLEE. That is why, it is not possible to determine the analytical solutions of this equation. At this stage, numerical techniques are only an arena to study the acoustic wave phenomena described by this type of non-integrable NLEE. Using the numerical techniques, one can study the acoustic wave phenomena with dynamical behaviours by forming a planar dynamical system from Eq. (6.24).

6.5 FORMATION OF PLANAR DYNAMICAL SYSTEM

To drive the dynamical system, one can consider the following wave transformation:

$$x = \xi - \lambda_s \tau. \quad (6.26)$$

Eq. (6.24) can then be converted to

$$\frac{d^2 \psi^{(1)}}{dx^2} = -\frac{\lambda_s}{Q} \frac{d\psi^{(1)}}{dx} + \frac{\Xi R}{Q} \left\{ \psi^{(1)} \right\}^2 \frac{d\psi^{(1)}}{dx} + \frac{P}{Q} \left\{ \psi^{(1)} \right\}^3 \frac{d\psi^{(1)}}{dx}. \quad (6.27)$$

Simplifying Eq. (6.24) along with the conditions $\psi^{(1)} \rightarrow 0$, $\frac{d\psi^{(1)}}{dx} \rightarrow 0$, $\frac{d^2 \psi^{(1)}}{dx^2} \rightarrow 0$ as $x = \pm\infty$, the following planar dynamical system (PDS) is formulated:

$$\begin{aligned} \frac{d\psi}{dx} &= y \\ \frac{dy}{dx} &= \frac{P^2}{4Q^2} \psi^7 + \frac{7P\Xi R}{Q^2} \psi^6 + \frac{\Xi^2 R^2}{3Q^2} \psi^5 - \frac{5\lambda_s P}{4Q^2} \psi^4 - \frac{4\lambda_s \Xi R}{3Q^2} \psi^3 + \frac{\lambda_s^2}{Q^2} \psi, \end{aligned} \quad (6.28)$$

where $\psi^{(1)} \sim \psi$ (for simplicity). Eq. (6.19) can also be converted to the following PDS by substituting $R = 0$ in Eq. (6.28):

$$\begin{aligned}\frac{d\psi}{dx} &= y \\ \frac{dy}{dx} &= \frac{P^2}{4Q^2}\psi^7 - \frac{5\lambda_s P}{4Q^2}\psi^4 + \frac{\lambda_s^2}{Q^2}\psi.\end{aligned}\quad (6.29)$$

It is obviously found from Eq. (6.28) that Eq. (6.24) is completely non-integrable because no one can determine the equilibrium points by solving analytically. Whereas, one can easily determine the real equilibrium points as $(0, 0)$, $\left((4\lambda_s/P)^{1/3}, 0\right)$ and $\left((4\lambda_s/P)^{1/3}, 0\right)$ and four complex equilibrium points from Eq. (6.29) by considering $\frac{d\psi}{dX} = 0$ and $\frac{dy}{dX} = 0$. Now, one can determine the Jacobian matrix for the system as in Eq. (6.29) as

$$J = \begin{pmatrix} 0 & 1 \\ \frac{P^2}{4Q^2}\psi^7 - \frac{5\lambda_s P}{4Q^2}\psi^4 + \frac{\lambda_s^2}{Q^2}\psi & 0 \end{pmatrix}. \quad (6.30)$$

Based the Eq. (6.30), it is found that (i) $(0, 0)$ is a saddle point as the eigenvalues at this point are $\lambda_1 = \frac{\lambda_s}{Q}$ and $\lambda_2 = -\frac{\lambda_s}{Q}$, which are two real eigenvalues having the opposite signs. (ii) $\left((4\lambda_s/P)^{1/3}, 0\right)$ is a saddle point as the eigenvalues at this point are $\lambda_1 = \frac{3\lambda_s}{Q}$ and $\lambda_2 = -\frac{3\lambda_s}{Q}$, which are two real eigenvalues having the opposite signs. (iii) $\left((4\lambda_s/P)^{1/3}, 0\right)$ is a center as eigenvalues at this point are $\lambda_1 = \frac{3\lambda_s}{2Q}i$ and $\lambda_2 = -\frac{3\lambda_s}{2Q}i$, which are two complex eigenvalues with real part equal to zero. Since the system Eq. (6.29) is integrable, one can study HIA wave phenomena by directing integrating Eq. (6.19). However, the system as in Eq. (6.28) can be studied only by using the numerical techniques. The dynamical analysis of HIA waves based on the non-integrable system is described in the next section.

6.6 DYNAMICAL ANALYSIS OF HIA WAVES

Nonlinear dynamical analysis plays a crucial role in offering a deeper and more precise insight into complex systems. It aids in forecasting and controlling system behaviors in diverse fields [153, 190–194].

6.6.1 PHASE PORTRAIT ANALYSIS

Bifurcation, an important concept in dynamical systems theory, refers to the abrupt changes in the behavior of a system as a result of variations in its parameters. Even minor adjustments in parameters can cause dramatic shifts in the system's dynamics. This phenomenon is studied using bifurcation theory and phase plane analysis, where the trajectories represent the solutions of traveling waves [190],[153]. Figures 3 and 4 display the phase plane analysis of Eq. (6.28) in strongly coupled plasmas (SCPs), while Figure 4 focuses on weakly coupled plasmas (WCPs). It is observed that the system exhibits three real equilibrium points. From Figures 3 and 4, it is evident that in SCPs at $\Gamma = 6$, when the nonextensivity parameter $q < q_{sc}$, the system supports wave propagation. However, for $q > q_{sc}$, wave propagation is not supported. At $\Gamma = 4.5$, for $q > q_{sc}$, the wave propagation occurs in the modulated region, whereas at $\Gamma = 8$, wave propagation becomes unmodulated at the strongly coupled regime (SCV). In contrast, Figure 5 illustrates that in WCPs, the system supports wave propagation when $q > q_{sc}$. This demonstrates that in SCPs, for higher values of Γ , the system as in Eq. (6.28) does not support wave propagation at the SCV or for nonextensivity values exceeding the SCV, while in WCPs, the system supports wave propagation for nonextensivity values above the SCV. The equilibrium points $\mathcal{E}_0$ and $\mathcal{E}_2$ function as saddle points, with $\mathcal{E}_1$ acting as a center. As a result, the trajectory connecting one saddle point to the other, enclosing a fixed point, forms a nonlinear heteroclinic orbit (NHeO). Furthermore, the nonlinear periodic orbit (NPO) is contained within the NHeO. Since each orbit in the phase space corresponds to a wave solution, the NPO and NHeO represent periodic and shock wave solutions, respectively. To better understand the dynamic behavior of the system, we employed the fourth-order Runge-Kutta method for precise numerical calculations. The phase portrait is crucial for identifying and evaluating the stability of equilibrium points within the system. Additionally, our analysis reveals the presence of periodic orbits in the phase plane, indicating periodic wave solutions for HIA waves. These waves are described by a Burgers-type equation with both cubic and quartic nonlinearities. Since this completely a non-integrable equation, the numerical techniques are only an arena to study the aforementioned physical issues in the considered plasma environment.

