

SIMULATION OF HEAT TRANSFER ENHANCEMENT IN A PIPE USING RECTANGULAR CUT TWISTED TAPE INSERT: A FEM APPROACH



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Dedicated To My Family Members

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This is to certify that Onamika Ibnath Anika has carried out this research work under my supervision and that she has fulfilled the relevant Academic Ordinance of the Chittagong University of Engineering & Technology so that she is qualified to submit the following thesis in the application for the degree of MASTER OF PHILOSOPHY IN MATHEMATICS. Furthermore, the Thesis complies with the PLAGIARISM and ACADEMIC INTEGRITY regulations of CUET.

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Abstract

A simulation analysis has been conducted to investigate heat transfer enhancement using rectangular-cut twisted tape inserts in a U-shaped pipe. Numerical simulations have also been carried out for a plain tube and a tube with smooth twisted tape (TT) inserts, and the results have been compared across three configurations: rectangular-cut TT inserts, smooth TT inserts, and the plain tube. The analysis has utilized a U-bend pipe with a length of 1935.6 mm and an inner diameter of 26.6 mm. For non-isothermal turbulent flow, water has been selected as the working fluid, and the Reynolds number (Re) range has been considered from 5319.4 to 17288.05. The rectangular-cut twisted tape inserts have demonstrated superior heat transfer performance compared to the other two configurations due to enhanced fluid mixing in the turbulent flow regime. In this study, the dimensionless Nusselt number (Nu) has consistently been higher for the rectangular-cut twisted tape configuration and has shown a gradual increase with rising Reynolds numbers. The results have also indicated improved thermal performance (η), with a thermal performance factor of 1.04 and favorable friction factor values for the pipe fitted with rectangular cut twisted tape inserts.

বিমূর্ত

একটি U-আকৃতির পাইপে আয়তক্ষেত্রাকার-কাট টুইস্টেড টেপ সন্নিবেশ ব্যবহার করে তাপ স্থানান্তর বর্ধিত করণ পর্যবেক্ষণের জন্য একটি সিমুলেশন বিশ্লেষণ পরিচালিত হয়েছে। একটি প্লেইন টিউব এবং মসৃণ টুইস্টেড টেপ (TT) সন্নিবেশ সহ একটি টিউবের জন্য নিউমারিকেল মূল্যায়ন করা হয়েছে, এবং ফলাফলগুলি তিনটি ভিন্ন সন্নিবেশ জুড়ে তুলনা করা হয়েছে; আয়তক্ষেত্রাকার-কাট TT সন্নিবেশ, মসৃণ TT সন্নিবেশ এবং প্লেইন টিউব। বিশ্লেষণে 1935.6 মিমি দৈর্ঘ্য এবং 26.6 মিমি অভ্যন্তরীণ ব্যাস সহ একটি ইউ-বেন্ড পাইপ ব্যবহার করা হয়েছে। নন-আইসোর্থামাল অশান্ত প্রবাহের জন্য, জলকে কার্যকরী তরল হিসাবে নির্বাচিত করা হয়েছে, এবং রেনল্ডস নম্বর (Re) এর পরিসর 5319.4 থেকে 17288.05 পর্যন্ত বিবেচনা করা হয়েছে। সিমুলেশন হতে দেখা যায় আয়তক্ষেত্রাকার-কাটা TT সন্নিবেশগুলি অশান্ত প্রবাহ ব্যবস্থায় বর্ধিত তরল মিশ্রণের কারণে অন্য দুটি কনফিগারেশনের তুলনায় উচ্চতর তাপ স্থানান্তর কর্মক্ষমতা প্রদর্শন করেছে। এই গবেষণায়, আয়তক্ষেত্রাকার-কাট টুইস্টেড টেপ কনফিগারেশনের জন্য মাত্রাবিহীন নুসলেট নম্বর (Nu) ধারাবাহিকভাবে উচ্চতর হয়েছে এবং ক্রমবর্ধমান Re এর সংখ্যার সাথে ধীরে ধীরে বৃদ্ধি পেয়েছে। ফলাফলগুলি 1.04 তাপ কর্মক্ষমতা ফ্যাক্টর এবং আয়তক্ষেত্রাকার কাটা TT সন্নিবেশের সাথে লাগানো পাইপের জন্য অনুকূল ঘর্ষণ ফ্যাক্টর মান সহ উপযোগী তাপ কর্মক্ষমতা (η) নির্দেশ করেছে।

Table of Contents

Abstract	vii
Table of Contents	ix
List of Figures	xi
List of Tables	xii
Nomenclature	xiii
 Chapter 1:Introduction.....	 (1-5)
1.1 General Overview	1
1.2 Objectives With Specific Aims and Possible Outcomes	3
1.3 Thesis Outline	4
 Chapter 2: Literature Review	 (6-35)
2.1 Introduction	6
2.2 Types of Heat Transfer Method.....	7
2.2.1 Conduction.....	7
2.2.2 Convection	9
2.2.3 Radiation	9
2.2.4 Thermal conductivity.....	9
2.2.5 Heat flux.....	10
2.2.6 Heat transfer rate (Q)	10
2.2.7 Heat Transfer Co-efficient	11
2.2.8 Wall Temperature.....	11
2.2.9 Bulk Temperature.....	11
2.2.10 Specific heat.....	12
2.3 Heat Exchanger.....	12
2.3.1 Classification of Industrial Heat Exchanger	13
2.4 Fluid Properties.....	15
2.5. Shear stress and strain.....	16
2.6. Laminar Flow.....	16
2.7 Turbulent Flow.....	17
2.8 Reynolds Number	18
2.9 Velocity Boundary Layer	19
2.1.0 Newtonian Fluid.....	20
2.11. Non-Newtonian Fluid.....	20
2.12. Incompressible and Compressible Fluid.....	21
2.13. Nusselt Number.....	21
2.14. Pressure Drop.....	22

2.15 Friction Factor.....	23
2.16 Computational Fluid Dynamics.....	23
2.17. Finite Difference Method (FDM).....	26
2.18 Finite Volume Method (FVM).....	26
2.19. Finite Element Method (FEM).....	27
2.19.1 Applications of the Finite Element Method.....	28
2.20 Previous Studies.....	29
Chapter 3:Mathematical Modeling and Computational Domain... ..	(36-50)
3.1 Introduction.....	36
3.1.1 Flow Modeling.....	36
3.1.2 System and Control Volume.....	36
3.2 Substantial, Local, and Convective Derivatives.....	37
3.3 Relevance of Mass Conservation	38
3.3.1 Equation of Continuity.....	39
3.4 Motion Equation.....	41
3.5 Energy Equation.....	42
3.6 Mathematical Model for Simulation.....	44
3.7 Boundary Conditions.....	46
3.8 Computational Domain	47
3.9 Mesh Design.....	49
Chapter 4: Results and Discussion	(51-61)
4.1 Introduction.....	51
4.2 Numerical Results	51
4.3 Temperature Distribution.....	52
4.4 Nusselt Number.....	57
4.5 Nusselt Number Enhancement Index.....	58
4.6 Friction Factor.....	59
4.7 Friction Factor Enhancement Index.....	60
4.8 Thermal Performance Factor (TPF).....	61
Chapter 5: Conclusion.....	(62-64)
Suggestions for Further Study.....	64
Bibliography.....	65

List of Figures

Fig. No.	Figure Caption	Page No.
Fig. 2.1	Conduction, convection, radiation	7
Fig. 2.2	Conduction heat transfer	8
Fig. 2.3	Thermal conductivity of some materials	10
Fig. 2.4	Types of heat exchanger	13
Fig. 2.5	Heat exchanger's classification	14
Fig. 2.6	Laminar flow of fluid	17
Fig. 2.7	Turbulent fluid flow within a pipe	17
Fig. 2.8	Velocity and thermal boundary layer	20
Fig. 2.9	Flowchart of CFD method	25
Fig. 3.1	A differential fluid elements in cartesian co-ordinate system.....	37
Fig. 3.2	Control volume element in three dimensions.....	39
Fig. 3.3	(a) Smooth pipe,(b) pipe installed with smooth twisted tape inserts, (c) pipe installed with twisted tape inserts featuring rectangular cuts, (d) smooth twisted tape, (e) rectangular-cut twisted tape..	47
Fig. 3.4	Finer mesh configuration of (a) smooth twisted tape, (b) rectangular -cut twisted tape, (c) U-shaped pipe.	49
Fig. 4.1	Variation of temperature for various Reynolds numbers	53
Fig. 4.2	Cross-sectional view of (a) plain tube,(b) plain twisted tape,(c) rectangular cut twisted tape inserts.....	54
Fig. 4.3	Temperature variation in a U-loop pipe for the three setups (a),(b),(c).....	55
Fig. 4.4	Temperature for (a) plain tube,(b) plain twisted tape,(c) rect-cut twisted tape inserts.....	56
Fig. 4.5	Nusselt number vs Reynolds number.....	57
Fig. 4.6	Nusselt number enhancement index vs Reynolds number	58
Fig. 4.7	Friction factor vs Reynolds number	59
Fig. 4.8	Friction factor enhancement index vs Reynolds number	60
Fig. 4.9	Thermal performance factor vs Reynolds number.	61

List of Tables

Table No.	Table Caption	Page No.
Table 3.1	Comparison of three distinct inserts with finer mesh	50
Table 4.1	Temperature data at various Reynolds numbers for different inserts.....	52

Nomenclature

q	Heat flux (W/m ²)
k	Thermal conductivity (W/mk)
C_p	Specific heat at constant pressure (J/Kg.K)
T	Temperature (k)
T_w	Wall temperature
T_b	Bulk temperature
T_{in}	Inlet temperature
T_{out}	Outlet temperature
p	Pressure (pa)
F	Body force
μ	Dynamic viscosity
u	Internal enegy
$\gamma \cdot$	Strain rate
D	Diameter of a pipe
τ	Shear stress
ρ	Density (Kg/m ³)
I	Identity matrix
k	Turbulent kinetic energy
ω	Specific dissipation rate
μ_T	Eddy viscosity
F	Body force
Q	Heat source
Φ	Dissipation function

Chapter 1: Introduction

1.1 General Overview

The rate of heat transfer augmentation in heat exchangers (HX) plays a crucial role in modern industrial sectors. The heat exchanger is a significant device utilized in the industry to transfer thermal energy between two different mediums. It is frequently employed to regulate temperature, boost energy efficiency, enhance heat recovery, ensure safety, and protect equipment. Whenever energy is transformed from one form to another or transferred between devices, a portion of that energy inevitably converts to heat. As a result, heat transfer is a key factor in the operation of nearly all machinery and equipment. Heat exchange devices are essential components in power systems, utilities, and a wide range of technological applications [1].

Heat exchangers are commonly used across various industries, including process, power, petroleum, transportation, air conditioning, refrigeration, cryogenics, heat recovery, alternative fuels, and manufacturing. They also serve as crucial components in many industrial products available in the market [2].

Heat exchanger design can be broadly classified into two main categories: thermal and hydraulic design, and mechanical design. Thermal and hydraulic design primarily focuses on determining the appropriate surface area for transferring a specific amount of heat, along with evaluating pressure drop, pumping power requirements, and related factors. In contrast, mechanical design is centered on ensuring the mechanical integrity of the exchanger and designing both pressure-bearing and non-pressure components [3].

The primary challenge in designing a heat exchanger is to create a compact system that maximizes heat transfer efficiency while minimizing the required pumping power [4]. Besides, the choice of a specific heat exchanger configuration is influenced by various factors such as area requirements, maintenance needs, flow rates, and the phase of the fluids. Plate, shell-and-tube, and finned heat exchangers are common types of heat exchangers used in the industrial sector. The augmentation of the heat transfer rate can be attained by expanding the effectiveness of the heat exchanger.

Active, passive, and combined methods are employed to enhance heat transfer in tubular heat exchangers. Process industries focus on these various techniques to improve the heat transfer rate, aiming to reduce costs and optimize the performance of heat exchangers.

In the active method, mechanical aids are used to stir the fluid, either by mechanical means or by rotating the surface. Surface scraping, a common technique in batch processing of viscous liquids within the chemical process industry, is applied to the flow of various fluids, including high-viscosity. Commercial applications also incorporate equipment with rotating heat exchanger ducts. Surface vibration, whether at low or high frequencies, is primarily employed to enhance single-phase heat transfer. Among vibration-based methods, fluid vibration is the most feasible for practical use, considering the substantial mass of most heat exchangers.

In the passive method, swirl-flow devices include various geometric arrangements or tube inserts for forced flow, which create a rotating or secondary flow. Moreover, the combined method involves integrating both active and passive methods to enhance heat transfer. Of these, the passive method is favored because it does not rely on an external power source. Passive techniques employ various methods such as treated surfaces, extended surfaces, swirl-flow devices, twisted tapes with different cuts, and coiled tubes. Swirl-flow devices utilize a range of geometric configurations or tube inserts to induce secondary flows [5].

In fluids, heat transfer occurs via convection, involving the mixing of different fluid regions. The effectiveness of convective heat transfer is mainly determined by the fluid's motion. Scientists have been working relentlessly to optimize the performance of heat exchangers using water and air as working fluids by incorporating complex geometries. They studied the effects of twisted tape (TT) dimensions, twist ratio, helical inserts, multiple TT configurations, wire coils, and corrugations on heat transfer intensification and analyzed the outcomes. Some researchers conducted the impact of evenly spaced quadruple twisted tapes experimentally and numerically. They observed the best thermal enhancement factor was achieved at a lower space ratio in cross-arrangement than in full-length quadruple twisted tapes [6]. In a different trial, the effects of thermal intensification and pressure loss were

examined for V-cut and without V-cut twisted tape (TT) inserts in a round conduit. In contrast to a smooth pipe, the outcomes showed improvements in the thermal performance factor (TPF), friction factor, and Nusselt (Nu) number for reduced twisting ratios and width ratios [7]. Another research examined the thermohydraulic performance of circular tubes augmented with smooth twisted tapes and U-cut twisted tapes (UTT) at twisting ratios under turbulent flow conditions. Experimental findings reveal that UTT outperforms PTT and plain tubes in enhancing convective heat transfer, frictional resistance, and overall thermal efficiency. The superior performance of UTT is ascribed to intensified turbulence and secondary flow structures induced near the tube walls [8]. Besides, the X-shaped design significantly enhances heat transfer performance. Consequently, the study identifies the X-shaped tape as an efficacious design for optimizing the thermal performance of heat exchangers [9].

Researchers have found that various heat transfer enhancement techniques offer distinct advantages and limitations. These techniques differ in their geometric configurations and construction complexity while operating under varying flow and thermal conditions. Twisted tape, for example, is less effective when air is used as the working fluid due to the lower density of the liquid [10].

From the above discussion, it is evident that these passive intensification techniques significantly enhance the performance of heat transfer devices, making them highly beneficial and essential for applications in the industrial sector.

1.2 Objectives with Specific Aims

It is clear from the overview that turbulators and other enhancement methods significantly affect the intensification of heat transmission. However, the implications of rectangular cut twisted tape inserts in a straight conduit have been numerically and experimentally studied by some researchers[11],[12]. However, no research comparing rectangular cut twisted tape inserts in a U-loop pipe to smooth twisted tape and smooth tubes has been done. This research endeavors to numerically examine the phenomena of heat transfer augmentation in a U-loop pipe that has rectangular cut twisted tape inserts installed. Numerical simulations will also be conducted for

twisted tape without cut and smooth tubes. Furthermore, the effects of thermal performance index (η), pressure drop, heat transfer coefficient, and Reynolds numbers will be considered to perform this numerical analysis. The FEM-based software COMSOL MULTIPHYSICS [13] will be used to simulate the heat transfer phenomenon inside the designed computational domain.

The main goals of this research are –

- to study the heat augmentation occurrence in a U-loop circular tube using the Finite Element Method.
- to observe the effect of rectangular cuts in case of turbulent flow enclosed within the tube.
- to compare the heat transfer enhancement inside the pipe with smooth twisted tape (TT) and smooth pipe.
- to investigate the heat transfer parameters depending on the rectangular cut twisted tape inserts.

1.3 Thesis outlines

The thesis has the following structure:

Chapter 1 provides a general overview of heat exchangers, the application of heat exchangers in industrial areas, methods of improving heat transfer rate, and the objectives of the present work.

Chapter 2 presents an overview of fluid flow and heat transfer phenomena, covering fundamental concepts and key terminologies related to the finite element method, as well as fluid flow and heat transfer, as needed. Additionally, this chapter includes a brief discussion of relevant literature reviews connected to our study.

Chapter 3 reveals the mathematical model and computational domain of this study, which analyzes the flow characteristics of rectangular-cut twisted tape inserts. This chapter also illustrates the mesh design of rectangular cut twisted tape inserts, plain twisted tape inserts, and plain tubes.

Chapter 4 describes the simulation results which are obtained by COMSOL MULTIPHYSICS software for the turbulent flow region. Then it describes the heat transfer parameter including friction factor, friction factor enhancement index, Nusselt number, Nusselt number enhancement index, thermal performance factor, and temperature distribution with respect to Reynolds numbers for all three cases.