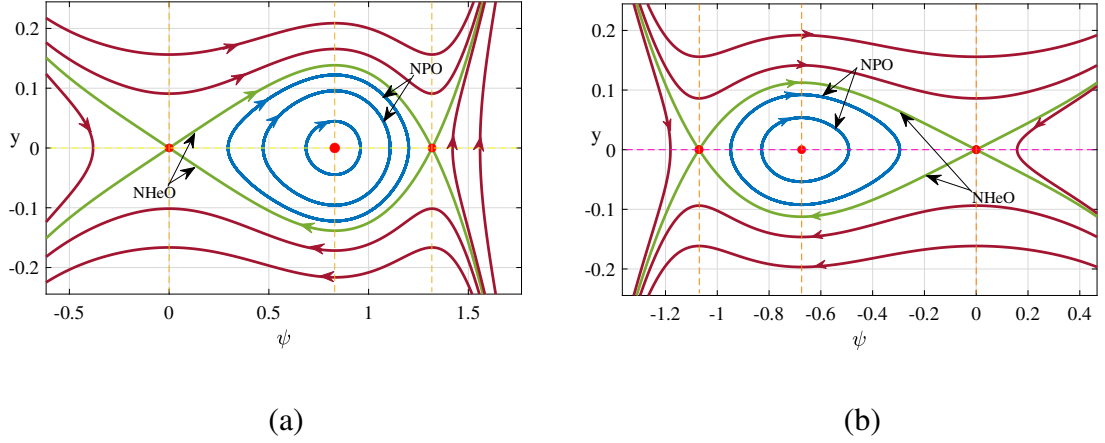


Fig. 6.3 Phase profile illustrations for (a) $\alpha = 0.01, \sigma = 0.1, q > q_{sc} = 0.65, \Gamma = 8, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0, \lambda_s = 0.1$, and $\eta = 0.9$ around $\mathcal{E}_0 = (0, 0)$, $\mathcal{E}_1 = (0.8303477374, 0)$ and $\mathcal{E}_2 = (1.318094871, 0)$; (b) for $\alpha = 0.01, \sigma = 0.1, q < q_{sc} = 0.6, \Gamma = 8, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0, \lambda_s = 0.1$, and $\eta = 0.9$ around all the real equilibrium points $\mathcal{E}_0 = (0, 0)$, $\mathcal{E}_1 = (-0.6743630150 + 4.944114701 \times 10^{-28}I, 0)$ and $\mathcal{E}_2 = (-1.070484559 + 2.125529050 \times 10^{-28}I, 0)$.

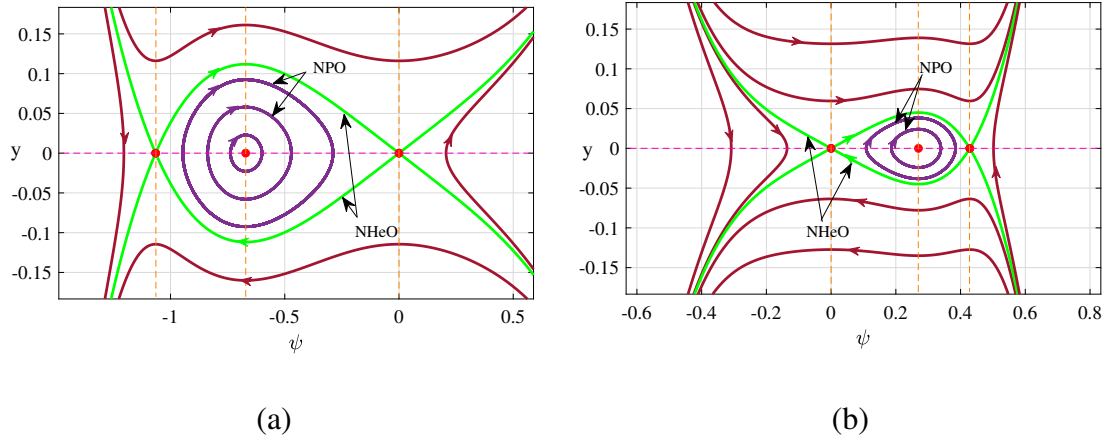


Fig. 6.4 Phase profile illustrations for (a) $\alpha = 0.01, \sigma = 0.1, q = 0.63, \Gamma = 8, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0, \lambda_s = 0.1$, and $\eta = 0.9$ around $\mathcal{E}_0 = (0, 0)$, $\mathcal{E}_1 = (-0.6710062662 - 1.940811233 \times 10^{-28}I, 0)$ and $\mathcal{E}_2 = (-1.065156053 - 1.248328103 \times 10^{-28}I, 0)$; (b) for $\alpha = 0.01, \sigma = 0.1, q < q_{sc} = 0.6, \Gamma = 6, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0, \lambda_s = 0.1$, and $\eta = 0.9$ around all the real equilibrium points $\mathcal{E}_0 = (0, 0)$, $\mathcal{E}_1 = (0.2699074289, 0)$, and $\mathcal{E}_2 = (0.4284513365, 0)$.

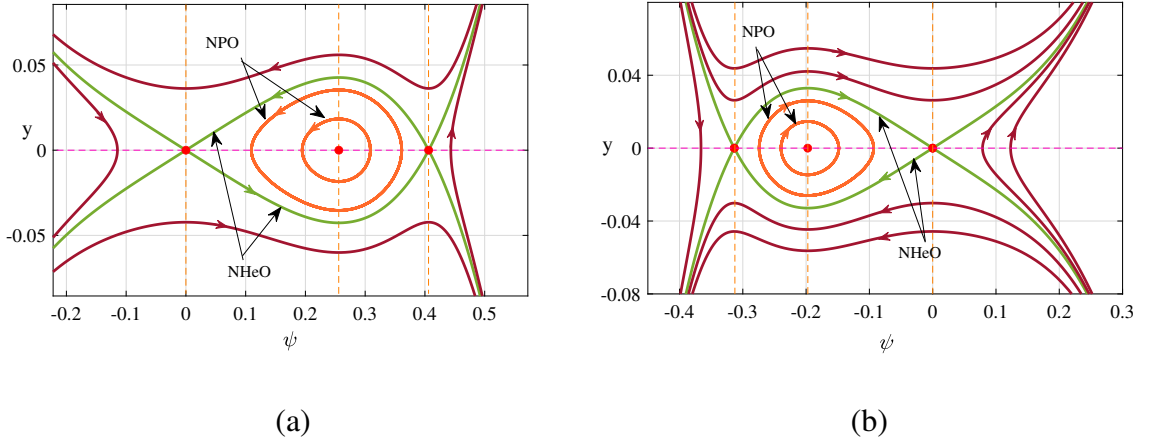
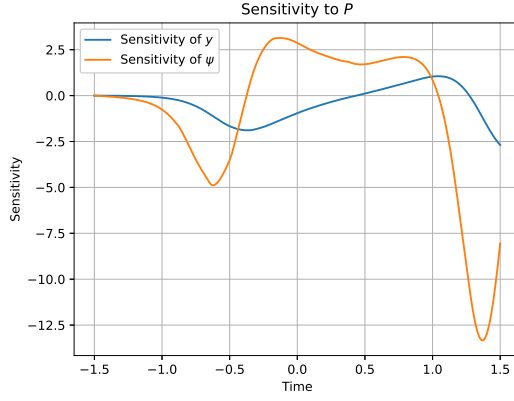