Chapter 5 describes the final analysis and the conclusion of this research work along with the possible recommendations for future research work which may add new dimensions to the area of the heat transfer intensification process.

Chapter 2: Literature Review

2.1 Introduction

Heat is the movement of energy between two objects caused by a difference in temperature. Thermal energy arises from an object's collision of atoms and molecules. Heat naturally flows from a warmer object (higher temperature) to a cooler one (reduced temperature) until thermal equilibrium is reached and both items reach an equal temperature. It is used across various industries for a broad range of purposes. Enhancing heat movement within a fluid is also indispensable in multiple sectors, as the fluid's properties impact the heat flow rate.

In fluids, heat transfer occurs through convection by mixing different fluid parts. The effectiveness of convective heat transfer is primarily influenced by the motion of the fluid. It is categorized as natural convection (caused by density differences within the fluid) and forced convection (requires an external source like a pump or fan to move the fluid). In natural convection, flow dynamics are lower compared to those in forced convection. As a result, the convection coefficient is typically lower, usually by one order of magnitude. This type of convection is commonly used in simple devices. Forced convection is also categorized as internal (if the fluid flow is bounded inside a pipe, duct, channel, capillary, etc.) and external forced convection (if the fluid flows on a solid wall, object, etc.). Additionally, the shape of the surface affects fluid flow and thus influences the heat transfer process [14].

This chapter discusses heat transfer modes, thermal conductivity, heat flux, heat transfer rate, bulk temperature, heat exchanger, classification of heat exchanger, fluid properties (temperature, density, pressure, compressibility, surface tension, viscosity), shear stress and strain, fluid flow behavior, laminar and turbulent flow, Reynolds number, velocity boundary layer, Newtonian and non-Newtonian fluid, compressible and incompressible fluid. We also discussed friction factor and pressure drop.

After that, covered some computational methods such as computational fluid dynamics, finite difference method (FDM), finite volume method (FVM), finite element method (FEM), etc. Then the chapter concludes with an exploration of related prior work.

2.2. Types of Heat Transfer Method

Heat can be transferred in three ways

- Conduction
- Convection
- Radiation

Every type of heat transmission move from areas of higher temperature to lower temperature and require a temperature gradient to occur.

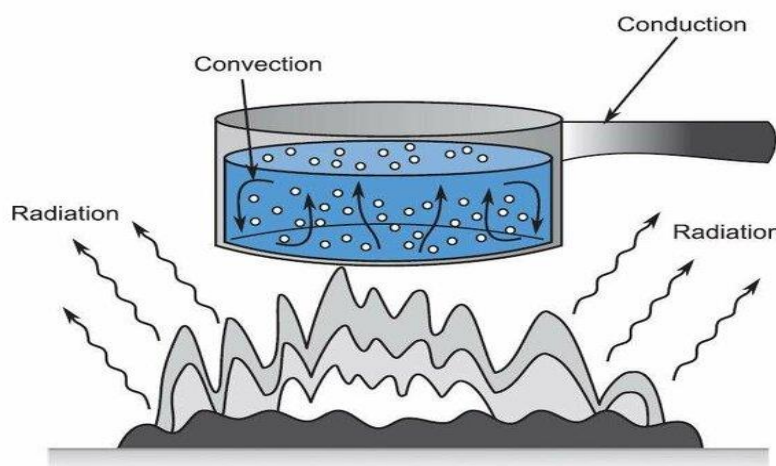


Figure 2.1: Conduction, convection, radiation.

2.2.1. Conduction

The microscopical exchange of kinetic energy between particles at the interface of two systems is known as conduction.

This heat transfer method depends on the temperature difference as a driving force and the thermal conductivity or resistance of the medium. Thermal conductivity varies according to the material's properties and size. Heat transfer problems always consider temperature gradient, geometry, and the physical characteristics of the object. In conduction cases, the object analyzed is generally a solid. There are numerous examples of conduction-based heat transfer. For instance, Significant loss of temperature occurs from a heated area to the cooler outdoor air on a chilly winter day.

This temperature loss mainly occurs because of conduction via the wall separating the room from the external environment. In heat conduction, the rate equation is referred to as Fourier's law.

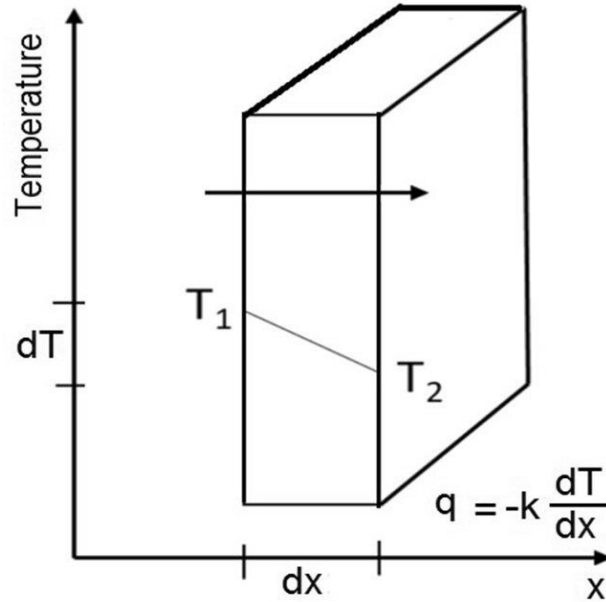


Figure 2.2: Conduction heat transfer.

The rate equation for a one-dimensional planar wall with a temperature distribution $T(x)$, as shown in Figure 2.2, is as follows:

$$q = -k \frac{dT}{dx}$$

The heat transfer rate in the x -direction per unit area, perpendicular to the flow direction, is known as the heat flux q (W/m^2), and it grows in proportion to the temperature gradient, dT/dx , in that direction.

One characteristic of the wall's material is the parameter k , also referred to as thermal conductivity ($\text{W}/\text{m}\cdot\text{K}$). Heat travels toward regions with lower temperatures, as indicated by the negative sign [16].

2.2.2 Convection

Heat transfer between a flowing fluid and its surface is known as convection.

It occurs at an object's surface when the surrounding fluid is warmed, leading it to transport energy away from the heat source. When an object's surface temperature differs from that of the surrounding fluid, convective heat transfer occurs [17].

2.2.3 Radiation

Radiation is a mode of heat transfer that operates without requiring physical contact between objects. It manifests as electromagnetic waves, commonly referred to as photons. Unlike conduction, which relies on direct contact, or convection, which depends on the movement of a fluid medium. Radiation is unique in that it can transfer energy through the emptiness of space without any intervening substance [18]. This remarkable property allows it to propagate at the speed of light in a vacuum, maintaining its full intensity without attenuation. This is the principle by which energy from the Sun travels across the vast void of space to reach the Earth.

All matter with a temperature above absolute zero inherently emits thermal radiation. While radiation is technically a volumetric phenomenon involving the emission, absorption, and transmission of energy within solids, liquids, and gases, it is often simplified as a surface-level process for opaque solids. Materials like metals, wood, and rocks are considered opaque to thermal radiation because the radiation generated within their interiors is unable to escape to the surface. Additionally, any radiation incident on these materials is typically absorbed within just a few microns of their outermost layer, further reinforcing the surface-focused perspective.

2.2.4 Thermal conductivity

A material's capacity to transfer heat is gauged by its thermal conductivity. A high thermal conductivity value signifies that a material is an efficient heat conductor. In contrast, a low number indicates that the substance is either an insulator or a poor heat conductor. At ambient temperature, pure copper's thermal conductivity is expressed as $k = 401 \text{ W/m}^\circ\text{C}$, meaning that a 1-meter solid wall of copper will transmit heat at a rate of 401 watts per square meter of surface area for each degree Celsius of

temperature difference across the wall. In addition to being superior electrical conductors, copper and silver have high thermal conductivity ratings and are effective heat conductors. However, rubber products, timber, and Styrofoam are examples of materials with low thermal conductivity and poor heat conductivity [19].

Material	Thermal conductivity k, W (m K) ⁻¹
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury	8.54
Water	0.613
Air	0.026

Figure 2.3: Thermal conductivity of some materials.

2.2.5 Heat flux

Heat flux, represented by the symbol q for average heat flux, refers to the rate of heat transfer per unit area perpendicular to the direction of heat flow.

$$q = \frac{Q}{A}$$

Here, A represents the area, and the unit of heat flux is expressed in watts per square meter (W/m²).

2.2.6 Heat transfer rate (Q)

Heat transfer is the mechanism of moving heat energy between two objects or more

objects with different temperatures. This transfer continues until thermal equilibrium is reached between the surroundings or the bodies involved. The rate of energy transferred over a given time is known as the heat transfer rate. When a fluid passes through a heated tube, the tube's surface transfers heat to the cooler fluid. The following is the definition of the fluid's heat transfer rate (Q):

$$Q = m C_p (T_{\text{out}} - T_{\text{in}})$$

In this context, T_{in} refers to the fluid's inlet temperature, T_{out} signifies the bulk fluid's outlet temperature, and m indicates the mass flow rate.

2.2.7 Heat Transfer Coefficient

The heat transfer coefficient (h) indicates the rate of heat transfer per unit temperature difference, area, and time between a fluid or fluids and a surface. Various factors, such as the thermodynamic and transport properties of the fluid, the flow characteristics, the surface geometry, and the current thermal conditions influence its value.

$$h = \frac{q}{T_w - T_b}$$

where q , T_w , and T_b are the heat flux, wall temperature, and average fluid temperature in the main flow respectively.

2.2.8 Wall Temperature

A thermocouple is used at various points during the experimental session to determine the fluid's average wall or surface temperature. We calculate it using the average temperature for every location on the fluid domain's surface during our simulation. Because the liquid surface and the heat exchanger's outer layer are so close, the surface temperature is excessively high in relation to the bulk temperature.

2.2.9 Bulk Temperature

Bulk temperature is also known as the average fluid temperature which is determined by the average outlet (T_{out}) and inlet (T_{in}) temperature. In simpler terms, the thermal

energy carried by the fluid's particles during motion is known as the fluid's mean or bulk temperature (T_b).

$$T_b = \frac{T_{out} + T_{in}}{2}$$

2.2.10 Specific heat

The specific heat of a substance is defined as the quantity of heat energy necessary to raise the temperature of a unit mass of the substance by a single degree. Specific heat can vary somewhat with the temperature at which it is measured and depends on the conditions allowed during the temperature increase [20].

2.3 Heat Exchanger

A heat exchanger is a specialized device engineered to facilitate the transfer of thermal energy between two or more fluids, a solid surface and a fluid, or solid particles and a fluid, all at varying temperatures.

This process typically occurs through direct or indirect thermal contact, without the need for external heat sources or mechanical work input. The fluids involved in the heat exchange process can consist of either pure substances or mixtures of different components.

It is commonly used in raising or lowering the temperature of a specific fluid stream, condensing or vaporizing single or multicomponent streams, and managing heat recovery or rejection in a system.

Some heat exchangers facilitate direct contact between the fluids exchanging heat, while in others, heat transfer occurs across a separating wall or through a wall in a changing manner. Typically, the fluids are divided by a thermal interface in most heat exchanger designs [21].

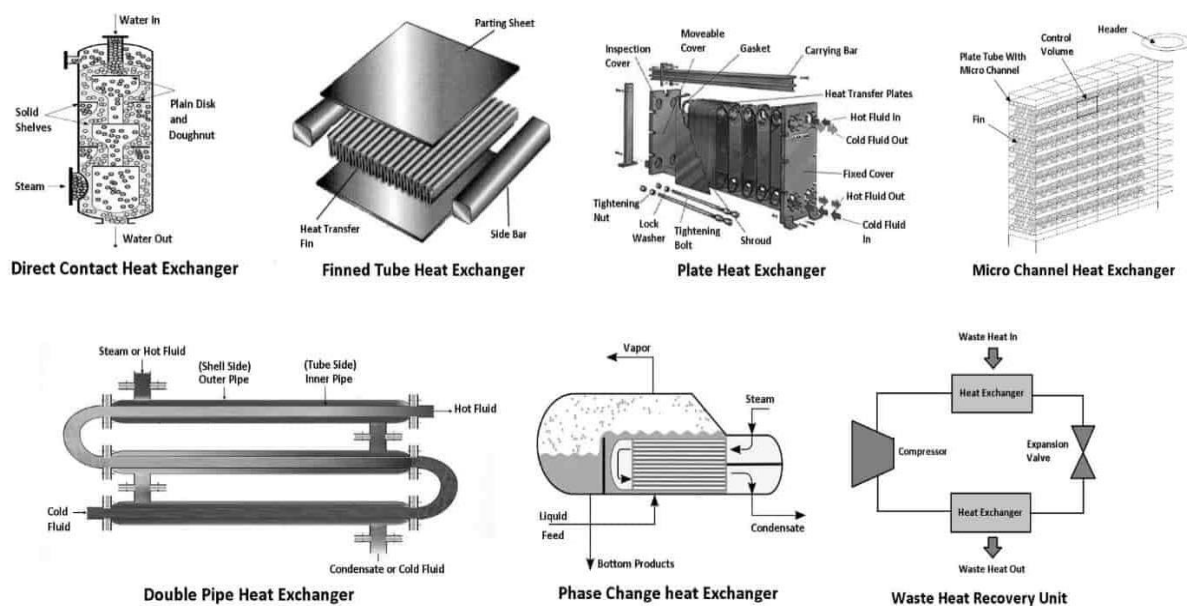


Figure 2.4: Types of heat exchangers [22].

2.3.1 Classification of Industrial Heat Exchanger

Industrial heat exchanger design should prioritize simplicity, especially when significant fouling is anticipated. Reducing the operating temperature, combined with selecting an appropriate fluid velocity and minimizing the concentration of foulant precursors, can help mitigate fouling risks. Additionally, the maximum flow velocity should be maintained within the limits imposed by pressure drop constraints and the potential for flow-induced erosion.

Industrial heat exchangers are employed across a wide range of applications, including energy generation, chemical engineering operations, transportation networks, sustainable fuel development, low-temperature technology i.e. cryogenics, cooling systems, refrigerant systems, energy recovery systems, and many other industries. Heat exchangers can be categorized in different approaches. In industrial systems, they are typically classified by their construction, heat transfer processes, compact surface area, flow configurations, pass configurations, and phase properties of the fluids [23].

The classification of heat exchanger is given below:

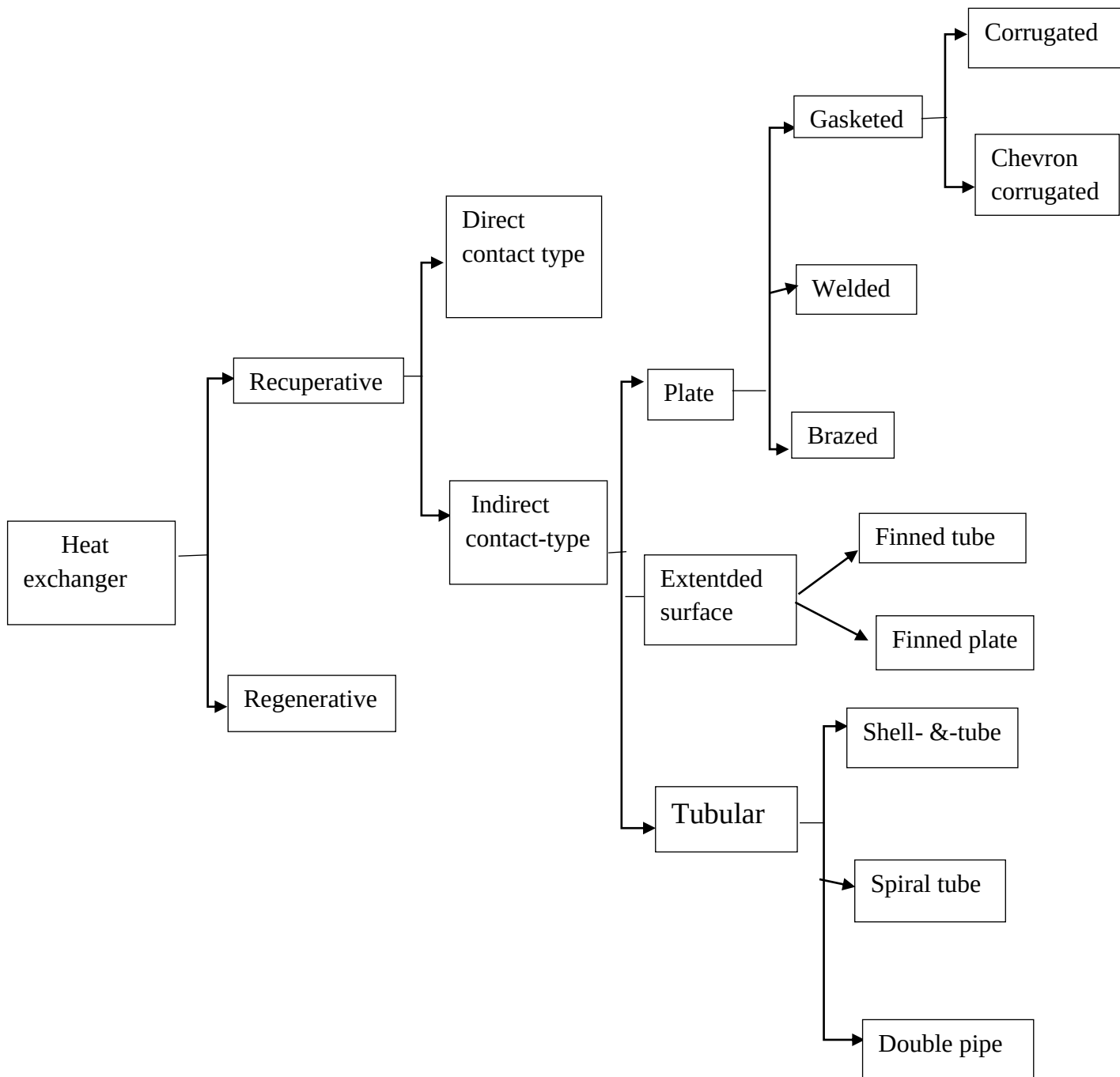


Figure 2.5: Heat exchanger's classification [24].