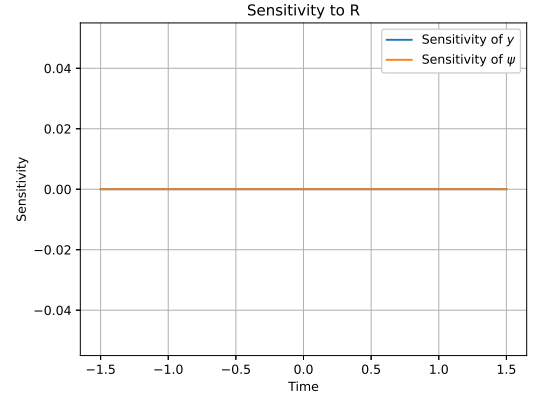
Fig. 6.5 Phase profile illustrations for (a) $\alpha = 0.01, \sigma = 0.1, q > q_{sc} = 0.65, \Gamma = 0.01, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0, \lambda_s = 0.1$, and $\eta = 0.9$ around $\mathcal{E}_0 = (0, 0)$, $\mathcal{E}_1 = (0.2559120448, 0)$ and $\mathcal{E}_2 = (0.4062350491, 0)$; (b) for $\alpha = 0.01, \sigma = 0.1, q < q_{sc} = 0.6, \Gamma = 0.01, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0, \lambda_s = -0.1$, and $\eta = 0.9$ around all the real equilibrium points $\mathcal{E}_0 = (0, 0)$, $\mathcal{E}_1 = (-0.1974620773 + 5.728000689 \times 10^{-31}I, 0)$ and $\mathcal{E}_2 = (-0.3134515092 - 5.600830394 \times 10^{-31}I, 0)$.

6.6.2 SENSITIVITY ANALYSIS

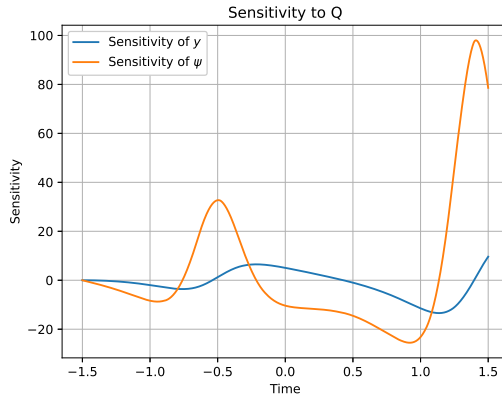
Sensitivity analysis is crucial in understanding the dynamical behavior of a system. In this context, consider the response function $F(y, \psi)$ described by Eq. (6.28), where changes in model parameters such as P , R , Q , Ξ , and $\lambda = \lambda_s$ (for simplicity) are analyzed. To study the effect of small perturbations, we perform a first-order Taylor series expansion for the perturbation ΔP . For numerical integration, we employ the Runge-Kutta-Fehlberg method (RK45), which provides an accurate solution to the ordinary differential equation (ODE) Eq. (6.28) over a specified time interval. Following this, the finite difference method is used for sensitivity analysis, quantifying the response to small perturbations of each individual parameter— P , R , Q , Ξ , and λ —based on the specific model parameter values: $\alpha = 0$, $\sigma = 0.1$, $q = 0.8$, $\Gamma = 80$, $r_2 = 0.3$, $r_1 = 0.55$, $\Xi = 1$, $R = 0.178$, and $\eta = 0.9$. Figures 6 illustrate the sensitivity of the system to small perturbations in ΔP , ΔR , ΔQ , and $\Delta \lambda$. From these figures, it is evident that, except for the parameter R , the other parameters— P , Q , and λ —are highly sensitive to small changes. The finite difference calculation of sensitivity begins with the initial values of $P = 0.4065$, $Q = 0.45$, and $\lambda = 1.1534$. This sensitivity analysis provides insight into how the system's behavior can be influenced by even minor variations in key parameters.



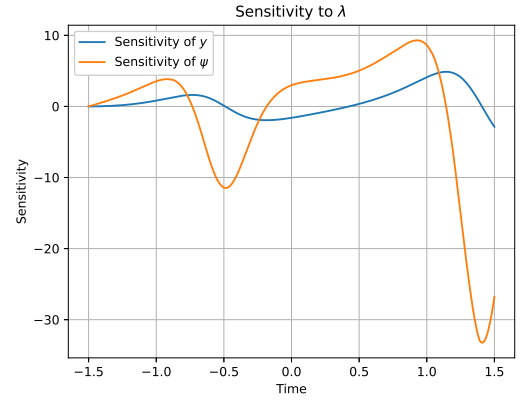
(a)



(b)



(c)



(d)

Fig. 6.6 Sensitivity analysis for (a) P with $\alpha = 0, \sigma = 0.1, q = 0.8, \Gamma = 80, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0.178$, and $\eta = 0.9$; (b) R with $\alpha = 0, \sigma = 0.1, q = 0.8, \Gamma = 80, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0.178$, and $\eta = 0.9$; (c) Q with $\alpha = 0, \sigma = 0.1, q = 0.8, \Gamma = 80, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0.178$, and $\eta = 0.9$; (d) with $\alpha = 0, \sigma = 0.1, q = 0.8, \Gamma = 80, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0.178$, and $\eta = 0.9$.