2.4 Fluid Properties

Fluid is a substance that always flows and deforms under an applied force. A fluid can be liquid or gas. It has different features such as kinematic features (velocity and acceleration), temperature-dependent features, and physical properties (physical condition of fluid i.e. color, smell). Some characteristics of fluids are discussed below

2.4.1 Temperature: Temperature is one of the thermodynamic properties of the fluid which measures the hotness or coldness of any fluid substance. It is generated by the molecular vibration in fluids. The units of temperature are Celsius, Fahrenheit, and Kelvin. Generally, Kelvin is the most used temperature unit.

2.4.2 Density: Mass of a substance per unit volume and denoted by Greek symbol ρ . Density sometimes varies with pressure and temperature such as gas density. The unit is kg/m^3 and expressed as

$$\rho = \frac{m}{v}$$

2.4.3 Compressibility: Compressibility is a fluid characteristic by which the fluid's volume can be changed under external pressure. Air and gas are examples of compressible fluids. The flow of natural gas through a pipeline, a typical method for heating, should be regarded as a compressible flow.

2.4.4 Pressure: Pressure is the amount of force applied to a specific area. It acts in all directions of the fluid. S.I. unit of pressure is newton per meter square, known as pascal (Pa). $1 \text{ Pa} = \text{N/m}^2$.

$$p = \frac{F}{A}$$

2.4.5 Surface Tension

Surface tension is indeed a fundamental property that defines the behavior of liquid surfaces, particularly at gas-liquid interfaces. At the interface between air and water, the boundary behaves as though it's under tension, much like the surface of a balloon or a soap bubble. This tension causes liquid droplets to adopt a spherical shape. This

effect arises from intermolecular attractive forces at the interface. The overall result of these attractive forces is that the interface tends to contract. It is defined as force per unit length, and the S.I. unit is N/m.

2.4.6 Viscosity

Viscosity is a measure of a fluid's resistance to flow or deformation when a force is applied at a certain rate. For example, oil is more viscous than water. The viscosity of a fluid changes with temperature, typically decreasing for liquids as temperature increases. A fluid with zero resistance is called an inviscid fluid. Mathematically, the viscosity of a liquid is defined as the ratio of the applied shear stress to the resulting rate of strain. Viscosity is expressed as

$$\mu = \frac{\sigma}{\dot{\gamma}}$$

2.5 Shear stress and strain

Stress refers to the internal distribution of forces within a material that balances and responds to the applied loads. Mathematically, stress is explained as the force divided by the area on which that force is applied.

$$\text{stress} = \frac{\text{force}}{\text{area}}$$

There are two types of stress such as normal stress and shear stress. Stress that is either tensile or compressive and acts perpendicular to a given plane is called normal stress. In contrast, stress that tends to deform the material by sliding parallel to the area acting in-plane and at right angles to compressive stresses is known as shear stress.

Strain is the deformation measured per unit of its original length, expressed as

$$\text{strain} = \frac{dL}{L}$$

2.6 Laminar Flow

The fluid flow which is layered and flows at a smaller flow rate in a regular path is defined as laminar flow. It is made up of thin layers that slide over one another and

are parallel to one another. The dimensionless Reynolds number determines laminar flows. Velocity remains constant at each point of the fluid and viscosity enhances due to the slow motion of the fluid.

A laminar flow can indeed transition to turbulent flow if the velocity and pipe diameter increase, as this would raise the Reynolds number. Blood flow and oil flow are examples of laminar flow.

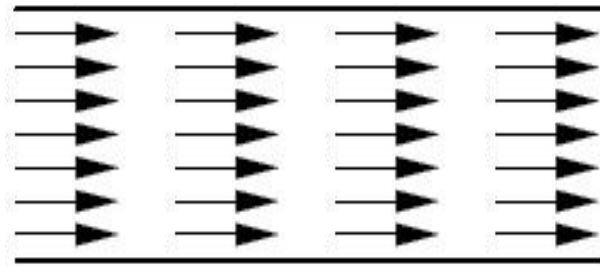


Figure 2.6: Laminar flow of fluid.

2.7 Turbulent Flow

Turbulent flow is a type of flow that is irregular and doesn't intersect with each other. This phenomenon occurs when the fluid flows chaotically, creating swirls, and vortices, which lead to significant turbulence.

Turbulent flow usually develops at high flow rates, the flow becomes irregular when the Reynolds (Re) number surpasses 4000 [25]. The flow-through pumps, and blood flow through arteries, atmosphere, and ocean currents are examples of turbulent flow.

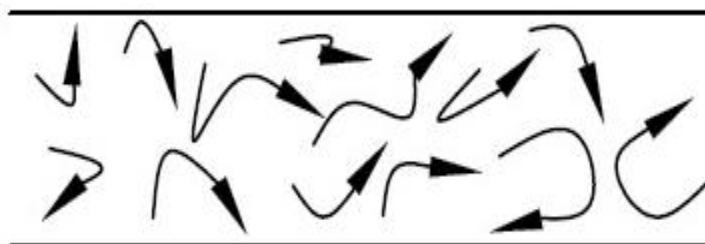


Figure 2.7: Turbulent fluid flow within a pipe.

2.8 Reynolds Number

In the 1880s, British engineer Osborne Reynolds conducted pivotal research on fluid flow, focusing on the shift between laminar (smooth and orderly) and turbulent (chaotic) flow in tubes [25]. Reynolds discovered that a specific parameter, now called the Reynolds number, could determine when this transition would occur.

The Reynolds number, denoted as Re , is a dimensionless quantity calculated by

$$Re = \frac{\rho V D}{\mu}$$

where:

ρ is the density of the fluid

V is the fluid velocity

D is the tube diameter,

μ is the dynamic viscosity.

More generally, for any flow geometry, it can be expressed as:

$$Re = \frac{\rho V L}{\mu}$$

with L representing a characteristic length that suits the specific flow configuration.

The Reynolds number signifies the relationship of inertial forces (which drive movement) to viscous forces (which resist movement). When the Reynolds number is large, inertial forces prevail, leading to turbulent flow.

Conversely, viscous forces prevail when the Reynolds number is lower, resulting in smooth, laminar flow. This principle makes the Reynolds number a key factor in predicting flow behavior across different fluid systems [26].

The range of Reynolds numbers helps to classify the flow regime

Laminar flow: Reynolds number $Re < 2000$

Transitional flow: Reynolds number $2000 < Re < 4000$

Turbulent flow: Reynolds number $Re > 4000$ [25].

2.9 Velocity Boundary Layer

Imagine a fluid flowing parallel to a flat plate, as shown in Fig. 2.8. Surfaces with slight curvatures, like turbine blades, can often be reasonably treated as flat plates for analysis. While y is measured perpendicular to the plate's surface, x is specified along the plate's surface, beginning at the leading edge and moving in the flow direction. The fluid moves toward the plate along the x -axis with a uniform incoming velocity that closely aligns with the free-stream velocity u_∞ over the plate, except near the surface (unlike in the case of cross-flow-like cylinders).

For illustration, imagine the fluid as a stack of adjacent layers. The speed of the particles within the layer closest to the surface drops to zero due to the no relative motion condition. This stationary zone, in turn, reduces the speed of the particles in the adjacent layer due to friction between fluid layers moving at different speeds. This effect propagates layer by layer, causing the plate's influence to extend up to a certain normal distance δ from the plate. Beyond this distance, the free-stream velocity u_∞ remains mostly unaffected. As a result, the x -component of the fluid flow velocity, u , transitions from zero at $y = 0$ to nearly u_∞ at $y = \delta$. The flow area situated above the plate, limited to a distance δ , where shear forces from viscosity are significant, is referred to as the velocity boundary layer [19].

Similarly, the thermal boundary layer forms when the fluid moves over a surface with a varying temperature, following the same principle as the velocity boundary layer. The figure of the velocity boundary layer and thermal boundary layer is shown in Fig. 2.8.

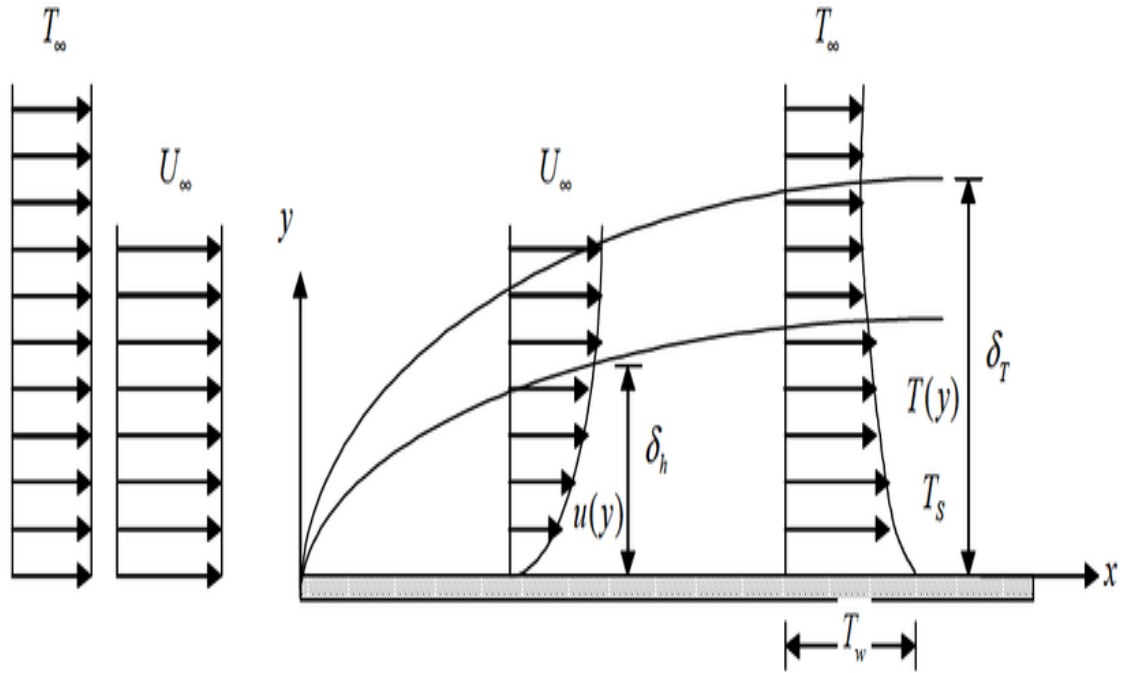


Figure 2.8: Velocity and thermal boundary layer.

2.10 Newtonian Fluid

A Newtonian fluid is a type of fluid that adheres to Newton's law of viscosity, which states that the rate of deformation (and thus the velocity gradient) is directly proportional to the shear stress (τ) applied to it. In simpler terms, the viscosity of a Newtonian fluid remains constant regardless of the forces acting on it, provided the temperature and pressure remain unchanged. Common examples of Newtonian fluids include water and air, which flow predictably and uniformly under varying levels of applied stress, without exhibiting changes in their inherent flow behavior. It is expressed as

$$\tau = \mu \left(\frac{du}{dy} \right)$$

Where μ is a dynamic viscosity (unit $\text{pa} \cdot \text{s}$) [30].

2.11 Non- Newtonian Fluid

A non-Newtonian fluid is a type of fluid that does not conform to Newton's law of viscosity, meaning its flow behavior does not exhibit a direct proportionality between shear stress and shear rate. In these fluids, viscosity depends on the shear rate, and the relationship between shear stress and shear rate varies and may also depend on time. As a result, the viscosity is not constant. Ketchup is an example of a non-Newtonian fluid.

2.12 Incompressible and Compressible Fluid

A flow is described as incompressible when the fluid density (ρ) remains constant throughout the flow field and it is termed as compressible if the density (ρ) varies with pressure. For example, the flow of natural gas through a pipeline should be treated as a compressible flow. This type of flow is common in many natural and technological applications, such as gas flow in the exhaust systems of internal combustion engines. This problem led to the development of the Fanno flow model. These types of flows are referred to as internal flows. Compressible flow also includes external flows, such as the airflow around objects like airplane wings [27].

A solid understanding of compressible flow is vital for designing various systems in manufacturing engineering. For example, control engineers using pneumatic systems work with compressed substances, while cooling processes in some manufacturing operations and refrigeration system designs also rely on principles of compressed air flow. These systems often require attention to the specific behaviors unique to compressible flow [28].

2.13 Nusselt Number

The Nusselt number, which derives its name from a German engineer Wilhelm Nusselt (1882-1957) [29], a pioneer in the study of convective heat transfer, is a dimensionless quantity that serves as the coefficient of convective heat transfer.

To understand the physical significance of the Nusselt number, imagine a fluid layer with a thickness L subjected to a temperature variation $\Delta T = T_2 - T_1$. When the fluid is in motion, heat transfer across the layer occurs by convection, whereas in a

stationary fluid layer, heat transfer takes place by conduction. For convection and conduction heat flux is

$$\dot{q}_{conv} = h \Delta T \quad (2.1)$$

$$\text{and} \quad \dot{q}_{cond} = k \frac{\Delta T}{L} \quad (2.2)$$

$$\text{So, the ratio is } \frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h \Delta T}{k \Delta T / L} = \frac{hL}{k} = Nu \quad (2.3)$$

For a tube, it is calculated as $Nu = \frac{hD}{k}$

Where D represents the diameter of the tube, the Nusselt number is a dimensionless quantity that quantifies the enhancement of heat transfer due to convection compared to conduction within a fluid. A higher Nusselt number signifies a more efficient convective heat transfer process, indicating that convection plays a more dominant role in the overall heat transfer compared to conduction [30].

2.14 Pressure Drop

The reduction in fluid pressure across two points in the flow area is referred to as pressure drop [31]. Viscous effects are usually responsible for pressure loss. In the absence of friction or resistance, this pressure drop would be nonexistent, as it depends on the fluid's viscosity. When fluid flows through a tube, frictional forces lead to a pressure decrease along the conduit. Pressure decrease refers to the frictional loss associated with the surface over which the fluid flows, indicating the pressure decreases as it moves from one point to another outlet within a conduit. A greater pressure reduction requires more blower power for the fluid to flow across the surface, especially if the surface is rough, where the pressure drop is even greater [32]. The friction factor can be determined by using the pressure loss. It is represented by Δp and p_i, p_o denote the inlet and outlet pressure respectively.

$$\Delta p = p_i - p_o$$

2.15 Friction Factor

The friction factor has been determined to be dependent on the flow's Reynolds number as well as the inner surface roughness of the pipe. Friction properties are determined in the chemical manufacturing sector using the friction factor. It is denoted by f .

$$f = \frac{\Delta p}{\frac{\rho L v^2}{2D}} \quad (2.4)$$

f = friction factor

ρ = fluid density (kg/m³)

v = fluid velocity in the tube (m/s)

D = pipe's internal diameter (m)

L = pipe length (m)

2.16 Computational Fluid Dynamics

Computational Fluid Dynamics (CFD) is a sophisticated and adaptable technology for analyzing complex flows. Advancements in computational methods and computer technology have made CFD widely applicable across science and industry to understand intricate phenomena in real-world problems. High-speed computers enable more effective solutions for practical applications.

The journey of Computational Fluid Dynamics (CFD) began in the 1970s, emerging as an acronym that represents a blend of physics, mathematical modeling, numerical methods, and, to some extent, computer science. These elements combined to enable the simulation of fluid flows. The development of CFD was spurred by the advent of increasingly powerful mainframe computers and significant advancements in methods for simulating transonic flows, particularly through the solution of non-linear potential equations.

By the early 1980s, the field progressed to solving first two-dimensional and later three-dimensional Euler equations. Enhanced computational speeds and advances in

numerical techniques, such as multigrid methods, made it feasible to compute inviscid flows. From the mid-1980s onward [33], attention shifted toward more complex simulations of viscous flows governed by the Navier-Stokes equations, reflecting the growing capabilities of computers and numerical methodologies.

CFD is the analysis of fluid flow prediction which solves mathematical equations by numerical methods using digital computers. It simulates fluid engineering systems by mathematical modeling and numerical approaches, including discretization, solvers, numerical parameters, and grid generation [34]. A flow simulation seeks to analyze the behavior of flow within a particular system based on a defined set of input and output parameters. These parameters are commonly referred to as boundary conditions.