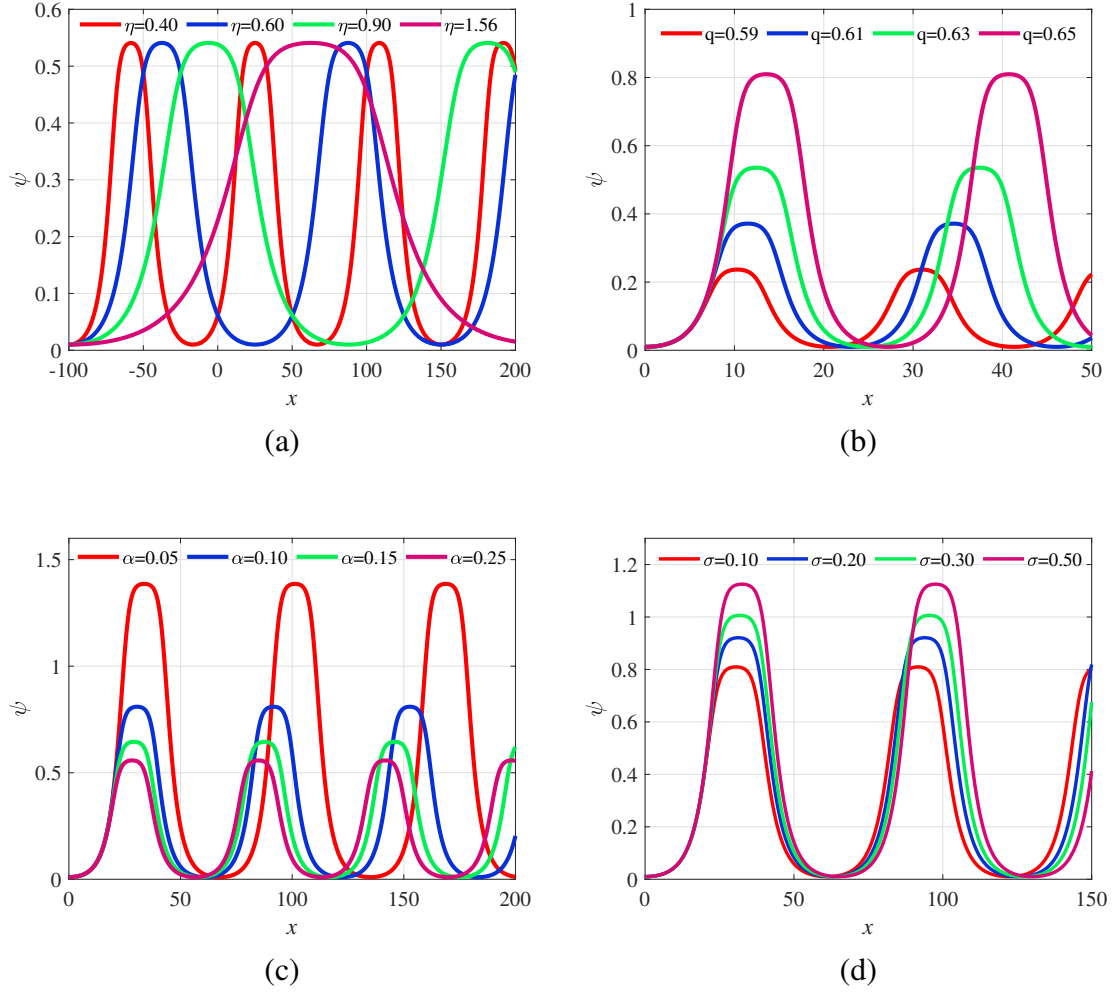


Fig. 6.7 Periodic wave analysis in SC plasmas for (a) η with $\alpha = 0.1, \sigma = 0.1, q > q_{sc} = 0.65, \Gamma = 6, r_2 = 0.3, r_1 = 0.55, \Xi = 1$; and $\lambda_s = 0.1$ (b) q with $\alpha = 0.1, \sigma = 0.1, \Gamma = 6, r_2 = 0.3, r_1 = 0.55, \Xi = 1, \lambda_s = 0.1$, and $\eta = 0.9$; (c) α with $r_2 = 0.3, \sigma = 0.1, q > q_{sc} = 0.65, \Gamma = 6, r_1 = 0.55, \Xi = 1, \lambda_s = 0.1$, and $\eta = 0.9$; (d) σ with $\alpha = 0.1, q > q_{sc} = 0.65, \Gamma = 6, r_2 = 0.3, r_1 = 0.55, \Xi = 1, \lambda_s = 0.1$, and $\eta = 0.9$.

6.7 PERIODIC WAVES GOVERNED BY THE NON-INTEGRABLE NLEE

Periodic waves, characterized by their repeating structures, are fundamental to a wide range of physical systems governed by nonlinear dynamics. Studying these waves is crucial for understanding the full range of wave behaviors, analyzing system stability, exploring nonlinear interactions, and applying theoretical principles to the real-world contexts. It also