For example, in a boiler designed to heat water for heating purposes, it might be essential to calculate the temperature and velocity of the water leaving the boiler, given a specific mass flow of water and energy input from the gas burner. Additionally, understanding the flow patterns and temperature distribution within the boiler could be crucial if design modifications are needed to enhance mixing or minimize energy loss through the walls. Due to the complex geometry of most boilers, obtaining an analytical solution to the flow equations is often difficult. It will be useful for all technical applications to understand the fundamental flow characteristics at a large number of discrete sites distributed across the boiler shape. This will provide sufficient insight into the flow behavior, allowing technical experts to obtain the necessary insights for operating or configuration needs.

The main concept behind CFD (Computational Fluid Dynamics) methods is to calculate the values of flow parameters at numerous locations throughout the system. CFD techniques are now often applied in automotive applications, aerospace applications, biomedical applications, and so on [35].

The algorithm of the CFD technique is shown in Fig. 2.9.

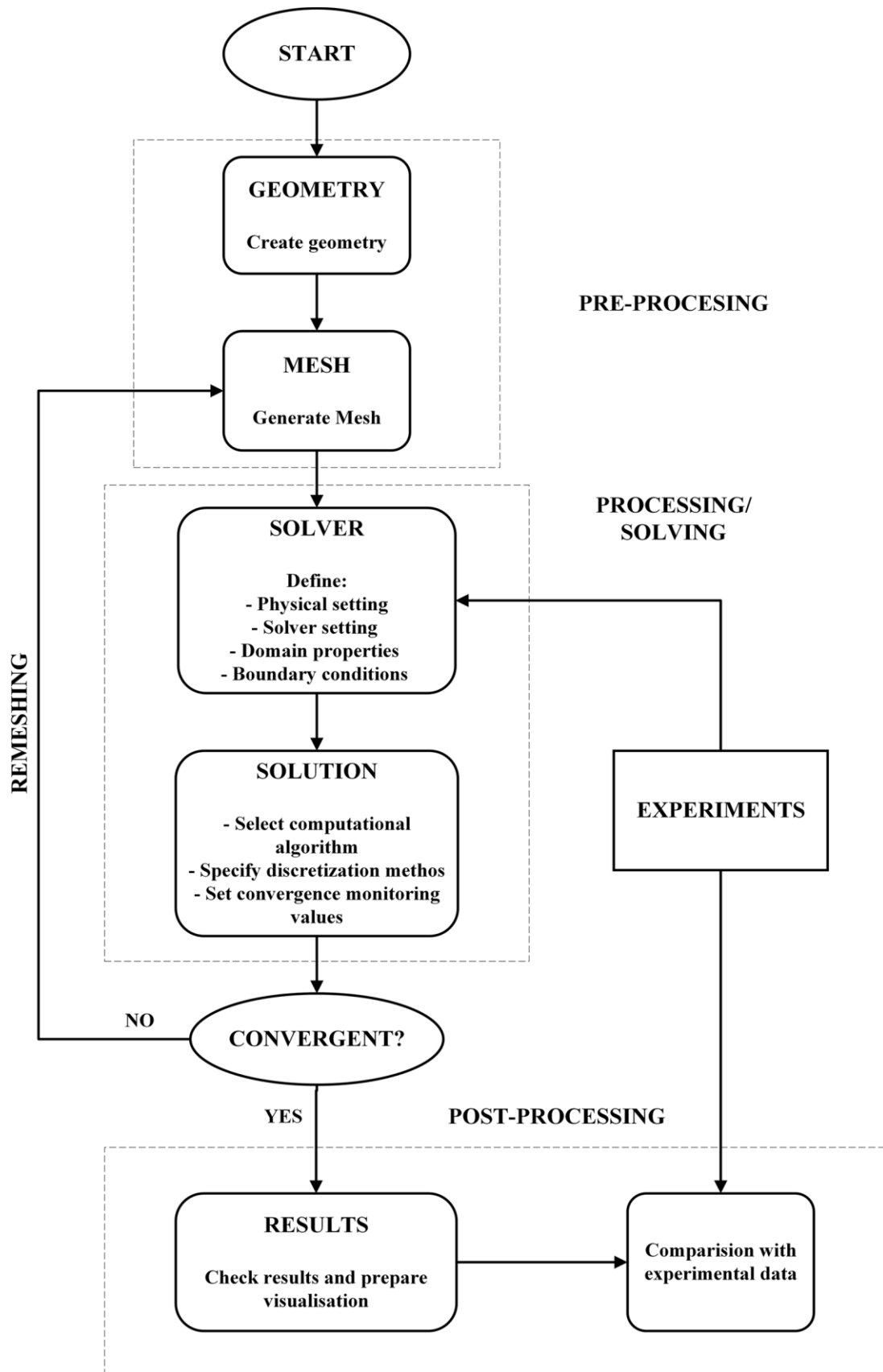


Figure 2.9 Flowchart of CFD method.

2.17 Finite Difference Method (FDM)

The Finite Difference Method (FDM) is the earliest approach developed, relying on a local Taylor series expansion to estimate differential equations. It employs a grid of intersecting paths in a square topology to discretize the partial differential equation (PDE). However, this grid structure can become a limitation when dealing with complex geometries in higher dimensions. This challenge led to exploring integral forms of PDEs, eventually paving the way for the finite element and finite volume methods [36].

2.18 Finite Volume Method (FVM)

The Finite Volume Method (FVM) is a computational approach used to solve fluid flow equations by integrating the governing equations over individual control volumes that together constitute the solution domain. It is extensively utilized in heat and mass transfer implementations and in fluid mechanics compatible with structured or unstructured meshes.

The Finite Volume Method (FVM) possesses the beneficial characteristic of local conservation in its numerical fluxes, ensuring that the flux transferred from one computational cell is exactly conserved in the adjacent cell. This property is especially valuable for modeling problems where flux conservation is critical, such as in fluid dynamics, and thermal and mass transport. Because FVM relies on a "balance" approach, it ensures local conservation by establishing a balance for each discretization cell, or "control volume." The divergence theorem is used to derive an integral formulation of the fluxes, which is then applied over the boundaries of the control volumes. The fluxes are then discretized at these boundaries with respect to the discrete unknowns.

From an industrial perspective, the Finite Volume Method (FVM) is recognized as a rugged and cost-effective approach for discretizing conservation laws. By "rugged," we intend that it performs reliably even with challenging equations, and hyperbolic equations in non-linear systems, and it can be readily adapted to further complex, real-world physical scenarios beyond standard academic cases. FVM is cost-effective because it enables concise and dependable computational coding, even for intricate

problems. In several instances, it is more suitable than the Finite Difference Method, which typically requires simple geometries. However, designing FVM schemes that achieve sufficient precision can sometimes be challenging [37].

2.19 Finite Element Method (FEM)

The Finite Element Method (FEM), also known as Finite Element Analysis (FEA), is a numerical approach employed to estimate solutions to boundary value problems for partial differential equations (PDEs). It involves partitioning a complex domain into smaller, more manageable sections known as finite elements. By applying variational methods from calculus, FEM minimizes a related error function to solve the problem. This approach integrates numerous simple equations defined over these small sub-domains to approximate the behavior of a more complex equation across the entire domain.

To solve a boundary value problem defined in the domain Ω using the finite element method, the following steps are carried out:

Divide the computational domain Ω into a finite number of elements with N nodes, considering the values of u at these nodes as the primary unknowns.

Convert the boundary value problem into a system of finite element equations.

Calculate the coefficient matrices for each individual element.

Combine the individual element matrices to build the global matrix.

Solve the global matrix equations which may involve a system of algebraic equations of $u_i (i = 1, N)$. Where N indicates the number of nodes of a computational domain.

Finite element equations are fundamentally derived using one of the ways listed below:

Direct approach: It can only be utilized for extremely simple problems.

Variational approach: It is a more general approach. This method can also be applied to problems involving the optimization of an associated functional.

Variational statement: The variational statement of a boundary value problem (BVP) is its integral representation. For a given boundary value problem with an approximate solution $u(x)$, the local residual error can be defined as

$$r(x) = L(u(x)) - f(x) \quad (2.1)$$

By multiplying the local residual error $r(x)$ with a weighting function $v(x)$ and integrating the resulting weighted residual error over the domain Ω we arrive at the total weighted residual error. Setting this total error to zero yields the following variational statement

$u(x) \in \tilde{H}$ such that

$$\int_{\Omega} v r(x) d\Omega = \int_{\Omega} v (L(u) - f(x)) d\Omega = 0, \forall v \in H \quad (2.2)$$

Here, v is referred to as the weighting function or test function and $\tilde{H}$ is a linear space. The expression in equation (2.2) is known as a variational statement because the weighting function v can vary arbitrarily. By applying integration by parts to the integral expression in equation (2.2), it is often possible to derive an expression involving lower-order derivatives of $u(x)$. This process reduces the order of derivatives of u , and equation (2.2) frequently transforms into

$$\int_{\Omega} P(u, v) d\Omega + \int_{\Omega} Q(u) v ds = 0 \quad (2.3)$$

This reduction weakens the smoothness requirements on $u(x)$ making the integral representation known as the weak statement.

Weighted residual approach: This is a more general and versatile approach, which is also suitable for problems where no functional is defined [38].

2.19.1 Applications of the Finite Element Method

The finite element method (FEM) is widely used for solving both structural and nonstructural problems. In structural applications, it is commonly employed for:

1. **Stress Analysis:** Includes tasks such as analyzing trusses, and frames, and identifying stress concentrations caused by features like holes, fillets, or abrupt geometric changes.
2. **Buckling Analysis:** Assessing structural stability under compressive forces.

3. **Vibration Analysis:** Investigating dynamic responses and natural frequencies of structures.

For nonstructural problems, FEM is applied to areas such as:

1. **Heat Transfer:** Addressing problems involving conduction, convection, and radiation.
2. **Fluid Flow:** Analyzing fluid movement.
3. **Electromagnetic Potential:** Studying the distribution of electric or magnetic potentials.

FEM also plays a key role in biomechanical engineering, where it is used to analyze stress and other factors in systems such as the human spine, skull, hip joints, dental implants, heart, and eyes [39].

2.20 Previous studies

Heat exchangers are essential devices used in numerous industries, playing a key role in optimizing energy efficiency, enhancing process performance, and reducing operational costs. The effectiveness of a heat exchanger device depends on fluid flow behavior, flow arrangements, flow distribution, geometry, and surface area. However, researchers are continuously investigating new designs, advanced materials, and cutting-edge technologies to enhance the efficiency, functionality, and environmental sustainability of heat exchangers. Generally, to enhance device efficiency and heat transfer augmentation rate, different techniques such as surface coating, wire coil inserts, surface irregularity modifications, and numerous designs of twisted tapes (TT) including plain TT, full-length TT, short-length TT, square-cut TT, and V-cut TT are widely utilized [11].

Murugesan *et al.* [40] explored the impact of heat transfer in a tube fitted with square-cut twisted tape (STT) inserts at twist ratios of 2.0, 4.4, and 6.0. Their findings showed that the highest Nusselt number (Nu), friction factor, and thermal performance factor (TPF) were observed in comparison to both a plain tube and a tube with plain twisted tape (PTT) inserts. In another study, Murugesan *et al.* [41] examined the effects of heat transfer and pressure loss in a circular pipe, both with and without a V-cut twisted tape (TT). Their results demonstrated an improvement in

TPF, friction factor, and Nusselt number at a lower twist ratio $y = 2.0$ width ratio $WR = 0.34$ and maximum depth ratio $DR = 0.43$ compared to a simple tube. Additionally, the use of Triangular Perforated Twisted Tape with a V-cut significantly enhanced the thermal performance factor (TPF), friction factor, Nusselt number, and overall heat transfer coefficient. A more recent study investigated the effect of pitch (50 mm, 100 mm, 120 mm) on TPTTV, showing that the highest heat transfer coefficient and the best TPF (1.49), along with improved Nusselt number and friction factor, were achieved with a reduced pitch of 50 mm, outperforming the PTT inserts [42]. Nakhchi *et al.* [43] conducted experimental research to investigate the impact of double-cut twisted tape inserts on heat exchanger performance. They considered cut ratio (0.25 to 0.90) for the Reynolds number 5000 to 15000 and observed some better heat transfer parameters. The findings reveal that the best thermal performance is 1.63, with the highest cut ratio for the Re number value being 5000. The Nusselt number and the friction factor were increased with the rise in the twisted tape's cut ratio achieving 177.4% and 489% more than the plain tube. Sabbir *et al.* [44] conducted a numerical simulation using the finite element approach to analyze heat transfer enhancement in a circular pipe with and without inserts. The inserts were placed inside the tube to disrupt fluid particles in laminar flow, thereby increasing the heat transfer rate. The simulation was performed for both time-dependent and steady-state conditions, considering Reynolds numbers ranging from 1600 to 2400.

A computational study was carried out to enhance the convective heat transfer coefficient by using single, double, and triple twisted tape inserts in a heat exchanger (HX). The findings revealed that all twisted tape inserts improved the heat transfer rate. However, the triple-twisted tape inserts resulted in a significant increase in fluid hindrance, ranging from 13.7 to 21.2 times superior to that observed with the single and dual-twisted tape inserts [45].

In a computational investigation, researchers found that varying the twist ratios (TR) of plain twisted tape (PTT) inserts can impact heat transfer characteristics. The results showed that heat transfer increased with a higher TR, with a TR of 3.5 yielding the ideal heat transfer rate [46]. In another experiment, Salam *et al.* [12] investigated a pipe fitted with rectangular-cut twisted tape inserts. Their findings revealed that the Nusselt number and friction factor improved by 39% to 80% as the Reynolds number

increased. Bhosale *et al.* [11] implemented a study on heat exchanger performance by introducing rectangular-cut twisted tape inserts into a pipe. Their results showed that the superior Nusselt number and heat transfer coefficient were achieved at a twist ratio (TR) of 3.426 with these inserts. Khudheyer and his team [47] performed a numerical and experimental analysis using new models of tapered twisted tape. The tapered section's length (l) and starting width (SW) were varied in this study. Moreover, airflow was used as working fluid for the range $10,000 \leq Re \leq 40,000$. The results indicate that the narrow initial width and the small length of the tapering section enhanced the Nusselt number and thermal performance in both experimental and numerical studies.

Rubbi *et al.*[48] examined heat transfer enhancement in a tube using twisted tape (TT) with a hemispherical extruded surface (HES) for turbulent flow at a TR of 4.0. The addition of HES improved swirl generation in the flow, leading to a 69.4 % increase in heat transfer rate compared to a plain tube. Their study also highlighted a positive effect on the thermal performance factor (TPF). In an experimental study, researchers claimed that corrugated tube with spring tape inserts, considering spring and pitch ratios in the turbulent flow region play a vital role in heat transfer enhancement.

It showed that the Nusselt number increases as the spring ratio ($s = 3.0$) decreases due to the swirl generation in the flow regime and the best η value observed as 2.8 [49].

Pimsarn *et al* [50] performed an experiment using inclined circular rings and twisted tape inserts in a heat exchanger. The effect of circular ring inclination angle $\alpha = 30^\circ, 45^\circ, 60^\circ, 90^\circ$, the quantity of twisted tapes (H) 2,4,6 and twisting ratios 1.0,2.0,3.0 were also taken into account for the Reynolds number range 6000 to 20000 to determine the aerothermal performance(η), friction factor, and Nusselt number. They observed better aerothermal performance $\eta = 1.6$ at $30^\circ, H = 2$, and twist ratio 3.0 and the heat transfer rate of the inclined circular rings with twisted tape was superior to the smooth tube.

Kumar *et al.* [51] investigated the influence of annular perforated ring structures within a cylindrical conduit. Variations in thickness, diameters, pitch ratios, and perforation indices were systematically analyzed across a spectrum of Reynolds

numbers (6500 to 23000) to elucidate their effects on pressure drop and thermal energy transfer efficiency. The experimental investigation shows that a reduction in the diameter ratio (DR) contributes to a significant intensification in heat transfer but also raises the friction factor due to increased flow disturbances.

The experimental and numerical analysis of rectangular winglet vortex generators was conducted by Zhiqiang *et al.* [52]. They examined the number of winglet vortex generators ($N = 4, 6, 8$) the height ratio (0.05, 0.1, 0.2), and the pitch ratio (1.57, 3.14, 4.71) to assess thermal performance. The maximum Nusselt number and friction factor ratio were observed at the highest height ratio ($HR = 0.2$) with the largest winglets ($N = 8$) and the lowest pitch ratio ($PR = 1.57$). Due to the generation of longitudinal vortices in the turbulent flow regime, superior thermal enhancement performance was also reported.