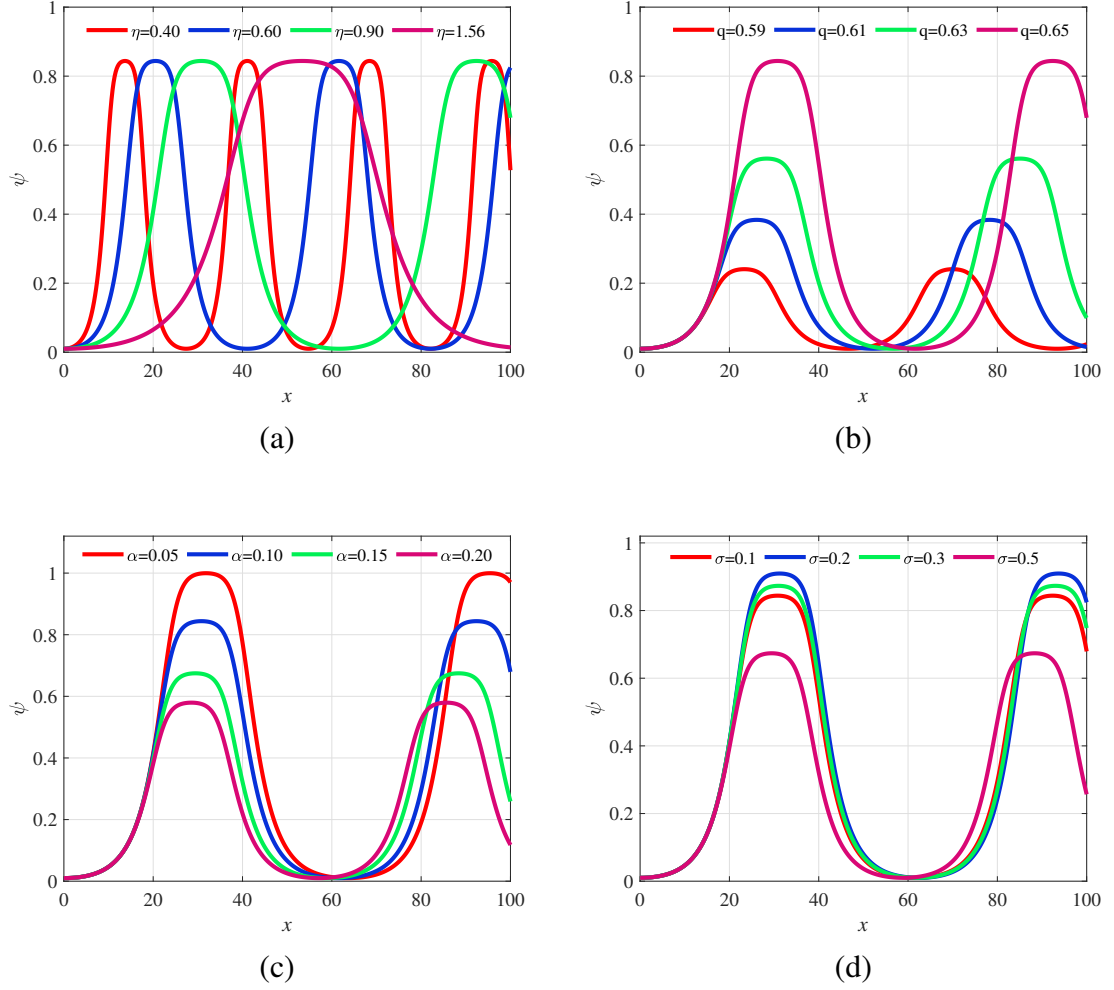


Fig. 6.8 Periodic wave analysis in WC plasmas for (a) η with $\alpha = 0.1, \sigma = 0.1, q > q_{sc} = 0.65, \Gamma = 6, r_2 = 0.3, r_1 = 0.55, \Xi = 1$; and $\lambda_s = 0.1$ (b) q with $\alpha = 0.1, \sigma = 0.1, \Gamma = 6, r_2 = 0.3, r_1 = 0.55, \Xi = 1, \lambda_s = 0.1$, and $\eta = 0.9$; (c) α with $r_2 = 0.3, \sigma = 0.1, q > q_{sc} = 0.65, \Gamma = 6, r_1 = 0.55, \Xi = 1, \lambda_s = 0.1$, and $\eta = 0.9$; (d) σ with $\alpha = 0.1, q > q_{sc} = 0.65, \Gamma = 6, r_2 = 0.3, r_1 = 0.55, \Xi = 1, \lambda_s = 0.1$, and $\eta = 0.9$.

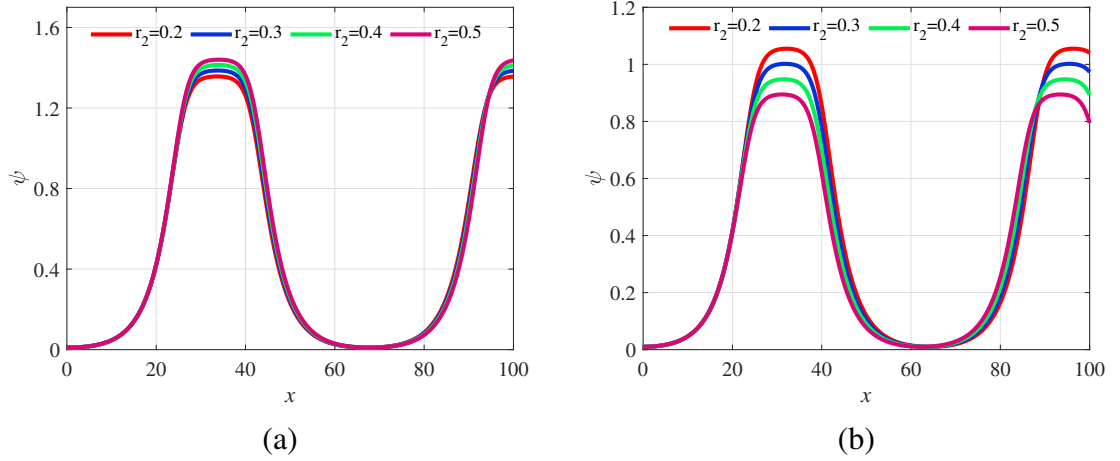


Fig. 6.9 Periodic wave analysis in SC plasmas for (a) r_2 with $\alpha = 0.05$, $\sigma = 0.1$, $q > q_{sc} = 0.65$, $\Gamma = 6$, $r_1 = 0.55$, $\Xi = 1$, $\lambda_s = 0.1$, and $\eta = 0.9$. In WC plasmas for (b) r_2 with $\alpha = 0.05$, $\sigma = 0.1$, $\Gamma = 0.01$, $r_1 = 0.55$, $\Xi = 1$, $\lambda_s = 0.1$, and $\eta = 0.9$.

helps in uncovering the mathematical properties of the governing equations. Figures 7-9 illustrate the behavior of periodic waves governed by the non-integrable equation as in Eq. (6.24) in both WCP and SCP, with varying parameters near SCV. In Figures 7(a) and 8(a), it is observed that the width of the shock waves increases in both WCP and SCP as the parameter η grows. As the nonextensivity parameter q increases, energy becomes more concentrated in smaller regions of the wave, resulting in steeper local gradients, as shown in Figures 7(b) and 8(b). Figures 7(c) and 8(c) reveal that in WCP, the wave amplitude decreases progressively as α increases, and a similar trend is observed in SCP. However, in SCP, the amplitude decreases more sharply with higher values of α . Figures 7(d) and 8(d) demonstrate that as the electron-to-ion temperature ratio σ increases, the amplitude of periodic waves grows in SCP, while it exhibits more complex variations in WCP. Figures 9(a) and 9(b) examine the effect of r_2 on SCP and WCP. In SCP, the amplitude increases gradually as r_1 increases, while in WCP, the amplitude decreases with increasing r_2 .

6.8 SHOCK WAVE EXCITATIONS GOVERNED BY BOTH INTEGRABLE AND NON-INTEGRABLE NLEES

The influence of plasma parameters on electrostatic HIA shocks governed by both integrable and non-integrable NLEEs is explored for both of SCP and WCP environments.