Moreover, inserting helical tape and nail rods in a heat exchanger significantly improved heat transfer, as shown in the study by Sadashiv and Madhukeshwara [53]. They found that an extended helical tape with a centered rod enhanced heat transfer by 160%, while a helical tape without a rod improved it by 150%, in comparison to a plain tube, in their numerical analysis of helical tape swirl generators, both with and without rods. A recent experimental study focused on heat enhancement in a dual-tube heat exchanger using twisted tape (TT) and helical tape inserts. This study reported a maximum thermal performance factor (TPF) of 3.06, with Nusselt number (Nu) increases ranging from 219% to 315%, depending on the twist ratios of both the TT and helical tape inserts [54]. Additionally, Marzouk *et al.* [55] conducted both numerical and experimental investigations into heat transfer enhancement using nail rod inserts in a dual-tube heat exchanger with varying pitch lengths (100 mm, 50 mm, and 25 mm). They found that the highest Nusselt number, pressure drop, and heat transfer improvements occurred featuring a decreased pitch length of 25 mm, as the smaller pitch allowed for a greater number of nails, improving fluid interaction and enhanced turbulence, which boosted the heat transfer rate. Kosker *et al.* [56] examined the effect of curvature across the section in twisted tapes (TTs) on thermal energy transfer enhancement in annular pipes. Six TT configurations, with two twist ratios ($TR = 3, 4$) and three curve ratios ($CR = 1.761, 2.252, 3.271$), were analyzed

numerically for Reynolds numbers (Re) ranging from 5840 to 30900. The results, validated against experimental data, show that the Nusselt number (Nu) increases with a higher curve ratio, lower twist ratio, and lower Re , providing a 30–90% improvement in heat transfer over smooth pipes.

San *et al.* [57] demonstrated the effects of wire coil configurations with varying wire diameters. They also examined the impact of the coiled pitch relative to the inner tube diameter. The results indicated that the Nusselt number increases with a higher wire diameter ratio. Furthermore, it was established that the reduction in the ratio of coiled pitch to internal tube diameter leads to an improvement in the Nusselt number.

Jafari *et al.* [58] investigated the influence of swirling flow injection on the thermal performance of a tube through experimental methods. The swirling flow mechanism utilized was a twisted coil tube with a cross-sectional profile comprising three lobes. To evaluate the thermal efficiency of the tubes equipped with the swirling generator, the study focused on the Nusselt number (Nu) and the friction factor as key performance indicators. The experimental analysis was carried out for a range of parameters, including varying rotational angles over a length of 1 to 4 meter. Furthermore, the Reynolds number was systematically altered within the interval of 5,000 to 30,000.

Promvonge *et al.* [59] conducted an experimental study on circular tubes equipped with cone-ring turbulators and twisted tape (TT) swirl generators to assess heat transfer, friction factor, and efficiency enhancement characteristics. The heat transfer experiments were carried out under constant boundary conditions maintained electrically. The study evaluated two heat transfer enhancement configurations: one involved a cone positioned within the testing conduit, whereas the other incorporated a twisted tape (TT) swirl generator positioned at the center of the cone loop. Different rotational ratios were applied to two twisted tapes for each test scenario. The results showed that tubes equipped with both cone loops and twisted tapes (TT) achieved Nusselt numbers 4–10% higher and efficiency gains of 4–8% compared to tubes with only cone loops. Efficiency increased with the use of cone loops and twisted tapes as the Reynolds number (Re) rose but started to decline consistently when Re exceeded 16,000. Additionally, the Nusselt number, friction factor(f), and comprehensive

performance assessment metrics all improved, demonstrating the significant benefits of using turbulators and swirl generators in tube systems.

In another experimental analysis, Rahimi and his team investigated the heat transfer behavior of a tube fitted with modified TT inserts. Their experimental analysis observed that the jagged twisted tape insert demonstrated a significantly higher Nusselt number and thermal-hydraulic performance compared to the other designs across the investigated Reynolds number range. Specifically, the jagged insert achieved up to a 31% elevation of the Nusselt number and a 22% enhancement in thermal performance relative to the classic twisted tape. CFD modeling was employed to explain these experimental results. The superior performance of the jagged twisted tape was attributed to the higher turbulence intensity generated near the tube wall. Notably, the enhanced performance of the jagged design at low Reynolds numbers and its ability to increase turbulence intensity near the wall were highlighted as key findings of this study[60].

In an experimental study by Chowdhuri M. A. K. *et al.* [61], the performance of rod-pin insert tubes was examined, and it was found that the rod-pin insert achieved optimal results when the pin distance was 150 mm, with a performance improvement of approximately 9.8 times compared to a plain tube.

Hayat and his team [62] performed a numerical study utilizing twisted tape inserts equipped with trapezoidal-shaped ribs, considering various rib inclination angles (from 30° to 60°) and Reynolds numbers ranging from 4000 to 12000. They discovered that the highest thermal performance factor occurred at a 30° angle, leading to superior heat transfer performance. In a recent study, Abdulhamed *et al.*[63] explored heat enhancement by inserting twisted tape (TT) inserts with large holes. Using air as the working fluid, they observed the highest Nusselt number and thermal performance factor with an increased number of holes.

However, no study has specifically investigated the performance of rectangular-cut twisted tape (TT) inserts in a U-loop-shaped tube, especially in comparison to the heat transfer rate of plain twisted tape (PTT) and a plain tube. Hence, this study seeks to investigate the heat transfer improvement achieved by inserting rectangular-cut TT inserts into a U-loop pipe, comparing it with PTT and a plain tube. The rectangular-

cut TT inserts enhance heat transfer efficiency by promoting fluid mixing near the pipe surface, potentially offering a cost-effective solution for industrial applications due to its simple manufacturing process.

In this research, water is chosen as the heat transfer fluid, and numerical simulations are carried out using the Finite Element Method (FEM) with COMSOL Multiphysics software. The simulations cover Reynolds numbers ranging from 5319.4 to 17,288.05, representing non-isothermal turbulent flow.

The study analyzes the thermal performance factor (TPF), Nusselt number (Nu), and friction factor for rectangular-cut TT inserts, PTT inserts, and plain tubes.

Chapter 3: Mathematical Modeling and Computational Domain

3.1 Introduction

In real-world problems, Computational Fluid Dynamics (CFD) serves as a modern approach to understanding complex phenomena related to fluid flow. The Navier-Stokes (N.S.) equations serve as the core mathematical framework for computational fluid dynamics (CFD), which explains fluid motion.

3.1.2 Flow Modeling

The initial step in solving a current complication involving the transport of momentum, thermal energy, and mass in fluid dynamics using physical systems is to formulate a collection of equations derived from fundamental conservation principles. These equations comprise:

- The conservation of mass (equation of continuity).

- The conservation law of momentum.

- The first thermodynamic law which governs the conservation of energy.

3.1.3 System and Control Volume

A system is outlined as a specific amount of substance or a designated area of space allocated for analysis. The area or matter beyond the system's boundaries is referred to as the surroundings.

Systems are classified as closed or open based on whether the focus is on a fixed amount of matter or a spatial region. A closed system contains a fixed amount of matter, and its boundary is impermeable to mass. However, energy in the form of thermal or mechanical work can pass through, and the volume of the system does not have to remain constant. If neither energy nor mass can pass through the margin, the system is considered isolated.

Example: Contemplate a Piston and cylinder mechanism where gas is heated. Here, the gas represents the system, and the internal faces of the piston and cylinder represent the margin. With no mass crossing the margin, the system is identified as closed, though energy can pass through, and parts of the margin, like the piston interface, may move. The surroundings include the external environment of the gas, such as the cylinder-piston unit.

A non-isolated system, also known as a control region, refers to a specific region in space where mass flux occurs, as in a nozzle, or pressurizer. In a control volume, both mass and energy can flow in and out. Many engineering applications, such as car radiators, water heaters, and compressors, include mass transfer and are best examined under open-system conditions rather than closed systems.

A control volume may be arbitrarily specified but choosing the right region simplifies the analysis. For instance, when studying air flow through a nozzle, marking the region inside the nozzle as the control region is ideal. Control volumes can have fixed or moving boundaries, though most have fixed boundaries. In addition to mass flow, they can also involve heat and work transfers, similar to closed systems [64].

3.2 Substantial, Local, and Convective Derivatives

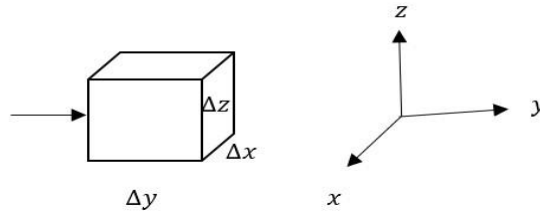


Figure 3.1: A differential fluid element in a cartesian co-ordinate system.

Consider a small volume of fluid that displaces from point $P(x,y,z)$ by time t to point $Q(x+\delta x, y+\delta y, z+\delta z)$ at time $t+\delta t$. Assume $f(x, y, z, t)$ represents a scalar function corresponding to a fluid property, such as pressure or density. The complete variation in f as the flow particle transitions from P to Q is denoted by δf . So, we have

$$\delta f = \left(\frac{\partial f}{\partial x}\right)\delta x + \left(\frac{\partial f}{\partial y}\right)\delta y + \left(\frac{\partial f}{\partial z}\right)\delta z + \left(\frac{\partial f}{\partial t}\right)\delta t$$

$$\text{Or, } \frac{\delta f}{\delta t} = \frac{\partial f}{\partial x} \frac{\delta x}{\delta t} + \frac{\partial f}{\partial y} \frac{\delta y}{\delta t} + \frac{\partial f}{\partial z} \frac{\delta z}{\delta t} + \frac{\partial f}{\partial t} \quad (3.1)$$

$$\text{Let, } \lim_{\delta t \rightarrow 0} \frac{\delta f}{\delta t} = \frac{Df}{Dt} \text{ or } \frac{df}{dt}, \lim_{\delta t \rightarrow 0} \frac{\delta x}{\delta t} = \frac{dx}{dt} = u, \lim_{\delta t \rightarrow 0} \frac{\delta y}{\delta t} = \frac{dy}{dt} = v, \lim_{\delta t \rightarrow 0} \frac{\delta z}{\delta t} = \frac{dz}{dt} = w \quad (3.2)$$

Fluid particle velocity is expressed as $\mathbf{q} = (u, v, w)$. From (3.2) and (3.1)

$$\frac{Df}{Dt} = u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} + w \frac{\partial f}{\partial z} + \frac{\partial f}{\partial t} \quad (3.3)$$

But we have,

$$\mathbf{q} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k} \quad (3.4)$$

$$\text{and } \nabla = \left(\frac{\partial}{\partial x}\right)\mathbf{i} + \left(\frac{\partial}{\partial y}\right)\mathbf{j} + \left(\frac{\partial}{\partial z}\right)\mathbf{k} \quad (3.5)$$

$$\text{Using (3.4) and (3.5) } \mathbf{q} \cdot \nabla = u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \quad (3.6)$$

$$\text{From (3.6) and (3.3) } \frac{Df}{Dt} = \frac{\partial f}{\partial t} + (\mathbf{q} \cdot \nabla)f \quad (3.7)$$

Let $\mathbf{g}(x, y, z, t)$ is a vector function,

$$\frac{D\mathbf{g}}{Dt} = \frac{\partial \mathbf{g}}{\partial t} + (\mathbf{q} \cdot \nabla)\mathbf{g} \quad (3.8)$$

Finally, we get

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla \quad (3.9)$$

The operator $\frac{D}{Dt}$ generally known as the material, or substantial derivative, represents the rate of variation observed as one moves with the fluid flow. The first component on the R.H.S of equation (3.9) is known as the local derivative and reflects time-dependent changes at a fixed point. The second term on the R.H.S, $\mathbf{q} \cdot \nabla$ is called the convective derivative and describes changes in a physical quantity caused by the motion of the fluid [65].

3.3 Relevance of Mass Conservation

According to the rule of mass conservation (equation of continuity), fluid mass cannot be produced or destroyed. The goal of the equation of continuity is to provide a mathematical expression for the principle of mass conservation. As a result, in a continuous flow, the equation of continuity states that the rise in the mass of the fluid within any surface enclosing a region of the fluid at any specific time, it must be equal to the excess of inflowing mass over outgoing mass.

3.3.1 Equation of Continuity

The control volume is depicted in Figure 3.1 In a fluid flow with three dimensions at a random, stationary location. In the Cartesian axis, the symbols u , v , and w stand for the velocity's directional components that are in parallel with the x , y , and z axes, correspondingly

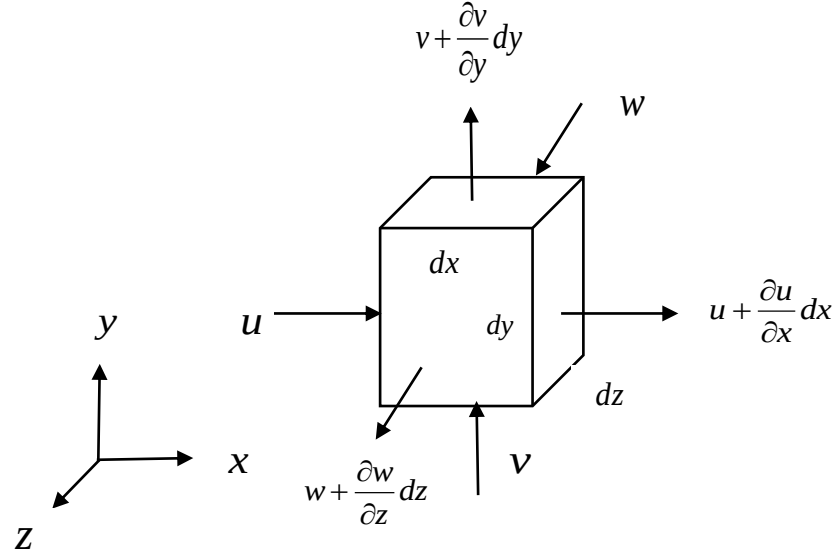


Figure 3.2: Control volume element in three dimensions.

The principle of mass conservation requires that the time rate of change of mass within a volume be equivalent to the net mass flux into it. The overall mass enclosed in the volume is ρdV .

As dV is fixed, we get

$$\frac{\partial \rho}{\partial t} dV = \sum (\text{mass flow in} - \text{mass flow out}). \quad (3.1)$$

Since density can change across time and place, the partial derivative is used here. The mass change rate in the control volume due to flow in the x -direction is

$$\dot{m}_x = \rho u dy dz - \left[\rho u + \frac{\partial(\rho u)}{\partial x} dx \right] dy dz \quad (3.2)$$

Similarly, the similar expressions that emerge from the flow in y and z directions includes

$$\dot{m}_y = \rho v dx dz - \left[\rho v + \frac{\partial(\rho v)}{\partial y} dy \right] dx dz \quad (3.3)$$

$$\dot{m}_z = \rho w dx dy - \left[\rho w + \frac{\partial(\rho w)}{\partial z} dz \right] dx dy \quad (3.4)$$

Thus, the rate at which mass changes becomes,

$$\frac{\partial \rho}{\partial t} dV = \dot{m}_x + \dot{m}_y + \dot{m}_z = - \left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} \right] dx dy dz \quad (3.5)$$

Since $dV = dx dy dz$, the expression in equation (3.5) can be rewritten

$$\text{as } \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} + \rho \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] = 0 \quad (3.6)$$

this equation represents the general continuity equation for 3D flow in xyz coordinates. There are numerous ways to express the continuity equation.

$$\frac{\partial \rho}{\partial t} = -(\nabla \cdot \rho \mathbf{V}) \quad (3.7)$$

$$\frac{D\rho}{Dt} = -\rho(\nabla \cdot \mathbf{V}) \quad (3.8)$$

where the derivative $\frac{D\rho}{Dt}$ is the rate of change of density following a fluid particle

[66]. In the case of an incompressible fluid, the equation reduces to

$$\nabla \cdot \mathbf{V} = 0 \quad (3.9)$$

3.4 Motion Equation

Under gravitational influence only, the momentum conservation law for a stationary volume element is formulated as

$$\frac{\partial \rho \mathbf{V}}{\partial t} = -(\nabla \cdot \rho \mathbf{V}) \mathbf{V} - \nabla P + \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{g} \quad (3.10)$$

The term $\frac{\partial \rho \mathbf{V}}{\partial t}$ represents the increment rate in momentum per unit volume, $(\nabla \cdot \rho \mathbf{V}) \mathbf{V}$ describes momentum enhancement due to the convection of each unit volume, ∇P corresponds to the force due to pressure acting on the per unit volume, $\nabla \cdot \boldsymbol{\tau}$ indicates the momentum transfer rate as a result of viscous forces acting on each unit volume, and $\rho \mathbf{g}$ indicates the gravitational attraction force acting on each unit volume of the element. By applying the equation of continuity, the following form can be obtained by reorganizing equation (3.10)

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla P + \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{g} \quad (3.11)$$

The second law of dynamics is expressed in the last equation, which can be written in the representation of

$$m \times a = \text{Total force.}$$

The equations of dynamics presented in two forms (3.10) and (3.11) are equivalent to the continuity equation in its respective forms (3.7) and (3.8). It is important to note that equations (3.10) and (3.11) account for gravity as the sole body force. Furthermore, equation (3.11) characterizes the movement of a Newtonian fluid with variable density and fixed viscosity.

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla P + \frac{1}{3} \mu \nabla (\nabla \cdot \mathbf{V}) + \mu \nabla^2 \mathbf{V} + \rho \mathbf{g} \quad (3.12)$$

If ρ and μ are constants, equation (3.12) may be simplified through applying the continuity equation ($\nabla \cdot \mathbf{V} = 0$) for a Newtonian fluid to yield

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla P + \mu \nabla^2 \mathbf{V} + \rho \mathbf{g} \quad (3.13)$$

This is the vector representation of the well-known Navier-Stokes equation.

3.5 Energy Equation

The following expression gives the energy balance equation for a fixed control volume in a fluid flow [21].

$$\frac{\partial}{\partial t} \rho \left(u + \frac{1}{2} V^2 \right) = -\nabla \cdot \rho \mathbf{V} \left(u + \frac{1}{2} V^2 \right) - \nabla \cdot \mathbf{q}'' + \rho (\mathbf{V} \cdot \mathbf{g}) - \nabla \cdot P \mathbf{V} + \nabla \cdot (\boldsymbol{\tau} \cdot \mathbf{V}) + q''' \quad (3.14)$$

Energy acquisition rate per volumetric unit $\frac{\partial}{\partial t} \rho \left(u + \frac{1}{2} V^2 \right)$, energy supplied per volumetric unit $\nabla \cdot \rho \mathbf{V} \left(u + \frac{1}{2} V^2 \right)$, energy supplied per unit of volume by conduction $\nabla \cdot \mathbf{q}''$, the energy transfer rate to the fluid per unit volume caused by gravity $\rho (\mathbf{V} \cdot \mathbf{g})$, The rate of work done on the fluid per unit volume by pressure forces $\nabla \cdot P \mathbf{V}$, the rate at which viscous forces exert work on a fluid in each volume element $\nabla \cdot (\boldsymbol{\tau} \cdot \mathbf{V})$, and the rate at which heat is produced per unit volume is expressed as q''' . In which u represents the system's internal energy, p is a pressure and q''' indicates heat generation. The work component on the right part of the equation, which signifies the rate at which internal and kinetic energy increase, incorporating the potential energy of the fluid. It is therefore missing from the equation's left side equation (3.14) can be rewritten using the continuity and motion equations to get

$$\rho \frac{Du}{Dt} = -\nabla \cdot \mathbf{q}'' - P(\nabla \cdot \mathbf{V}) + \nabla \mathbf{V} : \boldsymbol{\tau} + q''' \quad (3.15)$$

For a Newtonian fluid [21],

$$\nabla \mathbf{V} : \boldsymbol{\tau} = \mu \Phi \quad (3.16)$$

Here $\nabla \mathbf{V} : \boldsymbol{\tau}$ is a dissipation term, and Φ is a viscous dissipation function. Using enthalpy is often more practical than using internal energy. In terms of the definition of enthalpy $i = u + P/\rho$ and the conservation of mass equation, equation (3.8), and equation (3.15) may be arranged as

$$\rho \frac{Di}{Dt} = \nabla \cdot k \nabla T + \frac{DP}{Dt} + \mu \Phi + q''' \quad (3.17)$$

Expressing thermal energy through a fluid's temperature and heat capacity is more convenient for practical engineering purposes than using internal energy or enthalpy. In general, when it comes to pure materials,

$$\frac{Di}{Dt} = \left(\frac{\partial i}{\partial P} \right) T \frac{DP}{DT} + \left(\frac{\partial i}{\partial T} \right) P \frac{DT}{Dt} = \frac{1}{\rho} (1 - \beta T) \frac{DP}{Dt} + C_p \frac{DT}{Dt} \quad (3.18)$$

where $\beta = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_P$ changing this into equation (3.17) we get:

$$\rho C_p \frac{DT}{Dt} = \nabla \cdot k \nabla T + \beta T \frac{DP}{Dt} + \mu \Phi + q''' \quad (3.19)$$

In the case of an ideal gas, $\beta = \frac{1}{T}$ and

$$\rho C_p \frac{DT}{Dt} = \nabla \cdot k \nabla T + \frac{DP}{Dt} + \mu \Phi + q''' \quad (3.20)$$

It has been noted that C_p need not be constant. By emphasizing that for an ideal gas, $di = C_p dT$, equation (3.17) might have been utilized directly to obtain equation (3.20).

$$\frac{Di}{Dt} = C_p \frac{DT}{Dt} \quad (3.21)$$

Equation (3.15) ($du = C dt$) is applied to an incompressible fluid with a given specific heat $C = C_p = C_v$. This enables us to get

$$\rho C \frac{DT}{Dt} = \nabla \cdot k \nabla T + \mu \Phi + q''' \quad (3.22)$$

Equations (3.15), (3.17), and (3.19) can be conveniently expressed in terms of momentum and energy. For simplicity, assuming $q''' = 0$, the energy equation is given by equation (3.22). When working with solid materials, the density is typically considered constant, $\mathbf{V} = 0$, can be assumed, and equation (3.22) simplifies to:

$$\rho C \frac{DT}{Dt} = \nabla \cdot k \nabla T + q''' \quad (3.23)$$

It serves as the starting point for most heat conduction problems [21].

3.6 Mathematical Model for Simulation

The following equations are required to describe the heat transfer phenomena in our simulation.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (3.24)$$

$$\rho(\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot \left[-p\mathbf{I} + (\mu + \mu_T)(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3}(\mu + \mu_T)(\nabla \cdot \mathbf{u})\mathbf{I} - \frac{2}{3}\rho K\mathbf{I} \right] + \mathbf{F} \quad (3.25)$$

The term ρ represents density, $\mathbf{u}$ is velocity, $(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$ is deformation rate tensor, p indicates pressure, $\mathbf{I}$ indicates an identity matrix, and $\mathbf{F}$ is a body force. Moreover, dynamic viscosity and eddy viscosity are denoted by μ and μ_T respectively.

Since the domain is a tubular U-loop pipe, to achieve a more precise prediction of the flow behavior in the curved region, the k - ω turbulent model is used. The parameters turbulent kinetic energy (k) and specific dissipation rate (ω) are defined by the following equations:

$$\rho(\mathbf{u} \cdot \nabla) k = \nabla \cdot [(\mu + \mu_T \sigma_k^*) \nabla k] + p_k - \rho \beta^* k \omega \quad (3.26)$$

$$\rho(\mathbf{u} \cdot \nabla) \omega = \nabla \cdot [(\mu + \mu_T \sigma_\omega) \nabla \omega] + \alpha \frac{\omega}{k} p_k - \rho \beta \omega^2 \quad (3.27)$$

$$\mu_T = \rho \frac{k}{\omega} \quad (3.28)$$

$$p_k = \mu_T \left[\nabla u : (\nabla u + (\nabla u)^T) - \frac{2}{3}(\nabla \cdot \mathbf{u})^2 - \frac{2}{3}\rho K \nabla \cdot \mathbf{u} \right] \quad (3.29)$$

Equation (3.30) indicates the rate of heat transfer along the flow.

$$\rho C_p \left[\frac{\partial T}{\partial t} + \rho C_p (\mathbf{u} \cdot \nabla) T \right] = \nabla \cdot (k \nabla T) + Q \quad (3.30)$$

In (3.30) Q represents a heat source, while k denotes a thermal conductivity.

$$\rho C_p \frac{\partial T}{\partial t} + \nabla \cdot (-k \nabla T) = Q \quad (3.31)$$

$$h = \frac{q}{T_w - T_b} . \quad (3.32)$$

$$q = \frac{Q}{A} . \quad (3.33)$$

Here, q represents the constant heat flux, A is the tube's surface area, T_{wall} is the wall temperature, and T_b refers to the bulk temperature of the fluid flow.

$$T_b = \frac{T_{\text{out}} + T_{\text{in}}}{2} . \quad (3.34)$$

Here, T_{in} and T_{out} represent the average temperatures at the inlet and outlet, respectively. The Nusselt number (Nu) is calculated using the following equation:

$$Nu = \frac{hD}{k} \quad (3.35)$$

The Darcy friction factor [67] could be calculated as where the pressure drop and head loss are denoted by Δp and h, respectively

$$\Delta p = h \rho g \quad (3.36)$$

$$h = \frac{f l u^2}{2 g D} \quad (3.37)$$

Finally, using equations (3.36) and (3.37), we arrive at equation (3.38).

$$\Delta p = \frac{l}{D} \frac{f u^2 \rho}{2} \quad (3.38)$$

The thermal performance factor is determined using the equation

$$\eta = \frac{\frac{Nu}{Nu_0}}{\left(\frac{f}{f_0}\right)^{\frac{1}{3}}} \quad (3.39)$$

The Nusselt number and friction factor for a plain tube is represented as Nu_0 and f_0 respectively [68].

3.7 Boundary Conditions

The following boundary conditions are established at different velocities in this flow model. $T = T_{in} = 293.15$ K is the initial fluid temperature, $\mathbf{n}$ is a unit normal vector, and the boundary conditions are taken as

$$\mathbf{u} \cdot \mathbf{n} = 0 \quad (3.40)$$

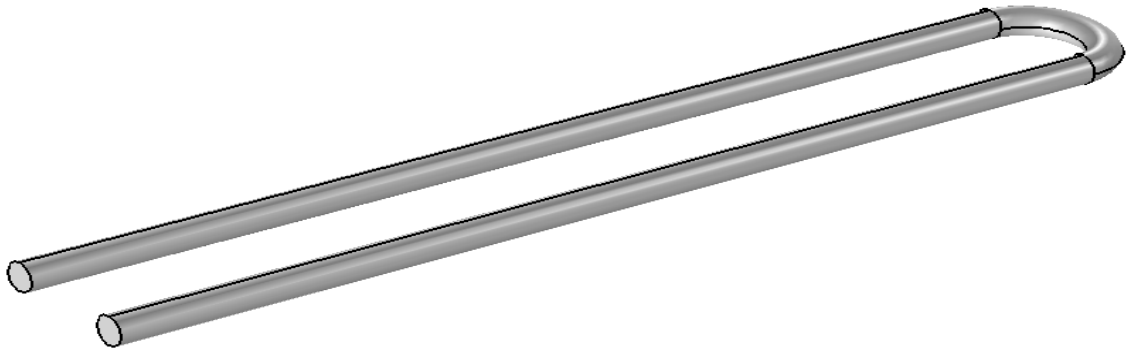
$$\left[(\mu + \mu_T)(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3}(\mu + \mu_T)(\nabla \cdot \mathbf{u})\mathbf{I} - \frac{2}{3}\rho K\mathbf{I} \right] \mathbf{n} = -\rho \frac{\mu_T}{\delta_w} \mathbf{u}_{tang} \quad (3.41)$$

$$\mathbf{u}_{tang} = \mathbf{u} - (\mathbf{u} \cdot \mathbf{n})\mathbf{n} \quad (3.42)$$

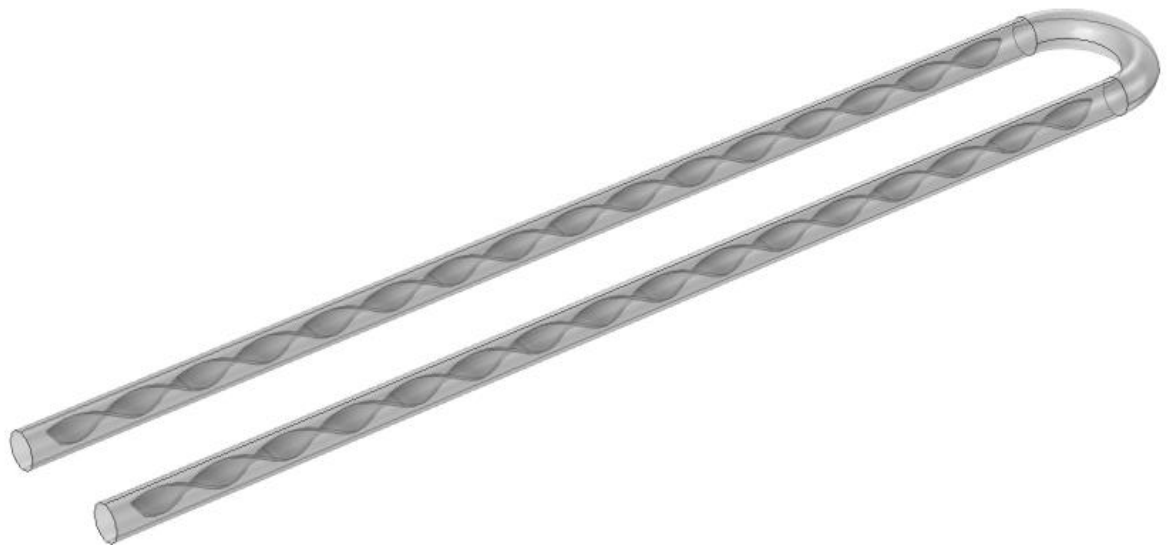
$$\nabla k \cdot \mathbf{n} = 0, \omega = -\rho \frac{c_\mu K^2}{K_V \delta_w \mu} \quad (3.43)$$

3.8 Computational Domain

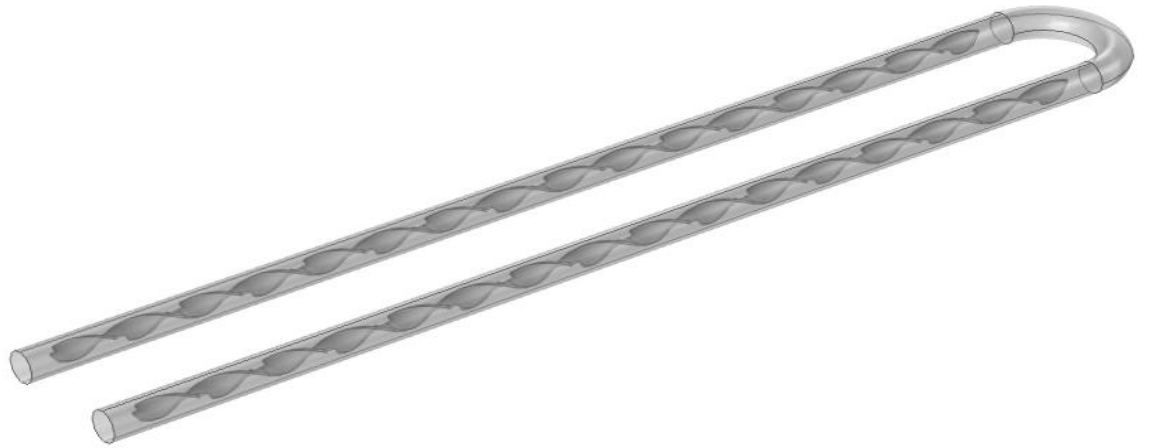
In this investigation, a U-shaped pipe featuring twisted tape inserts with the rectangular cut and smooth twisted tape inserts with a length of 1935.62 mm and an inner diameter of 26.597 mm was used.



(a)



(b)



(c)



(d)

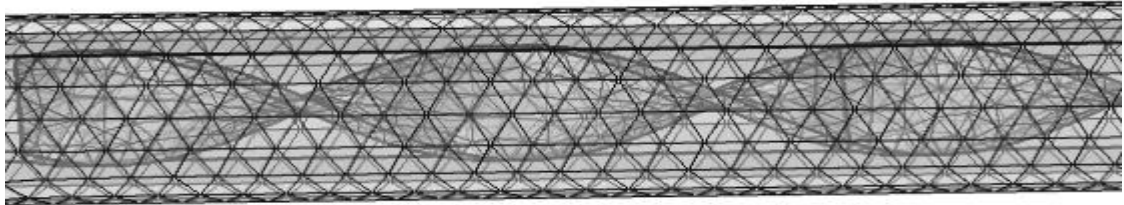


(e)

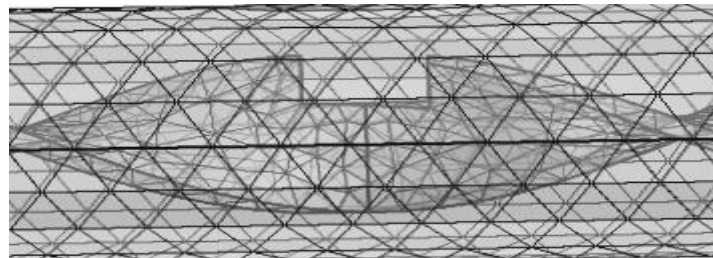
Figure 3.3: (a)Smooth pipe, (b) pipe installed with smooth twisted tape inserts, (c) pipe installed with twisted tape inserts featuring rectangular- cuts, (d) smooth twisted tape, (e) rectangular-cut twisted tape.

3.9 Mesh Design

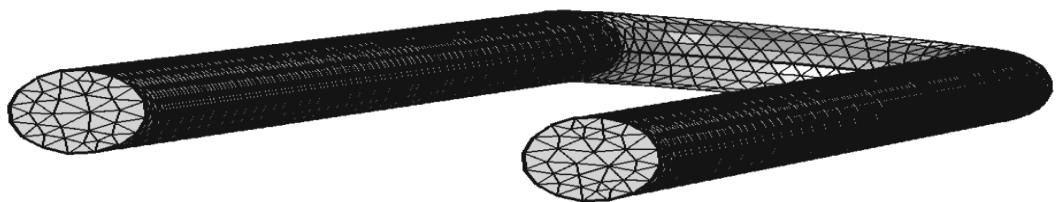
The mesh design is crucial for the correctness of computational findings. We adjusted the mesh design to improve simulation results. As a result, this computational model demonstrates that a finer mesh produces the greatest results.