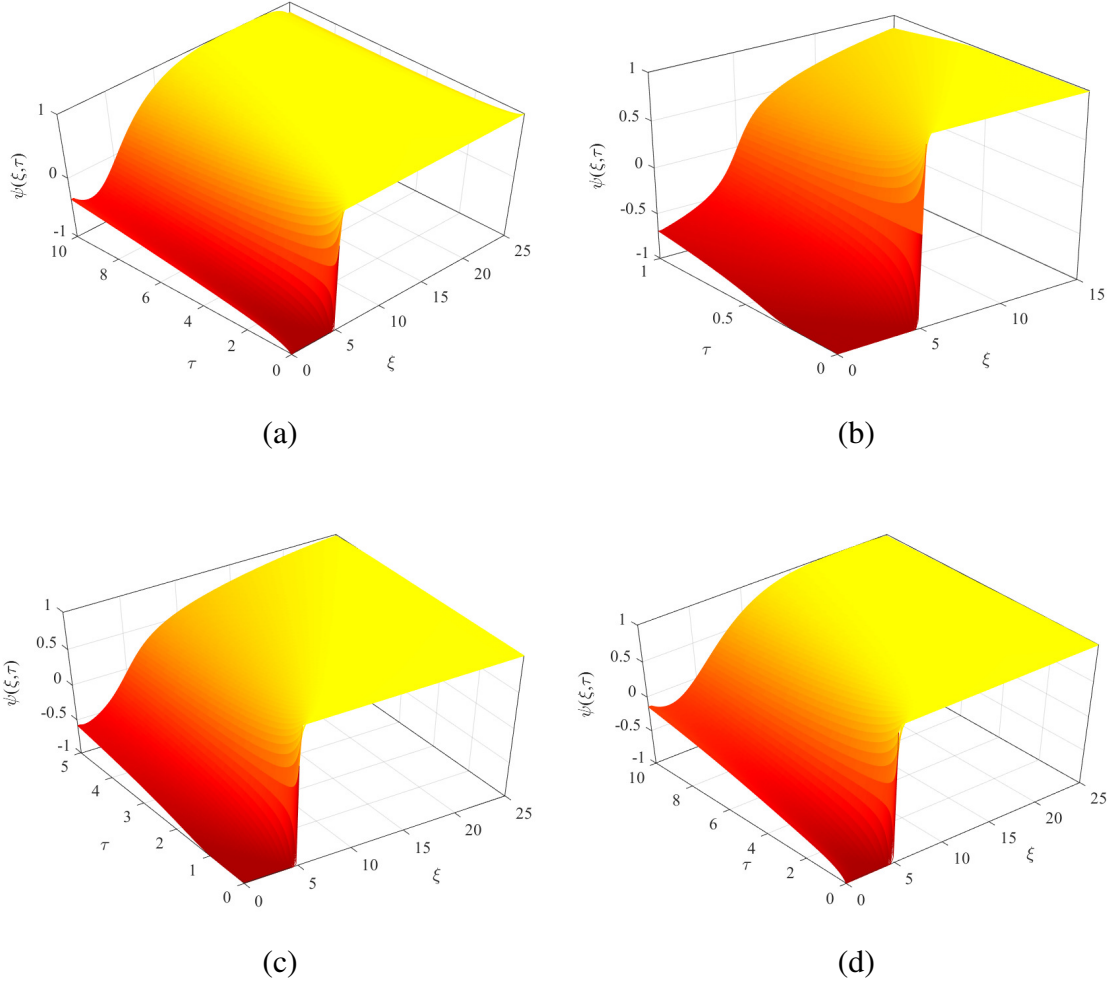


Fig. 6.10 Shocks in SCP governed by non-integrable NLEE for (a) $\alpha = 0.05, \sigma = 0.1, q > q_{sc} = 0.65, \Gamma = 6, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0.322$, and $\eta = 0.9$; (b) for $\alpha = 0.05, \sigma = 0.1, q < q_{sc} = 0.6, \Gamma = 6, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0.607$, and $\eta = 0.9$; (c) $\alpha = 0.25, \sigma = 0.1, q > q_{sc} = 0.65, \Gamma = 6, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0.525$, and $\eta = 0.9$; (d) $\alpha = 0.05, \sigma = 0.5, q > q_{sc} = 0.65, \Gamma = 6, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0.276$, and $\eta = 1.56$.

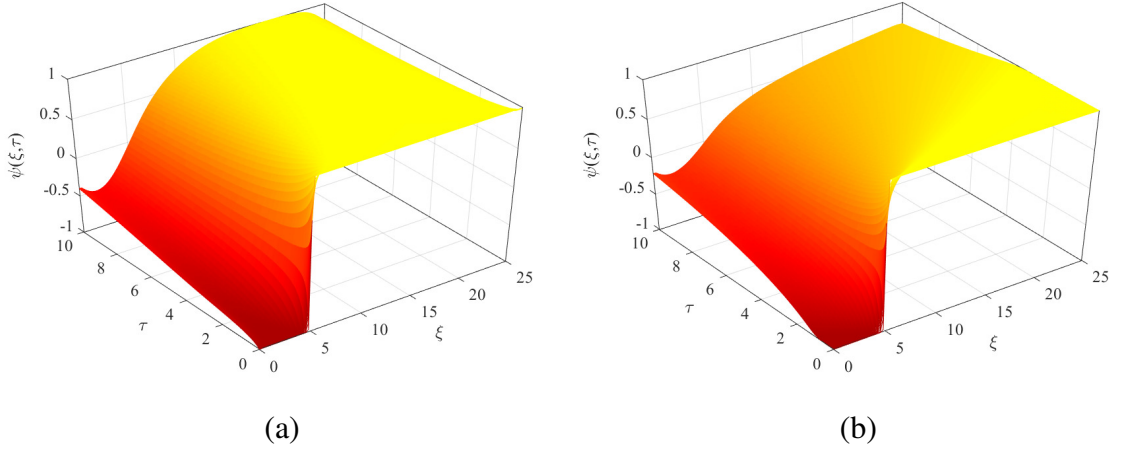


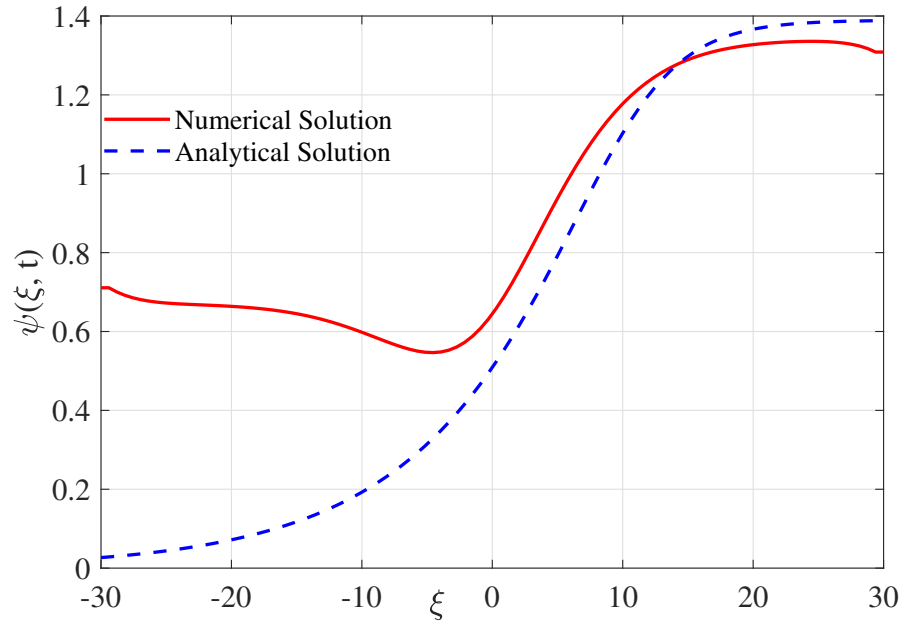
Fig. 6.11 Shocks in WCP governed by non-integrable NLEE for (a) $\alpha = 0.01, \sigma = 0.1, q > q_{sc} = 0.65, \Gamma = 0.01, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0.323$, and $\eta = 0.9$; (b) for $\alpha = 0.01, \sigma = 0.1, q < q_{sc} = 0.62, \Gamma = 0.01, r_2 = 0.3, r_1 = 0.55, \Xi = 1, R = 0.430$, and $\eta = 0.9$.