(a)



(b)



(c)

Figure 3.4: Finer mesh configuration of (a) smooth twisted tape, (b) rectangular -cut, twisted tape, (c) U-shaped pipe.

Table 3.1: Comparison of three distinct inserts with finer mesh.

Property Name	Plain tube	Plain twist	Rectangular cut
Tetrahedral elements	50273	87870	119455
Triangular elements	12842	23600	26550
Edge elements	764	2952	3472
Vertex elements	12	84	332
Minimum element quality	0.2337	0.07282	0.02637
Average element quality	0.7445	0.7048	0.7038
Element volume ratio	0.05427	0.01276	0.002795
Mesh volume	0.001011	9.604E-4	9.635E-4

Chapter 4: Results and Discussion

4.1 Introduction

This analysis delineates a comprehensive numerical analysis of heat transfer enhancement achieved through the utilization of rectangular cut twisted tape inserts. Furthermore, detailed simulations were conducted for plain twisted tape inserts as well as a plain tube. The findings from these simulations were compared to evaluate the performance differences between the rectangular cut twisted tapes, the plain twisted tapes, and the plain tube. In the present investigation, water is selected as the working fluid for heat transfer analysis, and the FEM (Finite Element Method) is utilized to perform detailed numerical simulations. The computational region is carefully designed, and all test runs are conducted using the COMSOL Multiphysics software. The Re (Reynolds number) is systematically varied within the range of 5319.4 to 17,288.05, characterizing the turbulent flow. A thorough numerical investigation is carried out to evaluate the influence of key parameters, including the thermal performance factor (TPF), Nusselt number (Nu), and friction factor, for three configurations: rectangular-cut twisted tapes (TT), plain twisted tapes (PTT), and a plain tube.

4.2 Numerical Results

We performed our simulation by altering the mesh design under some boundary conditions to achieve better results. Accordingly, Finer mesh confirms the best results including key heat transfer parameters for the used three setups. The results of this numerical evaluation were contrasted with the numerical findings of plain twisted tape [44] and experimental findings of rectangular cut twisted tape inserts [12]. In their experimental study, their data were analyzed within a Reynolds (Re) number range of 10,000 to 19,070 to achieve an enhanced heat transfer rate. They determined that the maximum Nusselt (Nu) number of 309 occurred at $Re = 19,070$ for a pipe equipped with rectangular cut twisted tape inserts. On the contrary, in our numerical study, the Reynolds number range was considered to span from 5,319.4 to 17,288.05, with the highest observed Nusselt number of 357.59 occurring at $Re = 17,288.05$ for the rectangular cut twisted tape inserts. Furthermore, the impact of the friction factor was compared to the experimental results of rectangular cut twisted tape inserts. who

noted a reduction in the friction factor with the rise in Reynolds number, across all twisted tape insert configurations. Both Salam *et al.*'s [12] experimental data and the present numerical study indicate that the rectangular cut twisted tape inserts result in an improved friction factor. Additionally, the Nusselt number values from the numerical analysis of plain twisted tape inserts are in good agreement with our simulated results, which demonstrate the Nu number range of 96 to 317 for plain twisted tape and 106 to 357 for rectangular cut twisted tape inserts. These comparative findings from both studies suggest that the rectangular cut twisted tape inserts significantly strengthen heat transfer performance facilitate improved fluid mixing, and generate fluid disturbances near the pipe's inner wall.

4.3 Temperature Distribution

The temperature distribution within a U-shaped pipe fitted with plain twisted tape, rectangular-cut twisted tape inserts, and a smooth tube is described and exhibited below. Each setup demonstrates various temperature variations within the pipe for turbulent flow.

Table 4.1: Temperature data at various Reynolds numbers for different inserts.

Reynolds number	Plain tube	Plain twist	Rectangular cut twisted tape inserts
5319.4	335.3518	354.7078	358.0877
6649.25	332.3802	352.5528	355.8829
7979.1	330.3946	351.289	354.5452
9308.95	328.9264	350.1803	353.6292
10638.8	327.757	349.4941	352.9339
11968.65	326.7863	348.9301	352.3946
13298.5	325.9586	348.699	351.934
14628.35	325.2386	348.1201	351.3141
15958.2	324.6029	347.9586	350.9649
17288.05	324.0356	347.6432	350.6525

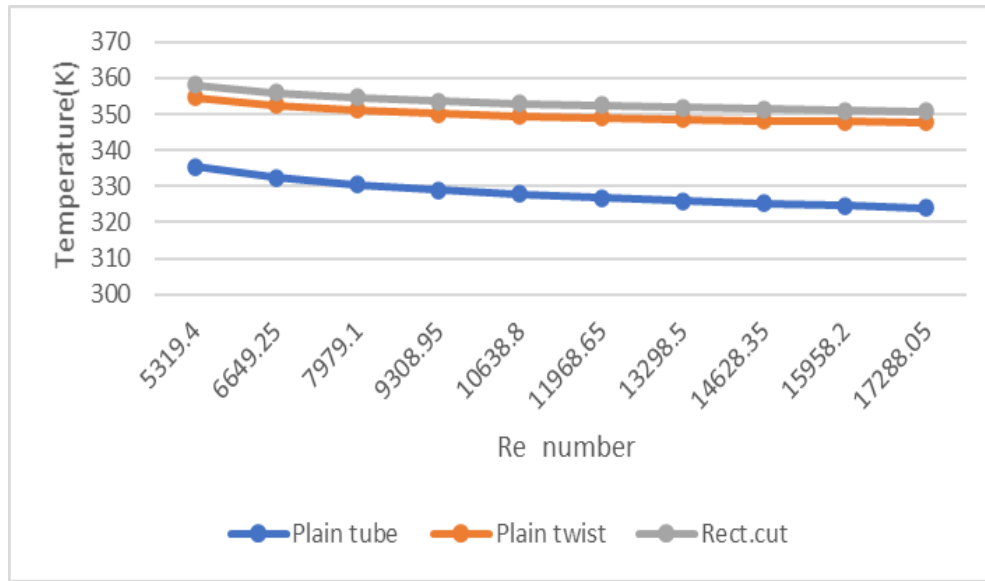


Figure 4.1: Variation of temperature for various Reynolds numbers.

Figure 4.1 illustrates the relationship between temperature fluctuation and the Reynolds number for a tube fitted with rectangular-cut twisted tape (TT) inserts, a plain twisted tape, and a smooth tube. It is observed that the temperature decreases as the Reynolds number increases, with the maximum temperature achieved for the configuration utilizing the rectangular-cut twisted tape inserts in comparison with the other setups.

The Fig. (4.2), describes the cross-sectional representation of a U-shaped domain characterized by a temperature distribution. The color gradient, changing from blue to red, indicates regions of gradually increasing thermal intensity. Besides, Fig (4.3) exhibits some temperature slices of the whole domain for the three setups.

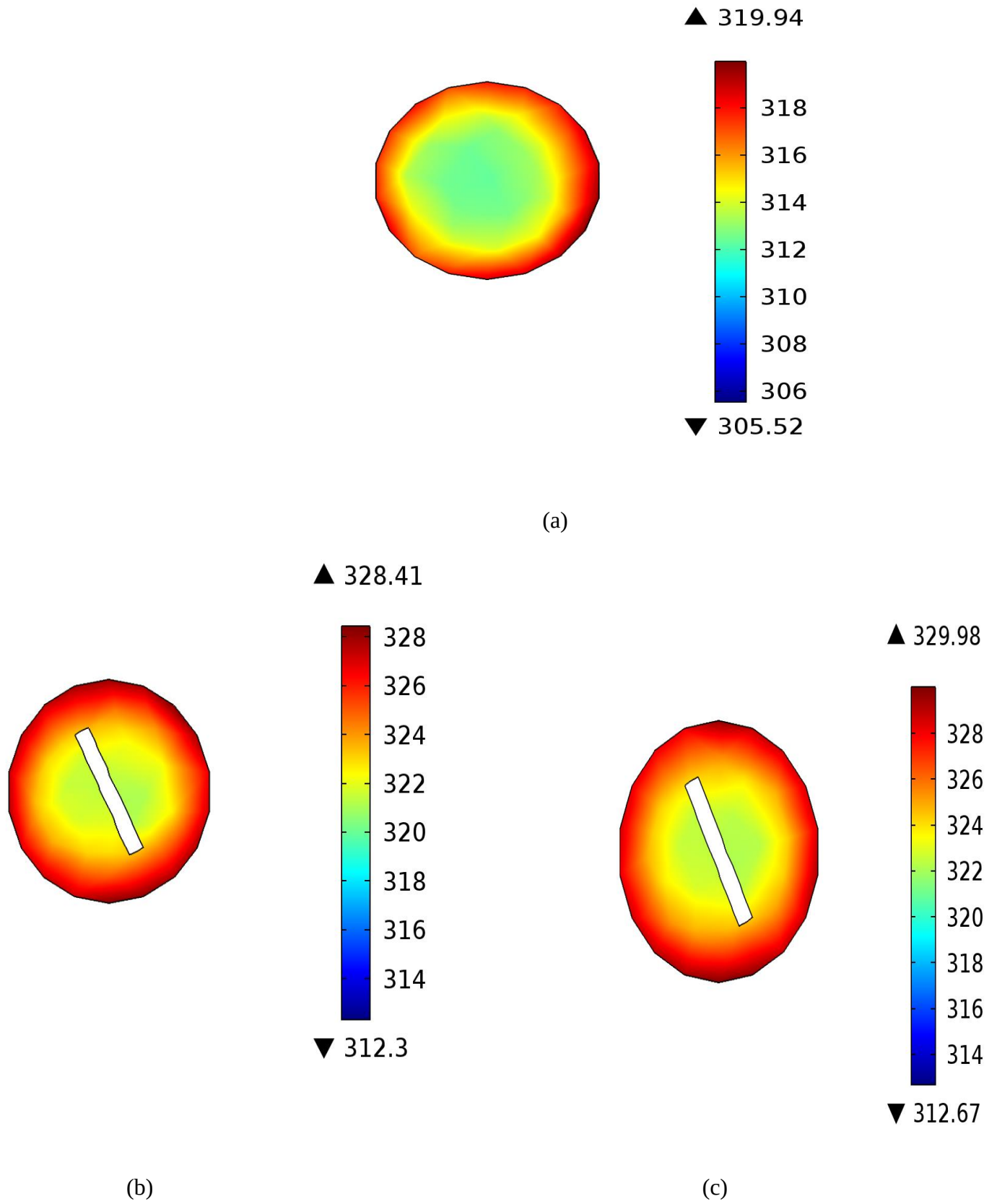


Figure 4.2: Cross-sectional view of (a) plain tube, (b) plain twisted tape, (c) rectangular cut twisted tape inserts.

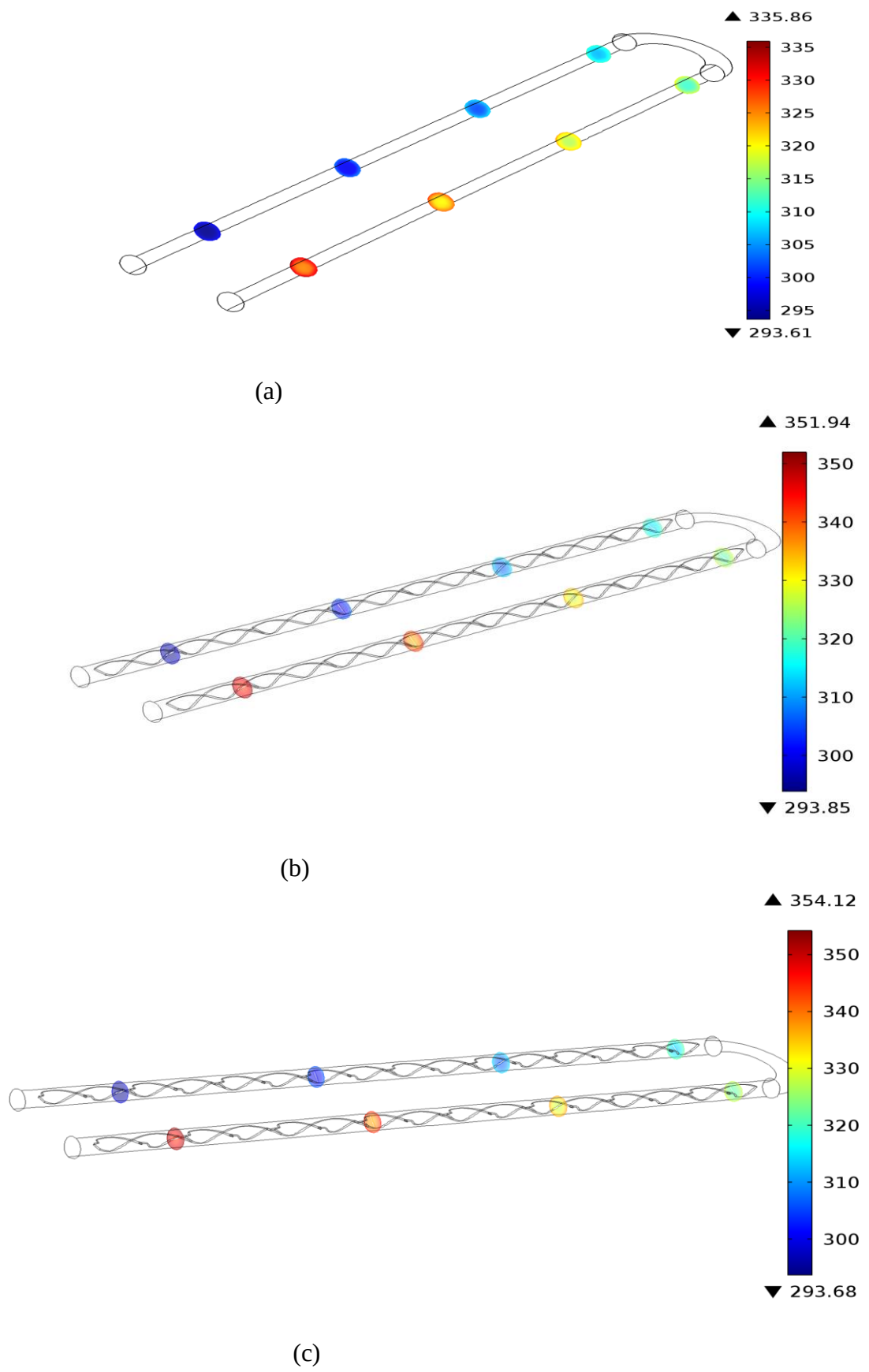
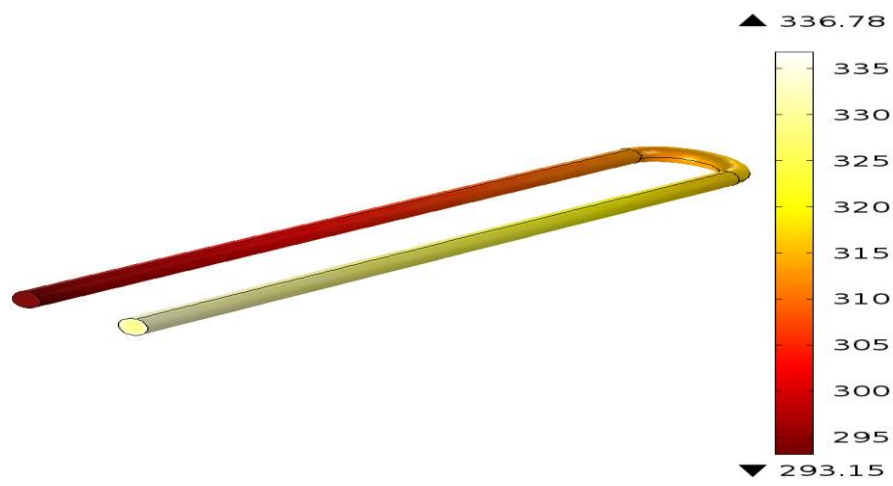


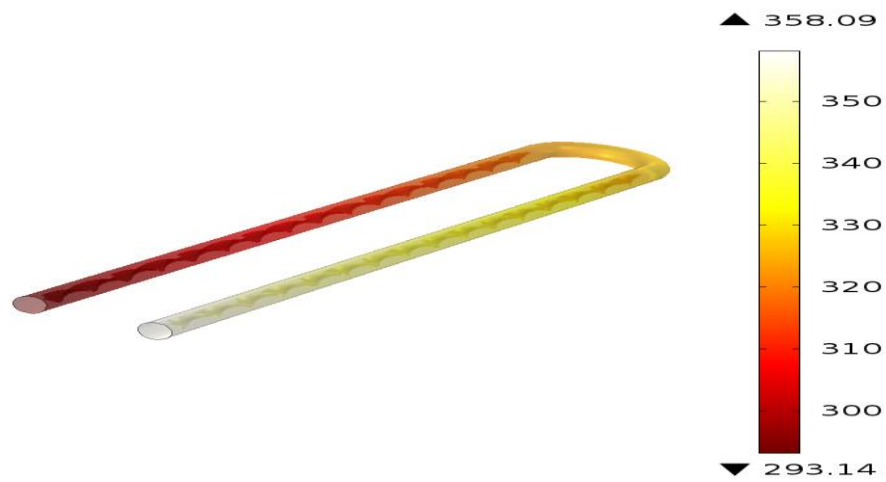
Figure 4.3: Temperature variation in a U-loop pipe for the three setups (a),(b),(c).