Using the analytical solution of the integrable equation provided in Eq. (6.19), the nonlinear propagation of electrostatic shocks has been analyzed by varying plasma parameters, as shown in Figure 2, along with their corresponding physical interpretations. Since Eq. (6.24) is non-integrable, finding an analytical solution to this equation is not feasible. Therefore, a numerical approach is employed to investigate the effect of plasma parameters around or at SCVs on HIA shocks, considering both SCP and WCP. Figure 10 demonstrates the impact of physical parameters on the nonlinear propagation of electrostatic shocks, as described numerically by Eq. (6.24), near the SCVs in SCPs. The figure shows that the amplitude of electrostatic shocks increases slightly when the nonextensivity index is close to its SCV, but the increase becomes more significant when the nonextensivity index is larger than the SCV. Additionally, the amplitude rises as the nonthermal electron population increases and as the electron-to-ion temperature ratio grows. Figure 11, on the other hand, illustrates the influence of physical parameters on the nonlinear propagation of electrostatic shocks near the SCVs in WCPs. Similar to the results in SCPs, it is observed that the amplitude of electrostatic shocks increases slightly as the nonextensivity index approaches the SCV, with a more pronounced increase when the nonextensivity index is larger than the SCV. It is important to note that the results from this study are valuable for advancing our understanding of shock wave evolution (SWEs) in plasmas, particularly those with strongly

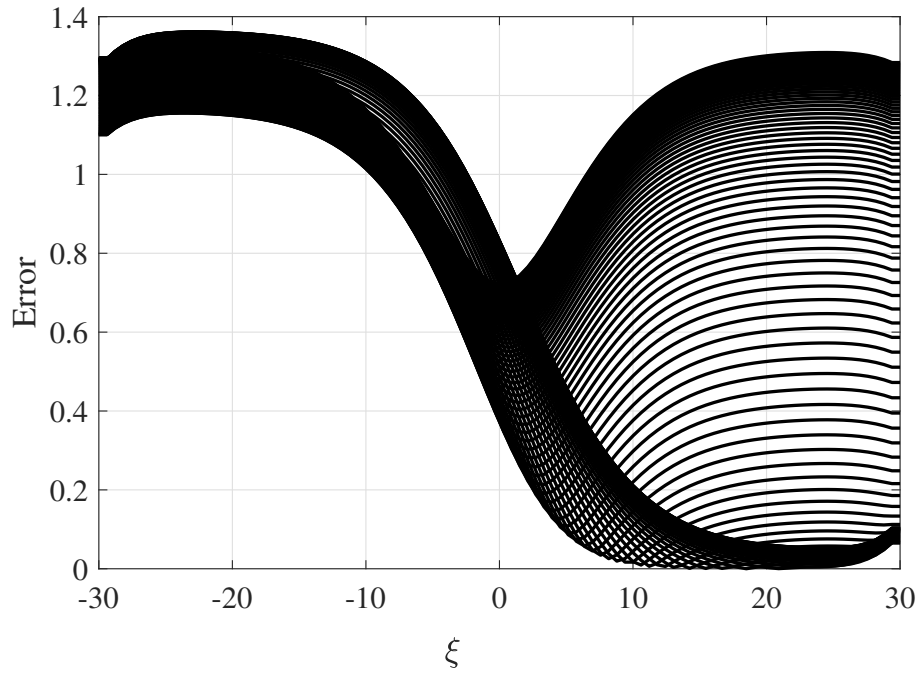
coupled pair-ions, and non-extensivity electrons.

6.9 COMPARISON AND NUMERICAL VALIDATION

In the previous investigation [182], the authors derived the BE with quadratic nonlinearity using the RPT to study the propagation of HIA shock waves in unmagnetized SCPs having inertial positively HIs fluid and inertia-less (α, q) -distributed electrons and Maxwellian light ions. When the BE with quadratic nonlinearity failed to adequately describe such wave phenomena, they extended the analysis by deriving the BE with cubic nonlinearity, incorporating higher-order corrections from the RPT. This approach was used to study the propagation of HIA shock waves around CVs in unmagnetized SCPs. The authors noted that HIA shock waves are not supported at the CVs, and that the density ratio r_1 must exceed 0.4459870178, i.e., $r_1 > 0.4459870178$, for the wave phenomena to occur. To overcome this limitation, we derived an integrable BE with quartic nonlinearity, considering the existence of SCVs by applying higher-order corrections to RPT. To derive this equation, we determined the solvability condition, which required both nonlinear coefficients of the BE with quadratic nonlinearity and cubic nonlinearity to be zero. Figure 1 clearly shows the SCV for the density ratio r_1 , where both nonlinear coefficients approach zero. It is important to note that the SCV for the density ratio r_1 is supported only under the influence of non-extensive electrons, specifically when $\alpha = 0$ and $q < 1$. This means that SCVs occur with the presence of superthermal electrons, where $q < 1$ and $\alpha = 0$. Based on stationary analytical shock and anti-shock wave solutions, the effect of plasma parameters on the electrostatic potential is displayed in Figure 2. The results presented in Figure 2 are relevant for understanding the propagation characteristics of shocks near CVs and SCVs, specifically when the BE with quadratic and cubic nonlinearity are invalid. Additionally, we derived a non-integrable BE with both cubic and quartic nonlinearity to study the impact of all plasma parameters on the propagation of HIA waves, not only around SCVs but also at the SCVs themselves. Since the BE with both cubic and quartic nonlinearity is non-integrable, we implemented numerical technique to study these physical phenomena. To determine the numerical solutions, we used the spatial derivative with the Lax-Friedrichs method, employing the Runge-Kutta 4th order (RK4) method to compute intermediate steps. To validate our numerical results, Figure 12(a) compares the electrostatic potential



(a)



(b)

Fig. 6.12 (a) Comparison of analytical and numerical shock wave solutions and, (b) error analysis of shocks with $\alpha = 0, \sigma = 0.1, q = 0.8 > q_s c = 0.65, \Gamma = 8, r_2 = 0.3, r_1 = 0.55, \Sigma = 1, R = 0, \text{ and } \eta = 0.5$

obtained from the analytical solution and the numerical solution of the BE with quartic nonlinearity. This figure initially shows a difference between the analytical and numerical solutions, which gradually decreases, indicating convergence. Furthermore, Figure 12(b) presents the error spectrum by randomly choosing different initial conditions for the numerical solution and comparing them with the analytical solution. Each consecutive time step shows the variation in error relative to the analytical solution. To minimize the errors for better accuracy, one may use any other numerical techniques, but beyond the scope of this study.

It is important to note that the results from this study are valuable for advancing our understanding of shock wave and periodic wave excitations in plasmas, particularly those with strongly coupled pair-ions, nonextensivity, and nonthermal electrons. These findings can be applied to various space environments, such as stellar polytropes, hadronic matter, quark-gluon plasma, and black halos, among others (see references [149, 186–188]). In conclusion, experimental verification, like in ref. [163] of the theoretical predictions presented in this article may be conducted in plasma laboratories, particularly when the BE involving either quadratic or cubic nonlinearity failed to address nonlinear electrostatic HIA wave propagation in the considered plasma environment.

6.10 CONCLUSION

In conclusion, this study provides a comprehensive investigation into the nonlinear dynamics of HIA shocks and periodic waves in both SCPs and WCPs, particularly focusing on the influence of plasma parameters around SCVs. The analysis reveals that various plasma parameters, including viscosity, nonextensivity, and the electron-to-ion temperature ratio, significantly affect the structure and behavior of electrostatic shocks and anti-shocks. For instance, increased viscosity depends the wave gradients, altering the shock width without substantially affecting its amplitude. Additionally, nonextensive effects and the density ratio between electrons and ions play crucial roles in modifying shock amplitudes, with increasing nonextensivity resulting in weaker compressive shocks and anti-shocks. Through bifurcation theory and phase plane analysis, the study further examines the dynamical behavior of these waves, demonstrating how small variations in plasma parameters lead to dramatic shifts in the system's behavior. The presence of periodic

orbits and shock wave solutions provides insight into the wave propagation characteristics, with the results being particularly relevant to understanding plasma behaviors in various environments, including stellar polytropes, hadronic matter, quark-gluon plasma, and black halos. Furthermore, the research highlights the importance of numerical methods, given the non-integrable nature of the governing equations, for investigating the dynamic behavior of electrostatic shocks in both SCP and WCP environments. The findings from this work contribute to a deeper understanding of shock waves and provide valuable theoretical insights for future experimental studies. These results could be experimentally verified in plasma laboratories, particularly in systems where adiabaticity, nonextensivity, and nonthermality are present, thus advancing our understanding of complex plasma phenomena.

Chapter 7: CONCLUDING REMARKS AND FUTURE ASPECTS

This chapter provides a summary of the findings from the preceding chapters, focusing on fully ionized, collisionless, unmagnetized, strongly and weakly coupled plasma systems. The study examined various plasma assumptions and explored the nonlinear dynamics of acoustic wave phenomena through both integrable and non-integrable nonlinear evolution equations.

Chapter 1 introduced the fundamental concepts relevant to multi-component, unmagnetized, strongly and weakly coupled plasmas, as found in astrophysical, space, and laboratory settings. It covered topics such as nonlinearity, wave phenomena, and outlined the objectives of the study.

Chapter 2 focused on the nonlinear propagation of dust-acoustic shock waves (DA SWs), oscillatory waves, and double layers in an unmagnetized coupled dusty plasma. Nonlinear evolution equations (NLEEs), incorporating various types of nonlinearity, were derived. By applying the GREMM, exact solutions for shock waves and oscillatory waves were obtained. The investigation examined how different parameters affect the nonlinear behavior of DA SWs, oscillatory waves, and double layers. The results demonstrated that GREMM is a highly effective mathematical tool for solving NLEEs, as it accurately determines wave solutions in terms of amplitude and width parameters.

Chapter 3 explored the nonlinear propagation of HISWs in an unmagnetized, strongly-coupled plasma. The study employed the Reduced Perturbation Technique (RPT) to simplify the governing fluid equations into nonlinear forms: BE, mBE, and mmBE, which were used to analyze shock wave excitations (SWEs) in the plasma. This chapter derived shock wave solutions for mBE and mmBE by directly integrating these equations and investigated how various plasma parameters affect these solutions. It was found that BE can support both positive and negative electrostatic SWEs, while mBE supports only positive SWEs around critical values. However, BE and mBE are not sufficient for examining key characteristics of SWEs, such as polarity, amplitude, and thickness, at critical values. Conversely, mmBE is suitable for studying SWEs at or near critical values

and for exploring double layers (DLs). For the first time, this study demonstrated that both SWEs and DLs can be supported at values close to critical values, offering new insights through the shock wave solutions.

Chapter 4 investigated the propagation of electrostatic dust-acoustic (DA) shock waves in an unmagnetized, strongly coupled dusty plasma consisting of negatively charged dust grains, Maxwellian ions, and electrons with an (α, q) -distribution, where ions and electrons are considered weakly coupled. The reductive perturbation technique was used to derive the Burgers equation with quadratic, cubic, and quartic nonlinearities from the theoretical model equations. This study found that DA waves described by the Burgers equation with quadratic nonlinearity can support both positive and negative electrostatic potentials in the presence of superthermal and nonthermal electrons. However, for some critical values of specific plasma parameters, the amplitude of the shocks approaches infinity, making the quadratic nonlinearity model inadequate for describing electrostatic shocks in these plasmas. In such cases, the Burgers equation with cubic nonlinearity is more suitable, as it supports compressive electrostatic DA shocks around critical values within certain limitations. This cubic model also supports positive electrostatic potentials around critical values for both superthermal and nonthermal electrons. To address the propagation of DA waves not only around critical values but also super-critical values (SCVs), the Burgers equation with quartic nonlinearity was derived. The solution to this equation reveals that DA acoustic shock waves with positive polarity can be observed both at critical values and around SCVs. These results provide valuable insights into the characteristics of localized electrostatic DA shock wave propagation in cosmic environments, compact astrophysical objects, and laboratory dusty plasmas.

Chapter 5 explored the characteristics of HIA nonlinear solitons and dressed soliton structures in both weakly and strongly coupled plasmas. By employing RPT, the study derived the Korteweg-de Vries (KdV) equation and inhomogeneous KdV-like equations from the plasma system. The research examined the nature of both HIA solitons and dressed solitons, focusing on the effects of higher-order contributions and various plasma parameters. These findings provide valuable insights into various nonlinear coherent structures observed in astrophysical and space environments, helping to better understand their formation and behavior.

In Chapter 6, we revisited the model equations from Chapter 3 and explored their implications. Using RPT under specific conditions, a new nonlinear evolution equation (NLEE) was derived. This equation proved to be non-integrable under certain conditions, presenting challenges for obtaining analytical solutions. As a result, numerical methods—specifically the Runge-Kutta-Fehlberg method (RK45) and the finite difference method—were employed to analyze the dynamic behavior of HIA waves in the plasma environments under study. The numerical simulations produced dynamic results, which were illustrated through phase portraits and detailed descriptions. These results indicated that the plasma system supports periodic HIA waves that resemble super-critical values across various plasma parameters. The behavior of these waves was characterized by a Burgers equation with cubic and quartic nonlinearities, as revealed by the numerical techniques. Additionally, sensitivity analyses were performed on the coefficients of the NLEE, highlighting the robustness of the numerical approach in exploring acoustic wave phenomena in the studied plasma environments. The findings provide valuable insights into the dynamics of acoustic waves in space and astrophysical environments or laboratory plasmas.

Future research directions should focus on refining numerical techniques, such as exploring higher-order implicit schemes or adaptive mesh refinement algorithms, to improve accuracy and efficiency in simulating acoustic wave phenomena in diverse plasma conditions. Expanding to multi-dimensional simulations and employing methods like finite element or spectral techniques will help capture complex wave interactions and spatial variations in plasmas. Comprehensive sensitivity analyses on the parameters of nonlinear evolution equations will enhance the understanding of how plasma parameters affect wave behavior and stability, which is vital for plasma diagnostics and experimental validation. Additionally, applying these findings to astrophysical environments, such as the solar corona and planetary magnetospheres, will provide insights into wave propagation and energy transport in extreme plasma conditions. Further investigation into nonlinear wave dynamics, including shock formation and solitons, will deepen the understanding of complex wave interactions and instabilities in strongly and weakly coupled plasmas, advancing both fundamental plasma science and practical technologies.

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