(a)



(b)



(c)

Figure 4.4: Temperature for (a) plain tube,(b) plain twisted tape,(c) rect-cut twisted tape inserts.

4.4 Nusselt Number

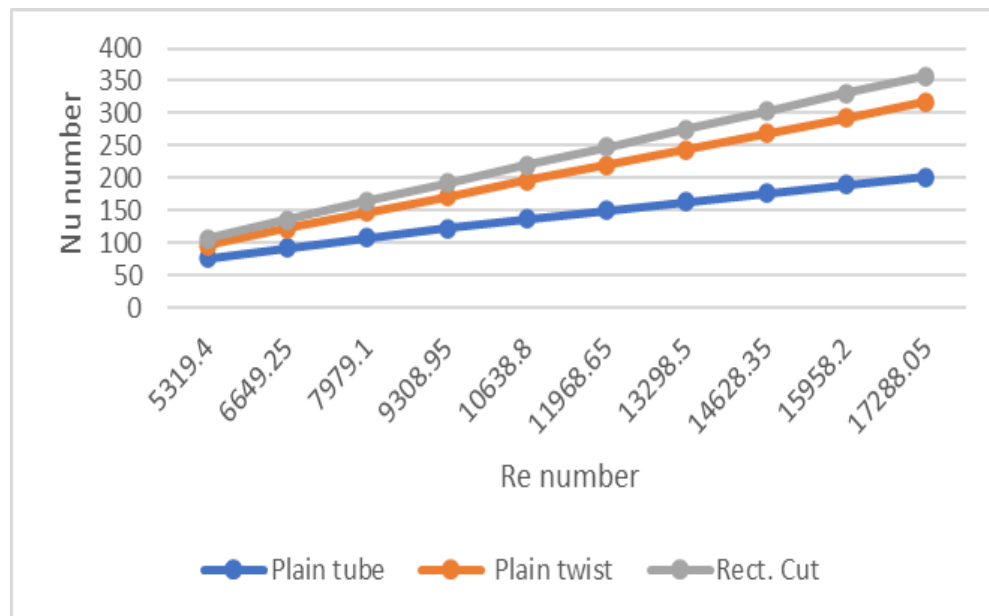


Figure 4.5: Nusselt number vs Reynolds number.

Figure 4.5, outlines the change of the Nusselt (Nu) number concerning the Reynolds (Re) number for three configurations: rectangular-cut twisted tape inserts, plain twisted tape inserts, and a plain tube. The outcomes indicate that the Nu number increases as the Reynolds number rises. Among the three setups, the tubular pipe installed with rectangular-cut twisted tape inserts demonstrates superior performance, achieving the highest Nu numbers compared to the other setups.

The Nusselt number is a dimensionless parameter that measures the ratio of convective to conductive heat transfer. Its increase with Reynolds number is attributed to the increased convective heat transfer coefficient observed in turbulent flow regions.

4.5 Nusselt Number Enhancement Index

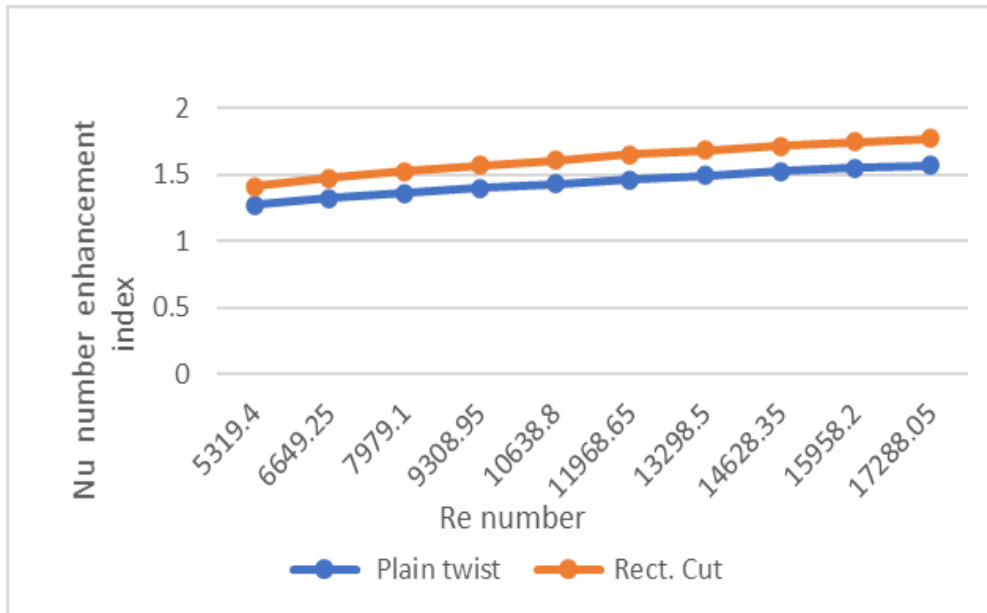


Figure 4.6: Nusselt number enhancement index vs Reynolds number.

Figure 4.6 presents the Nusselt (Nu) number enhancement index variation with the Reynolds (Re) number for three configurations: rectangular-cut twisted tape, plain twisted tape, and plain tube. The Nu number enhancement index, a dimensionless parameter, is defined as the quotient of the Nusselt number for the increased configuration to that of the smooth configuration. The findings indicate that the enhancement ratio for rectangular-cut twisted tape inserts is significantly higher than those of the plain tube and plain twisted tape inserts. Furthermore, the Nusselt number enhancement ratio progressively enhances with rising Reynolds numbers, emphasizing the positive effect of convective heat transfer within the turbulent flow regime of the tube fitted with rectangular-cut twisted tape inserts.

4.6 Friction Factor

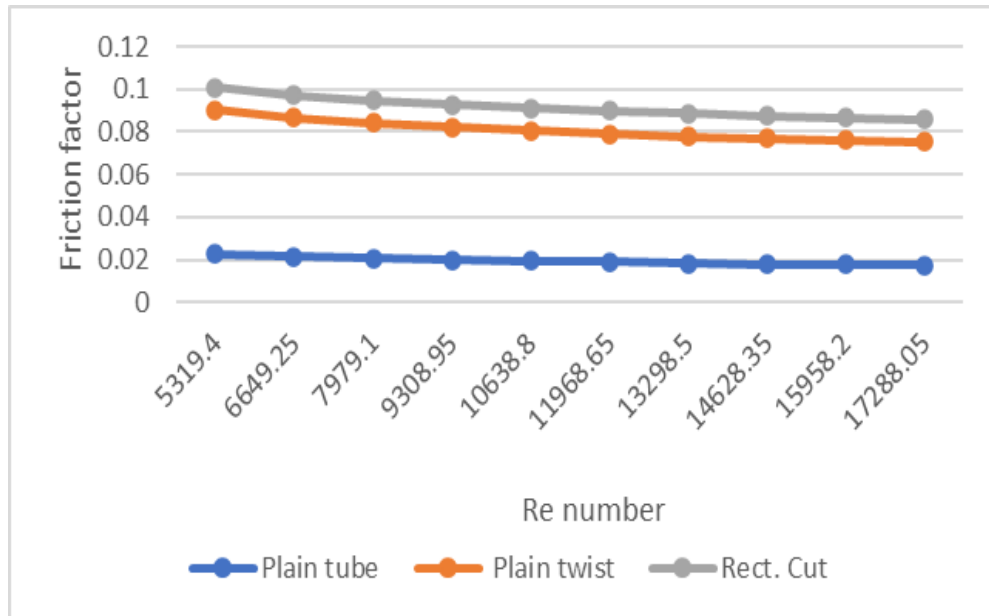


Figure 4.7: Friction factor vs Reynolds number.

Figure 4.7 demonstrates the reduction in the friction factor (f) as the Reynolds (Re) number increases for twisted tape with and without rectangular cut inserts, as well as for the plain tube. A friction factor of 0.1 is observed at $Re = 5319.4$ for the rectangular cut twisted tape inserts, which show more effective results compared to the plain twisted tape (PTT) and the plain tube. The graph indicates that the numerical value of the friction factor (f) decreases as the Reynolds number increases, due to the increased fluid disturbance near the pipe's inner wall, which reduces fluid resistance.

4.7 Friction Factor Enhancement Index

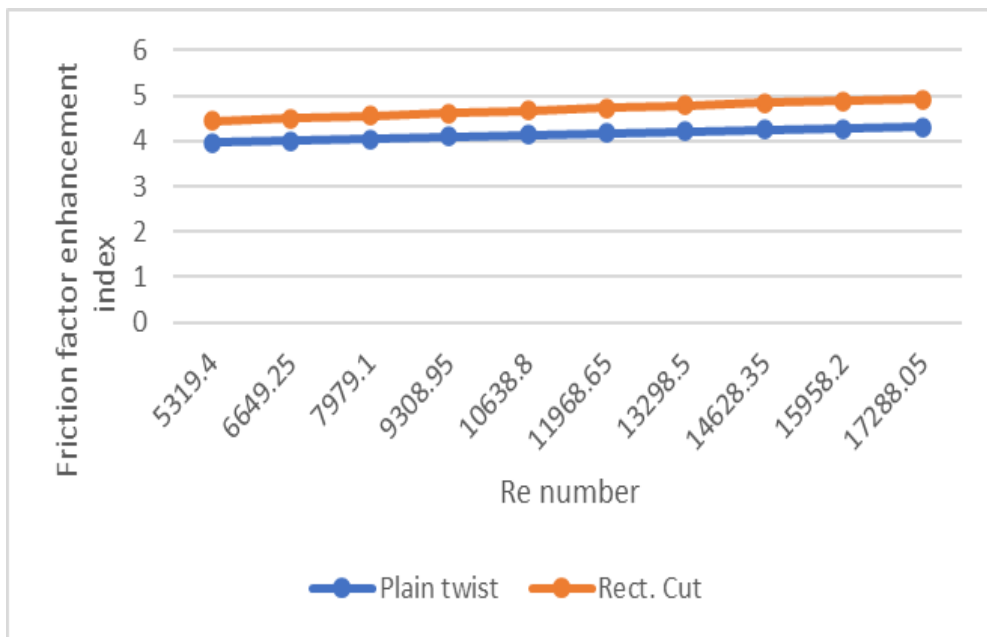


Figure 4.8: Friction factor enhancement index vs Reynolds number.

Figure 4.8 shows the variation of the friction factor enhancement index. It is observed that the friction factor enhancement index for rectangular cut twisted tape inserts has a more significant impact compared to the other two inserts (plain twisted tape and plain tube). A higher friction factor enhancement index indicates reduced pressure loss in the flow area, which is essential for improving heat transfer efficiency.

4.8 Thermal Performance Factor (TPF)

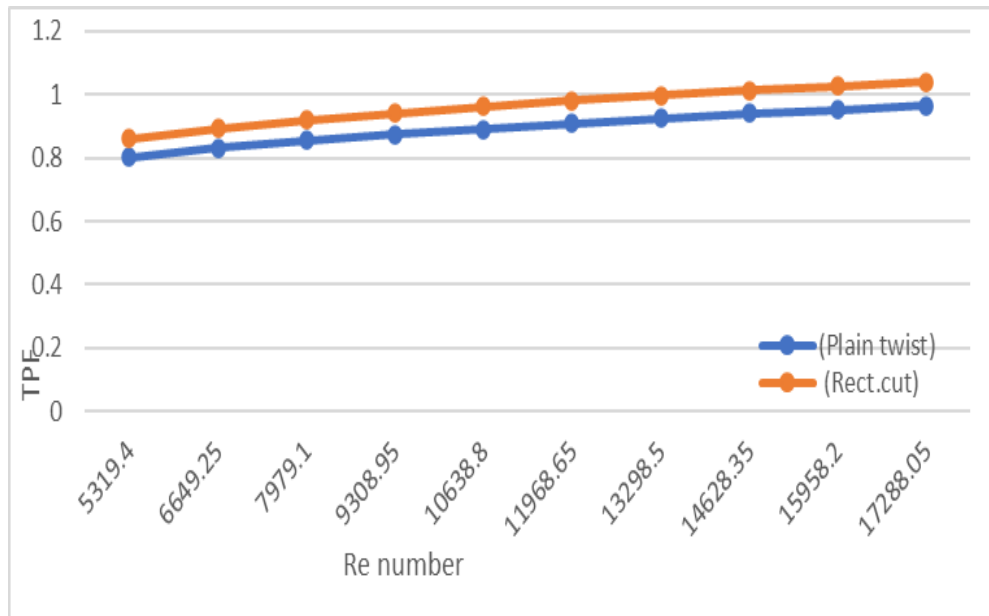


Figure 4.9: Thermal performance factor vs Reynolds number.

Figure 4.8 presents a comparison of thermal performance at various Reynolds numbers for different inserts (rectangular cut twisted tape, plain twisted tape, and plain tube). The results clearly indicate that the thermal performance factor (η) is considerably higher for the rectangular cut twisted tape inserts compared to both the plain twisted tape and the plain tube. Furthermore, the highest efficiency ($\eta = 1.04$) is observed at Reynolds number $Re = 17,288.05$ for the tube with rectangular cut twisted tape inserts. This suggests that the swirling flow induced by the rectangular cut twisted tape inserts significantly enhances heat transfer, leading to a notable improvement in the overall efficiency of the system.

Chapter 5: Conclusion

In the present world, heat transfer augmentation processes incorporating various turbulators are crucial for industrial applications. These turbulators are widely used in heat exchangers, including twisted tape inserts with various shapes and cuts, which are well-recognized by engineers. By creating swirl flows in the flow region, these inserts enhance heat transfer and increase the effectiveness of heat exchangers. Moreover, swirl flows generated by turbulators play a significant role in optimizing the manufacturing cost and overall efficiency of heat exchangers.

This study investigates the phenomena of heat transfer enhancement within a U-bend pipe equipped with rectangular-cut twisted tape inserts, using numerical simulation techniques. Additionally, simulations were performed on two other configurations for comparison: one featuring plain twisted tape inserts and another consisting of a smooth pipe without inserts. The simulations were conducted using a U-shaped pipe with a total length of 1935.62 mm and an inner diameter of 26.6 mm.

The outcomes obtained from the numerical simulations demonstrate that the U-shaped pipe employing rectangular-cut twisted tape inserts exhibited a notable improvement in heat transfer performance compared to the configurations with plain twisted tape inserts and the smooth pipe. This enhancement is due to the disruption of the boundary layer in the flow region caused by the rectangular-cut twisted tape inserts.

In our numerical simulation, it has been noticed that heat transfer parameters, particularly the Nusselt number, are significantly higher for the rectangular cut twisted tape inserts when compared to the other two configurations.

Specifically, for the rectangular cut twisted tape inserts, the Nusselt number reaches a value of 357.59 at the highest Reynolds number. In contrast, the plain twisted tape inserts and the plain tube exhibit Nusselt numbers of 317.23 and 202.06, respectively, at a Reynolds number of 17,288.05, within the turbulent flow range.

Furthermore, the enhancement index for the Nusselt number is more prominent for the rectangular cut twisted tape inserts, indicating a superior outcome compared to both the plain twisted tape inserts and the plain tube.

Moreover, we observed the impact of the friction factor in the turbulent flow regime, where the rectangular cut twisted tape inserts demonstrated a more favorable friction factor at lower Reynolds numbers ($Re = 5319.4$) compared to the other configurations. As the Reynolds number increased, the friction factor was found to decrease.

Additionally, the enhancement index for the friction factor was notably higher for the rectangular cut twisted tape inserts, indicating an improved performance in terms of frictional resistance compared to the other configurations.

We also observed that the thermal performance factor in the turbulent flow region was higher for the rectangular cut twisted tape inserts, with a value of 1.04, compared to both the plain twisted tape inserts and the plain tube. From our results, it is noted that the value of the thermal performance factor is more than one.

This indicates that the rectangular cut twisted tape inserts provide superior thermal performance in the turbulent flow zone.

Suggestions For Further Study

This investigation employed numerical methods to inspect the enhancement of heat transfer using rectangular-cut twisted tape inserts within a U-shaped pipe, comparing their performance against plain twisted tape inserts and a smooth tube. Water served as the working fluid for the simulations, with the analysis concentrated on the turbulent flow regime.

To advance the scope of this research, subsequent studies could:

1. Apply the application of alternative turbulence modeling methods,
2. Consider the impact of varying pipe geometries, and
3. Attempt experimental study to validate the experimental results with numerical outcomes, yielding better results.

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